

Timings for Numerical Experiments on Benchmark Examples for Fixed Order \mathcal{H}^∞ Controller Design

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1 Introduction

We applied HIFOO to various benchmark examples and published our results in the paper “Fixed-Order \mathcal{H}^∞ Controller Design via HIFOO, a Specialized Nonsmooth Optimization Package” at the American Control Conference 2008. Due to limited space in the paper, we would like to give further information on our benchmarks in this note, particularly the time required for obtaining the results.

2 Benchmark Examples

The rest of the note presents the data from numerical experiments. For each benchmark example and prescribed controller order, we made 10 runs with HIFOO. These runs generate different answers because of the different random initialization used for each run. The notation used in the tables has the following meaning:

n_K : The order of the controller. This is a design parameter which is selected by the user,

γ_{min} : The minimum closed-loop \mathcal{H}^∞ norm obtained by HIFOO stabilizing the closed-loop system, taking the best result of all 10 runs,

$t_{\gamma_{min}}$: The time (seconds) required for this particular run of HIFOO to find this minimizing controller ,

t_{median} : The median time (seconds) required by the 10 runs,

t_{min} : The minimum time (seconds) required by the 10 runs,

t_{max} : The maximum time (seconds) required by the 10 runs.

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All runs are limited to 5 minutes by setting the HIFOO parameter, `options.cpumax`, to 300.

2.1 AC8 Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	2.0050	2.1397	2.0790	1.3506	5.1695
1	1.6506	17.2758	16.7005	8.3927	23.0091
2	1.6265	19.2324	19.3389	15.4062	31.6364
3	1.6193	27.5180	34.2479	25.2236	59.9712
4	1.6181	39.3433	43.7620	12.5696	57.3440

2.2 HE1 Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	0.1539	0.2129	0.2054	0.1864	0.2685
1	0.1094	2.3244	1.6294	0.8506	6.6601
2	0.0756	10.5370	16.6939	7.3850	25.1700
3	0.0748	7.5326	18.6933	6.9720	20.7946
4	0.0747	6.3728	25.6477	6.3728	41.1314

2.3 VTOL Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	0.1540	0.1956	0.1993	0.1548	0.2684
1	0.1095	2.7171	2.1708	0.7638	8.3349
2	0.0765	6.3967	6.4105	2.7822	21.3587
3	0.0744	5.8577	7.6330	5.8577	14.6840
4	0.0754	11.6537	9.2270	6.1511	16.3581

2.4 REA2 Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	1.1491	0.3790	0.2995	0.1833	0.6370
1	1.1341	0.2675	0.2518	0.1940	0.5342
2	1.1341	0.4756	0.4883	0.2515	0.9387
3	1.1341	0.3694	0.5351	0.2784	2.9685
4	1.1341	0.5739	0.5552	0.2917	1.7293

2.5 CR Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	1.1701	0.2298	0.0752	0.0672	0.2298
1	1.1349	0.7195	0.1695	0.0998	0.7195
2	1.1349	0.2241	0.2679	0.1548	0.3989
3	1.1349	0.5858	0.3913	0.1961	1.3692
4	1.1349	0.3441	0.3708	0.2272	0.4859

2.6 AC10 Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	12.8345	71.0081	48.6483	18.5161	94.1052
1	12.2717	292.5691	137.8344	48.4410	292.5691
2	12.2717	199.1149	135.4956	66.3201	199.1149
3	12.2717	255.6541	166.1411	59.5749	290.7286
4	12.2717	229.4382	113.9840	82.0513	242.1450

2.7 BDT2 Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	0.6647	301.2754	301.0320	300.8271	302.5348
1	0.6293	301.5381	301.1877	300.4650	302.5534
2	0.6271	300.6057	300.8962	300.3513	303.9708
3	0.5587	292.8205	293.5649	288.0708	302.7044
4	0.5372	296.3040	295.4697	292.7954	303.9197

2.8 HF1 Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	0.4472	18.4462	20.8732	8.6184	32.6441
1	0.4472	15.8372	16.8512	10.0736	30.2944
2	0.4472	11.4081	13.7675	9.8403	40.2104
3	0.4472	36.2971	15.4096	10.7159	79.0604
4	0.4472	21.2617	18.4195	10.9682	25.4676

2.9 CM4 Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	0.8165	287.3214	294.1095	287.3214	309.6527
1	0.8165	457.9293	332.4497	296.8757	469.3365
2	0.8165	410.7657	306.7944	295.0812	410.7657
3	0.8165	300.7284	302.7653	299.7085	322.4197
4	0.8165	317.3836	302.4777	294.1026	325.6753

2.10 PA Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	4.3858 10^{-5}	9.7379	0.7306	0.3304	9.7379
1	3.6063 10^{-7}	0.8406	0.7738	0.6054	45.6288
2	1.1289 10^{-6}	1.0073	1.0659	0.6704	15.4695
3	5.5570 10^{-6}	1.5107	10.9355	1.5107	22.0346
4	8.0330 10^{-5}	8.3938	22.8104	8.3938	24.6496

2.11 HIMAT Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	5.6534	6.6543	2.7047	2.0403	6.6543
1	2.7818	18.0078	11.2623	7.0069	18.0078
2	2.2196	54.7353	63.9310	34.4310	125.2049
3	1.1668	62.1430	79.2856	60.4603	147.3301
4	1.1160	94.8554	144.9755	77.7079	179.6705

2.12 VSC Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	8.7042	0.0740	0.0740	0.0735	0.0843
1	3.4130	1.7615	1.3683	0.6103	13.1106
2	3.3288	23.1516	3.6145	1.8585	23.1516
3	3.2461	34.2347	9.1545	3.9442	34.2347
4	3.3408	40.8726	9.4010	6.5856	40.8726

2.13 AUV Example

2.13.1 AUV Example - Speed

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	2.5006	0.0799	0.0804	0.0766	0.1724
1	0.9541	10.0354	1.4057	1.1174	16.1110
2	0.9542	11.1521	10.7869	1.4308	22.7866
3	0.9541	3.5548	4.2924	1.9296	8.1485

2.13.2 AUV Example - Heading

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	0.9548	2.0983	1.9730	1.6082	2.6491
1	0.9537	2.7569	3.6883	2.1283	7.8052
2	0.9537	23.1055	7.3719	3.7201	23.1055
3	0.9537	6.3668	6.7760	5.2375	10.6797
4	0.9537	15.9393	8.4645	6.4282	15.9393

2.13.3 AUV Example - Depth

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	2.1377	0.9531	0.1100	0.0769	1.0482
1	0.9573	17.7180	1.7393	0.7961	18.7752
2	0.9568	15.2788	2.4984	0.9701	25.2766
3	0.9559	1.7144	4.2588	1.7144	39.2718
4	0.9551	4.9690	9.4653	3.4297	38.9183

2.14 Enn's Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	5386.9877	0.2644	0.3116	0.2644	0.7280
1	1.4256	2.0525	2.0691	1.5275	10.0925
2	1.2438	4.8615	13.7392	4.8615	24.9909
3	1.1921	30.0469	23.1429	19.1214	30.0469
4	1.1936	32.3732	35.3742	31.6322	40.4079

2.15 Wang's Example

n_K	γ_{min}	$t_{\gamma_{min}}$	t_{median}	t_{min}	t_{max}
0	50.8795	0.0679	0.0581	0.0412	0.0730
1	50.6450	1.1229	1.4341	0.7261	2.0335
2	50.6419	2.0711	1.4475	1.0642	2.9528
3	50.6400	3.2566	2.6642	1.5904	3.5854
4	50.6399	4.4033	3.9533	2.3145	5.1570