LOGIC
HW #1
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15 October 2013 (due in 2 weeks)

Q1. [10] Augment the signature $\{\neg, \wedge\}$ by $\vee$ and prove the completeness and soundness of the calculus obtained by supplementing the basic rules used so far with the rules:

$$(\vee 1)\frac{X \vdash \alpha}{X \vdash \alpha \vee \beta, \beta \vee \alpha}; \quad (\vee 2)\frac{X, \alpha \vdash \gamma \mid X, \beta \vdash \gamma}{X, \alpha \vee \beta \vdash \gamma}$$

Q2. [10] Prove: (**Finiteness Theorem for** $\models$) *If* $X \models \alpha$*, then so too* $X_0 \models \alpha$ *for some finite subset* $X_0 \subset X$.

Q3. [10] Using the preceding theorem, prove that *if* $X \cup \{\neg\alpha | \alpha \in Y\}$ *is inconsistent and* Y *is nonempty, then there exist formulas* $\alpha_0, \ldots, \alpha_n \in Y$ *in* Y *such that*

$$X \vdash \alpha_0 \vee \cdots \vee \alpha_n.$$