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Signaling games

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see Skyrms, *Signals* and Gintis, *Game theory evolving*

Vervet monkeys

- Vervet monkeys have distinct alarm calls for different predators
- Eagle: “cough” $\rightsquigarrow$ hide in the underbush
- Leopard: “bark” $\rightsquigarrow$ climb on a tree
- Snake: “chutter” $\rightsquigarrow$ watch out for snake

- Even inter-species communication exists
- How can such systems come about?
- How can meaning evolve?
- Can we give an explanation that is simple enough to apply even to bacteria and cells?

Sender-receiver games

- Introduced by David Lewis (1969) to explain convention and meaning
- "Worst-case scenario" in which natural salience is absent and signaling is purely conventional
- Two players: sender, receiver

- Sender has a "type" (state, private information)
- Sender chooses a signal (signals have no intrinsic meaning)
- Receiver responds by choosing an action
- Payoffs depend on type and action (and signal)

- A sender strategy maps types to signals
- A receiver strategy maps signals to actions
- An equilibrium is a pair of strategies such that neither can improve by deviating

Basic definitions

- Set of types T, signals S, actions A
- Probability distribution $\tau \in \Delta T$
- Sender strategy $\sigma : T \to \Delta S$
- Receiver strategy $\rho : S \to \Delta A$
- Payoff for sender: $u(t, s, a)$, for receiver: $v(t, s, a)$
- Equilibrium: pair of strategies σ, ρ such that

$$\sum_{t,s,a} u(t,s,a)\tau(t)\sigma(s|t)\rho(a|s) \geq \sum_{t,s,a} u(t,s,a)\tau(t)\sigma'(s|t)\rho(a|s)$$

and

$$\sum_{t,s,a} u(t,s,a)\tau(t)\sigma(s|t)\rho(a|s) \geq \sum_{t,s,a} u(t,s,a)\tau(t)\sigma(s|t)\rho'(a|s)$$

for all σ', ρ'

Simple example

		Receiver action		
		a_1	a_2	a_3
Sender type	t_1	$1, 1$	$0, 0$	$0, 0$
	t_2	$0, 0$	$1, 1$	$0, 0$
	t_3	$0, 0$	$0, 0$	$1, 1$

- One “right” action for each type
- Coordination game
- Signals costless

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$$\begin{tikzcd}
t_1 \arrow[r] & s_1 \arrow[r] & a_1 \\
t_2 \arrow[r] & s_2 \arrow[r] & a_2 \\
t_3 \arrow[r] & s_3 \arrow[r] & a_3
\end{tikzcd}$$

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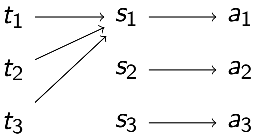

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  - Pooling

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- One “right” action for each type
- Coordination game
- Signals costless
- Types of equilibria:
  - Separating (“signaling system”)
  - Pooling
  - Partial Pooling

Information

- View the information of a signal as how it changes probabilities
- Signals involve two kinds of information:
  - What state the sender has observed
  - What action the receiver will take
- Information content and quantity
- Information is maximal in signaling system
  (but also in perfectly *mis*coordinating systems)

Information quantity

- Intuition: should compare probability with vs without observation
- Information quantity of signal s “in favor of” state (type) t:

$$\log \frac{\sigma(t|s)}{\tau(t)}$$

- Overall information quantity of signal s:

$$\sum_{t \in T} \sigma(t|s) \log \frac{\sigma(t|s)}{\tau(t)}$$

  (Kullback-Leibler divergence)
- Information quantity of signal about act is analogous

Example

- Consider two equiprobable states t_1, t_2 and two signals s_1, s_2
- Consider separating sender strategy $\sigma(t_1) = s_1$, $\sigma(t_2) = s_2$
- Information quantity of s_1:

$$\begin{aligned} &\sigma(t_1|s_1)\log\frac{\sigma(t_1|s_1)}{\tau(t_1)} + \sigma(t_2|s_1)\log\frac{\sigma(t_2|s_1)}{\tau(t_2)} \\ &= 1\log\frac{1}{0.5} + 0\log\frac{1}{0.5} = 1 \text{ (bit)} \end{aligned}$$

- Consider pooling sender strategy $\sigma(t_1) = s_1$, $\sigma(t_2) = s_1$
- Information quantity of s_1:

$$\begin{aligned} &\sigma(t_1|s_1)\log\frac{\sigma(t_1|s_1)}{\tau(t_1)} + \sigma(t_2|s_1)\log\frac{\sigma(t_2|s_1)}{\tau(t_2)} \\ &= \tau(t_1)\log\frac{\tau(t_1)}{\tau(t_1)} + \tau(t_1)\log\frac{\tau(t_1)}{\tau(t_1)} = 0 \text{ (bit)} \end{aligned}$$

Information content

- “Meaning” of signal s
- Its information quantity in favor of each respective state

$$\langle \log \frac{\sigma(t_1|s)}{\tau(t_1)}, \ldots, \log \frac{\sigma(t_n|s)}{\tau(t_n)} \rangle$$

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- Information content of s_1:

$$\langle 1, -\infty \rangle$$

Evolution

- Replicator dynamics as simple model of evolution
- Differential replication according to Darwinian fitness
- Discrete version proceeds in generations
- Equation to determine new proportion of individuals with strategy s:

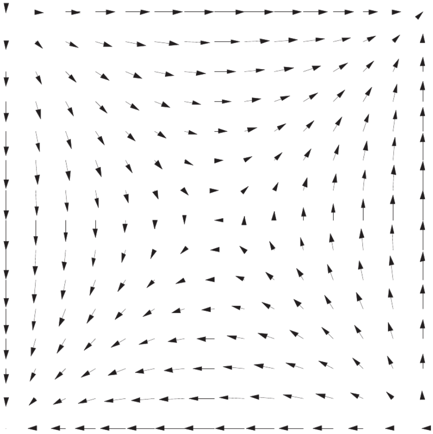
$$x_{t+1}(s) = x_t(s)\frac{\text{Fitness}(s)}{\text{Average fitness}}$$

- Continuous version:

$$\dot{x}(s) = x \cdot (\text{Fitness}(s) - \text{Average fitness})$$

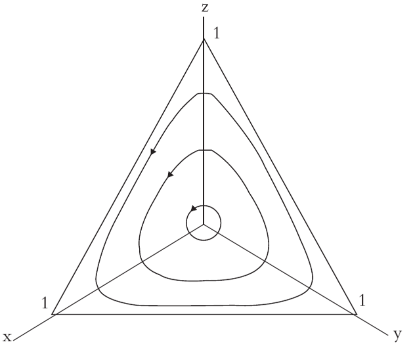
- Fitness in the simplest case is payoff of random pairing
- For cooperation to evolve, correlation is needed
- For symmetry breaking and exploration, add random mutation

Depiction of replicator dynamics



- Unstable states, rest points, stable and strongly stable states
- Illustrating with Hawk-Dove, Prisoner's dilemma, Inconsequential actions

Rock, scissors, paper



- Each pure strategy is equilibrium, but unstable
- Completely mixed state is stable, but not strongly
- No population that is not already in equilibrium converges

Evolution in signaling games

- Simplest case: two equiprobable types, two signals, two acts
- Sender and receiver have 4 strategies each, or 16 combined
- Signaling system always evolves
- All pooling equilibria are unstable
- Randomness breaks symmetry and creates information

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- The greater the inequality, the more likely
- On the other hand, the smaller the impact on the welfare is

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- Details depend on the exact payoffs, probabilities and mutation rates
- Correlation can destabilize pooling

Deception

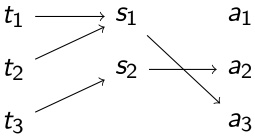
- Deception is ubiquitous in nature (e.g. Photuris vs Photinus)
- How can we define it, and how can it be sustainable?

- Deception is only meaningful in the context of an existing signaling convention
- Take the information content of a signal to be its agreed-upon meaning
- A signal whose information content does not reflect the type is misinformation (e.g., alarm call when no predator present)
- A misinformative signal benefitting the sender (and harming the receiver) is deceptive (e.g., Photuris)

- Systematic deception changes the convention (again, Photuris)

Successful deception in equilibrium

	a_1	a_2	a_3
t_1	$2, 10$	$0, 0$	$10, 8$
t_2	$0, 0$	$2, 10$	$10, 8$
t_3	$0, 0$	$10, 10$	$0, 0$

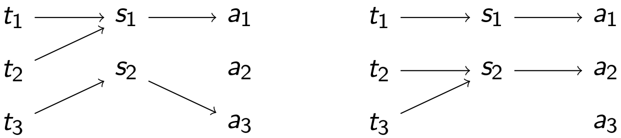


- Sender always manipulates receiver with "half-truths":
  - In t_1, the sender's signal raises the probability of t_2
  - In t_2, the sender's signal raises the probability of t_1
- These half-truths induce receiver to choose a_3 in t_1 and t_2
- Sender benefits at expense of receiver (who prefers a_1 or a_2)

- Deception can even be seen as "morally good":
- Sender gains 8, receiver loses only 2
- If you don't know your role in advance (or you alternate), you would choose the deceptive equilibrium as universal law

Information bottlenecks can impact efficiency

	a_1	a_2	a_3
t_1	$7, 7$	$0, 0$	$2, 2$
t_2	$4, 4$	$6, 6$	$0, 0$
t_3	$0, 0$	$5, 5$	$10, 10$



- Both are evolutionarily stable, although the right one is worse

Inventing new signals

- Chinese restaurant process:
  - Restaurant with infinite number of tables
  - Guests enter one by one
  - If N guests are there, each new guest joins the table of any of them with probability $\frac{1}{N+1}$
  - With probability $\frac{1}{N+1}$, he starts a new table
- Pólya urn process:
  - Urn with various colored balls
  - Draw a ball at random, put back two of that color
  - "Neutral" evolution (without selection pressure)
  - Converges to random color
- Hoppe-Pólya urn:
  - Add a black "mutator" ball to Pólya's urn
  - If it is drawn, put it back and add one with a new color
  - Equivalent to Chinese restaurant
  - Model for neutral evolution with invention

Inventing new signals

- Use a Hoppe-Pólya urn to model sender strategy
- Reinforcement learning: add balls depending on communication success (payoff)
- If receiver receives a new signal, he acts at random
- On success, the new signal is reinforced, otherwise removed
- Noisy forgetting to keep number of signals from exploding: at each step remove some ball at random
- In experiments, efficient signaling evolves quite robustly

Further topics

- Logic and information processing
- Complex signals and compositionality
- Teamwork
    - Quorum sensing (e.g. Vibrio fisheri)
    - Myxococcus xanthus
    - Multicellular organisms
- Learning to network
- Cheap talk