Reusable Software Infrastructure for Stream Processing

by

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Abstract

Developers increasingly use streaming languages to write their data processing applications. While a variety of streaming languages exist, each targeting a particular application domain, they are all similar in that they represent a program as a graph of streams (i.e. sequences of data items) and operators (i.e. data transformers). They are also similar in that they must process large volumes of data with high throughput. To meet this requirement, compilers of streaming languages must provide a variety of streaming-specific optimizations, including automatic parallelization. Traditionally, when many languages share a set of optimizations, language implementors translate the source languages into a common representation called an intermediate language (IL). Because optimizations can modify the IL directly, they can be re-used by all of the source languages, reducing the overall engineering effort. However, traditional ILs and their associated optimizations target single-machine, single-process programs. In contrast, the kinds of optimizations that compilers must perform in the streaming domain are quite different, and often involve reasoning across multiple machines. Consequently, existing ILs are not suited to streaming languages.

This thesis addresses the problem of how to provide a reusable infrastructure for stream processing languages. Central to the approach is the design of an intermediate language specifically for streaming languages and optimizations. The hypothesis is that an intermediate language designed to meet the requirements of stream processing can assure implementation correctness; reduce overall implementation effort; and serve as a common substrate for critical optimizations. In evidence, this thesis provides the following contributions: (1) a catalog of common streaming optimizations that helps define the requirements of a streaming IL; (2) a calculus that enables reasoning about the correctness of source language translation and streaming optimizations; and (3) an intermediate language that preserves the semantics of the calculus, while addressing the implementation issues omitted from the calculus. This work significantly reduces the effort it takes to develop stream processing languages by making optimizations reusable across languages, and jump-starts innovation in language and optimization design.

Acknowledgments

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This thesis focuses on language support for distributed stream processing. I was introduced to streaming during my internship at IBM Research, and my interest grew during my two and half years spent as a co-op working with the System S team. This thesis would not have been possible without my co-authors: Henrique Andrade, Buğra Gedik, Vibhore Kumar, Scott Schneider, Huayong Wang, Kun-Lung Wu, and Qiong Zou. I would particularly like to thank Buğra for sharing his experience with System S, and for serving on my thesis proposal committee. John Field, Yoonho Park, Rodric Rabbah, and Martin Vechev provided me with invaluable feedback on early versions of this work. I would also like to thank Nagui Halim for giving me the opportunity to work with such a talented group of people, as well as for his encouragement of my research.

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1 Introduction

Stream processing applications are everywhere. In entertainment, people increasingly consume music and movies as streaming media through internet services such as Spotify and Netflix. Netflix alone accounts for nearly 30% of downstream internet traffic during peak hours [98]. In finance, high-frequency trading programs federate live data feeds from independent exchanges to complete transactions. Indeed, high-frequency trading accounts for 50–60% of all trades in the United States [75]. In healthcare, streaming systems monitor patients to predict the onset of critical situations, allowing doctors to quickly respond to life-threatening events. In the neonatal intensive care unit, streaming systems can predict the onset of sepsis in premature babies 24 hours sooner than experienced ICU nurses [58].

Stream processing sits at the intersection of two converging trends. First, as the amount of digital information grows [2], there is a greater demand for data-centric applications. Second, as multicore machines and cluster computing become commonplace [1], applications are expected to utilize available resources by running on multiple processors or multiple machines. These trends have resulted in a paradigm shift that has a profound impact on the design of both programming languages and optimizations.

First, to encourage (and enforce) this new paradigm, *stream processing languages* represent an application as a data-flow graph of streams and operators, where each *stream* is an infinite sequence of data items, and each *operator* transforms data. A growing body of research explores, from the language design perspective, how to best build streaming applications [8, 11, 19, 25, 27, 28, 42, 54, 73, 77, 84, 93, 112, 123]. These languages tend to be tailored to specific classes of applications, for example, by building on relational algebra to filter, project, join, and aggregate streams of records (CQL [8]), or designed to support certain optimizations, such as by enforcing static data transfer to enable double-buffering and operator fission (StreamIt [112]).

Second, while traditional compiler optimizations, such as function inlining, loop invariant code motion, and register allocation, seek to improve performance on a single process or machine, *stream processing optimizations* aim to maximize resource utilization on large multi-processors and clusters of workstations, often by rewriting an application's dataflow graph [7, 10, 12, 14, 18, 20, 23, 25, 32, 42, 45, 46, 57, 63, 73, 92, 95, 99, 101, 103, 102, 108, 116, 117, 119, 122]. Examples of important streaming optimizations include operator placement, fusion, and fission (or replication), which are all implemented in Sawzall [93] and the cluster back-end for StreamIt [112]. Non-distributed streaming languages, such as CQL [8], will need to implement these core optimizations if they expect to scale with increased workloads.

To continue to advance the state of the art for both languages and optimizations, language implementors need the proper infrastructure. An *intermediate language* (IL) provides a platform-independent target for source language translation. There are several notable examples of compilers or virtual machines that use ILs to decouple source languages from the target platform, including SUIF [5] the JVM [72], and the CLR [52]. This decoupling improves source language portability, and reduces engineering effort by providing a common substrate for optimization. Unfortunately, because streaming optimizations involve reasoning about an entire distributed application, existing ILs are ill-suited for stream processing languages. Given the lack of an appropriate IL, language designers have been forced to use make-shift solutions. For example, several languages [24, 44, 88] , somewhat surprisingly, utilize MapReduce [29] as an intermediate language. However, MapReduce is limited in that it supports only batch processing, not continuous streaming. Alternatively, Dryad [60] provides a more general execution model than MapReduce's two-phased execution, but still supports only batch processing. The reality is that an intermediate language for stream processing does not exist.

1.1 This Dissertation

This thesis addresses the problem of how to provide a reusable infrastructure for stream processing languages. Central to the approach is the design of an intermediate language specifically for streaming languages and optimizations. The hypothesis is that **an intermediate language designed to meet the requirements of stream processing can serve as a common substrate for critical optimizations; assure implementation correctness; and reduce overall implementation effort**. There are three components to this work that support the hypothesis.

First, this thesis systematically explores the requirements for stream processing by creating a catalog of common streaming optimizations (§ 2). There is an abundance of prior work on streaming optimizations. The problem is that many different research communities have independently arrived at stream processing as a programming model for high-performance and parallel computation, including digital signal processing, databases, operating systems, and complex event processing. As a result, each of these communities has developed some of the same optimizations, but often with conflicting terminology and unstated assumptions. This catalog both consolidates terminology and makes assumptions explicit. In the process, it clarifies what information a streaming IL needs to provide in order to support streaming optimizations.

Second, the requirements inform the design of the Brooklet calculus for stream processing (§ 3). Brooklet defines a core minimal language that can represent a diverse set of streaming languages, and allows us to reason about the correctness of optimizations. To facilitate that reasoning, it makes explicit those things that require special machinery in distributed systems. It has a small-step operational semantics, which models execution as a sequence of atomic operator firings. It does not, however, define the order of the firings. The non-deterministic choice reflects the fact that ordered execution in a distributed system requires additional machinery, such as a sequencer. All uses of state are explicit, since keeping state consistent across nodes in a distributed system requires explicit machinery, such as two-phase commit. Finally, all communication is explicit, and one-to-one (i.e., connect an output of exactly one operator with an input of exactly one operator), because any form of many-to-many communication in a distributed system requires, again, explicit machinery, such as application-level multicast. This emphasis on distributed implementation distinguishes Brooklet from prior work on stream processing semantics [22, 50, 56, 62, 70, 79]. Brooklet provides a formal foundation for the design of the IL.

Finally, this thesis presents the River intermediate language (§ 4). River builds on Brooklet by addressing the real-world details that the calculus elides. Notably, River provides an implementation language for operator implementations; maximizes the concurrent execution of operators while preserving the sequential semantics of Brooklet; and uses back-pressure to avoid buffer overflows in the presence of bounded queues. Because every River program can be trivially abstracted into a Brooklet program, River is a practical intermediate language with a rigorously defined semantics. Moreover, this thesis explores techniques for making language development economic, through the use of modular parsers, type checkers, and code generators. The result is a set of tools and artifacts for developing River compilers. Collectively, the River IL and associated tools provide a reusable software infrastructure for stream processing.

1.2 Evaluation

River defines an intermediate language for stream processing based on the Brooklet calculus. We define formal translations from three representative languages, CQL, Sawzall, and StreamIt, into Brooklet (§ 3.3) , and proofs for the safety of three vital streaming optimizations, operator fusion, fission, and placement, in Brooklet(§ 3.4) . Every River program can be trivially abstracted into a Brooklet program and every River execution also is a Brooklet execution. Consequently, the IL has a well-defined formal foundation, making it possible to rigorously reason about the correctness of translations and optimizations.

To verify that River is able to support a diversity of streaming languages, we implemented language translators for CQL, StreamIt, and Sawzall, as well as illustrative benchmark applications (§ 4.2). The benchmarks exercise a significant portion of each language, demonstrating that River is expressive enough to support a wide variety of streaming languages.

To verify that River is extensible enough to support a diverse set of streaming optimizations, we implemented three critical streaming optimizations: operator fusion, fission, and placement that operate on the IL directly (§ 4.3). These optimizations demonstrate that River can serve a common substrate for critical optimizations.

Finally, we wrote a back-end for River on System S [42], a high-performance distributed streaming runtime. We then evaluated the effects of applying the three high-level optimizations to the benchmark applications written in the three different streaming languages (§ 4.5). River effectively decouples the optimizations from the language front-ends, and thus makes them reusable across front-ends. Because of this reuse, River reduces the overall implementation effort.

1.3 Research Contributions

This dissertation makes the following contributions:

1. It provides a **systematic exploration of the requirements for an intermediate language for stream processing** by developing a catalog of common streaming optimizations.

2. It develops a **formal foundation for the design of the intermediate language for stream processing** with a calculus that enables reasoning about the correctness of source language translation and streaming optimizations.

3. It presents an **intermediate language for stream processing with a rigorously defined semantic that decouples language front-ends from optimizations**.

4. It defines the **first formal semantics for the Sawzall language** as a byproduct of the Sawzall-to-Brooklet translation.

5. It reports the **first distributed implementation of CQL** as a product of our CQL-to-River translation and the River-to-System S [42] backend.

In short, this thesis provides a reusable software infrastructure for stream processing. It increases the portability of stream processing languages, and enables the reuse of common streaming optimizations. This work helps to support and encourage future innovation in language and optimization design.

2 Stream Processing Optimizations

Streaming applications are programs that process continuous data streams. These applications have become ubiquitous due to increased automation in telecommunications, health-care, transportation, retail, science, security, emergency response, and finance. As a result, various research communities have independently developed programming models for streaming. While there are differences both at the language level and at the system level, each of these communities ultimately represents streaming applications as a graph of streams and operators, where each *stream* is a conceptually infinite sequence of data items, and each *operator* consumes data items from incoming streams and produces data items on outgoing streams. Since many streaming applications require extreme performance, each community has developed a number of optimizations. The communities that have focused the most on streaming optimizations are digital signal processing, operating systems and networks, databases, and complex event processing. The latter discipline, for those unfamiliar with it, uses temporal patterns over sequences of events (i.e., data items), and reports each match as a complex event.

Unfortunately, while there is plenty of literature on streaming optimizations, the literature uses inconsistent terminology. For instance, what we refer to as an *operator* is called operator in CQL [8], filter in StreamIt [112], box in Aurora and Borealis [4, 3], stage in Seda [114], actor in Flextream [57], and module in River [10]. As another example for inconsistent terminology, push-down in databases and hoisting in compilers are essentially the same optimization, and therefore, we advocate the more neutral term *operator reordering.* To establish common vocabulary, we took inspiration from catalogs for design patterns [40] and for refactorings [38]. Those catalogs have done a great service to practitioners and researchers alike by raising awareness and using consistent terminology. This chapter is a catalog of the stream processing optimizations listed in Table 2.1.

Besides inconsistent terminology, this chapter is further motivated by unstated assumptions: certain communities take things for granted that other communities do not. For example, while StreamSQL assumes that stream graphs are forests (acyclic sets of trees), StreamIt assumes

Table 2.1: The optimizations cataloged in this survey. Column "Graph" indicates whether or not the optimization changes the topology of the stream graph. Column "Semantics" indicates whether or not the optimization changes the semantics, i.e., the input/output behavior. Column "Dynamic" indicates whether the optimization happens statically (before runtime) or dynamically (during runtime). Entries labeled "(depends)" indicate that both alternatives are well-represented in the literature.

Section	Optimization	Graph	Semantics	Dynamic
2.1.	Operator reordering	changed	unchanged	(depends)
2.2.	Redundancy elimination	changed	unchanged	(depends)
2.3.	Operator separation	changed	unchanged	static
2.4.	Fusion	changed	unchanged	(depends)
2.5.	Fission	changed	(depends)	(depends)
2.6.	Placement	unchanged	unchanged	(depends)
2.7.	Load balancing	unchanged	unchanged	(depends)
2.8.	State sharing	unchanged	unchanged	static
2.9.	Batching	unchanged	unchanged	(depends)
2.10.	Algorithm selection	unchanged	(depends)	(depends)
2.11.	Load shedding	unchanged	changed	dynamic

that stream graphs are possibly cyclic single-entry, single-exit regions. We have observed stream graphs in practice that fit neither mold, for example, trading applications with multiple input feeds and feedback. Additionally, several papers focus on one aspect of a problem, such as formulating a mathematical model for the profitability trade-offs of an optimization, while leaving other aspects unstated, such as the conditions under which the optimization is safe. Furthermore, whereas some papers assume shared memory, other papers assume a distributed system, where state sharing is more difficult and communication is more expensive, since it involves the network. This chapter describes optimizations for many different kinds of streaming systems, including shared-memory and distributed, acyclic and cyclic, among other variations. For each optimization, this chapter explicitly lists both safety and profitability considerations.

Each optimization is presented in a section by itself, and each section is structured as follows:

- *Tag-line and figure* gives a quick intuition for what the optimization does.
- *Example* describes a concrete real-world application, which illustrates what the optimization does and motivates why it is useful. Taken together, the example subsections for all the optimizations paint a picture of the landscape of modern stream processing domains and applications.

- *Profitability* describes the conditions under which the optimization improves performance. To illustrate the main trade-offs in a concrete and realistic manner, each profitability subsection is based on a micro-benchmark. All experiments were done on a real stream processing system (System S [6]), and each chart shows error bars indicating the standard deviation over multiple runs. The micro-benchmarks serve as an existence proof for a case where the optimization improves performance. They can also serve as a blue-print for testing the optimization in a new application or system.

- *Safety* lists the conditions necessary for the optimization to preserve correctness. Formally, the optimization is only safe if the conjunction of the conditions is true. But beyond that, we intentionally kept the conditions informal to make them easier to read, and to make it easier to state side conditions without having to introduce too much notation.

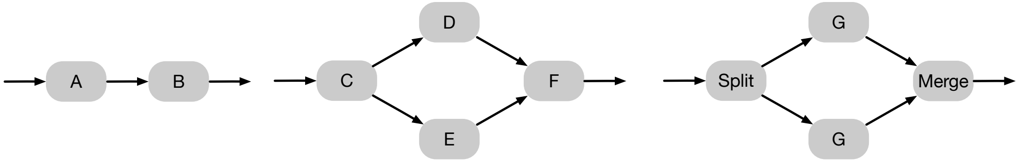
- *Variations* surveys the most influential and unique work on this optimization in the literature. The interested reader can use this as a starting point for further study.

- *Dynamism* identifies established approaches for applying the optimization at runtime instead of statically, i.e., at compile time.

Existing surveys on stream processing do not focus on optimizations [106, 13, 61], and existing catalogs of optimizations do not focus on stream processing. This chapter provides both: it presents a catalog of stream processing optimizations, and makes them approachable to users, implementers, and researchers.

2.0.1 Background

This section clarifies the terminology used in this chapter. A streaming *application* is represented by a stream *graph*, which is a directed graph whose vertices are operators and whose edges are streams. A streaming *system* is a runtime system that can execute stream graphs. In general, stream graphs might be cyclic, though some systems only support acyclic graphs. Streaming systems implement streams as FIFO (first-in, first-out) queues. Whereas a *stream* is a possibly infinite sequence of data items, at any given point in time, a *queue* contains a finite sequence



(a) Pipeline-parallel A || B. (b) Task-parallel D || E. (c) Data-parallel G || G.

Figure 2.1: Pipeline, task, and data parallelism in stream graphs.

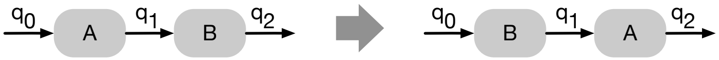
of in-flight data items. The *data item* is the unit of communication in a streaming application. Different communities have different notions of data items, including samples in digital signal processing, tuples in databases, or events in complex event processing; this chapter merely assumes that data items can contain *attributes*, which are smaller units of data. Streaming systems are designed for data in motion and computation at rest, meaning that data items continuously flow through the edges and operators of the graph, whereas the topology of the graph rarely changes. The most common cause for topology changes is *multi-tenancy*, where a single streaming system runs multiple applications that come and go. Another cause for topology change is fault tolerance, where back-up operators and streams take over when their primaries fail.

An *operator* is a continuous stream transformer: each operator transforms its input streams to its output streams, and operators may execute in parallel. It is up to the streaming system to determine when an operator *fires*; for instance, the system could schedule downstream operators to execute before upstream operators, or execute an operator whenever a data item becomes available in one of its input queues. Operators may or may not have *state*, which is data that the operator remembers between firings. Depending on the streaming system, state might be shared between operators. The *selectivity* of an operator is its data rate measured in output data items per input data item. For example, an operator that produces one output data item for every two input data items has a selectivity of 0.5. An operator with fan-out, i.e., multiple output streams, is called a *split*, and an operator with fan-in, i.e., multiple input streams, is called a *merge*. Many split or merge operators forward data items unmodified, but a relational *join* is an example for a merge operator that includes a non-trivial transformation.

It is often useful to employ specific terminology for the various flavors of parallelism among the operators in a stream graph. Fig. 2.1 illustrates these flavors. *Pipeline parallelism* is the concurrent execution of a producer A with a consumer B. *Task parallelism* is the concurrent execution of different operators D and E that do not constitute a pipeline. And *data parallelism* is the concurrent execution of multiple replicas of the same operator G on different portions of the same data. The architecture community refers to data parallelism as SIMD (single instruction, multiple data).

2.1 Operator Reordering **(a.k.a. hoisting, sinking, rotation, pushdown)**

Move more selective operators upstream to filter data early.

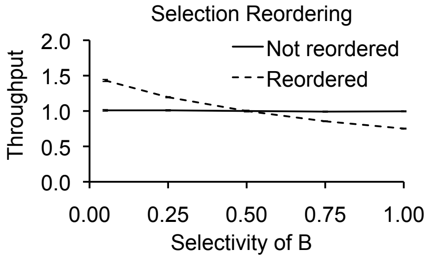


2.1.1 Example

Consider a healthcare application that continuously monitors patients, alerting physicians when it detects that a patient requires immediate medical assistance. The input stream contains patient identification and real-time vital signs. A first operator A enriches each data item with the full patient name and the result of the last exam by a nurse. The next operator B is a selection operator, which only forwards data items with alarming vital signs. In this ordering, many data items will be enriched by operator A and will be sent on stream q_1 only to be dropped by operator B. Hoisting B in front of A eliminates this unnecessary overhead.

2.1.2 Profitability

Reordering is profitable if it moves selective operators before costly operators. The selectivity of an operator is the number of output data items per input data item. For example, an operator that drops 70% of all data items outputs only 30% and thus has selectivity 0.3. The chart shows throughput given two operators A and B of equal cost, where the selectivity of A is fixed at 0.5. If A comes before B, then independently of the selectivity of B, A processes all data and B processes 50% of the data, so the performance does not change. If B comes before A, then B processes all data, but the amount of data processed by A depends on the selectivity of B, and overall throughput is higher when B drops more data. The cross-over point is when both are equally selective.



2.1.3 Safety

Operator reordering is safe if the following conditions hold:

- *Ensure commutativity.* The result of executing B before A must be the same as the result of executing A before B. In other words, A and B must commute. A sufficient condition for commutativity is if both A and B are stateless. However, there are also cases where reordering is safe past stateful operators; for instance, in some cases, an aggregation can be moved before a split.
- *Ensure attribute availability.* The second operator B must only rely on attributes of the data item that are already available before the first operator A. In other words, the set of attributes that B reads from a data item must be disjoint from the set of attributes that A writes to a data item.

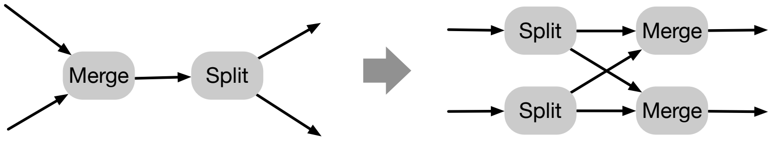
2.1.4 Variations

Algebraic reorderings

Operator reordering is popular in streaming systems built around the relational model, such as the Stream system [8]. These systems establish the safety of reordering based on the formal semantics of relational operators, using algebraic equivalences between different operator orderings. Such equivalences can be found in standard texts on database systems, such as [41]: besides moving selection operators early to reduce the number of data items, another common optimization moves projection operators (operators that strip away some attributes from data items) early to reduce the size of each data item. And a related optimization picks a relative ordering of relational join operators to minimize intermediate result sizes: by moving the more selective join first, the other join has less work. Some streaming systems reorder operators based on extended algebras that go beyond the relational model. For example, Galax uses nested-relational algebra for XML processing [95], and Sase uses a custom algebra for finding temporal patterns across sequences of data items [118]. Finally, commutativity analysis on operator implementations could be used to discover reorderings even without an operator-level algebra [97]. A practical consideration is whether or not to treat floating point arithmetic as commutative, since floating-point rounding can lead to different results after reordering.

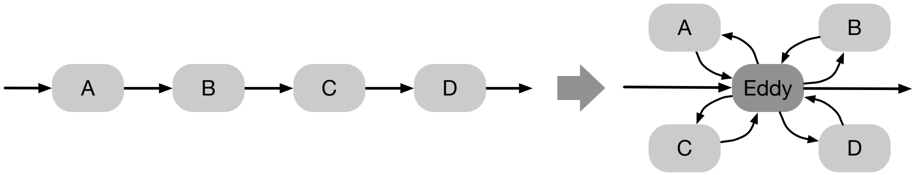
Synergies with other optimizations

While operator reordering yields benefits of its own, it also interacts with several of the streaming optimizations cataloged in the rest of this paper. Redundancy elimination (Section 2.2) can be viewed as a special case of operator reordering, where a Split operator followed by redundant copies of an operator A is reordered into a single copy of A followed by the Split. Operator separation (Section 2.3) can be used to separate an operator B into two operators B_1 and B_2; this can enable a reordering of one of the operators B_i with a neighboring operator A. After reordering operators, they can end up near other operators where fusion (Section 2.4) becomes beneficial; for instance, a selection operator can be fused with a Cartesian-product operator into a relational join, which is faster because it never needs to create all tuples in the product. Fission (Section 2.5) introduces parallel segments; when two parallel segments are back-to-back, reordering the Merge and Split eliminates a serialization bottleneck, as in the Exchange operator in Volcano [47]. The following figure illustrates this Split/Merge rotation:



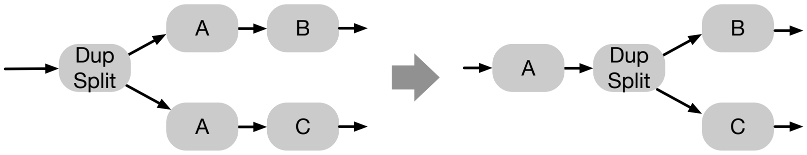
2.1.5 Dynamism

The optimal ordering of operators is often dependent on the input data. Therefore, it is useful to be able to change the ordering at runtime. The Eddy operator enables a dynamic version of the operator-reordering optimization with a static graph transformation [12]. As shown in the figure below, an Eddy operator is connected to every other operator in the pipeline, and dynamically routes data after measuring which ordering would be the most profitable. This has the advantage that selectivity need not be known ahead of time, but incurs some extra overhead for tuple routing.



2.2 Redundancy Elimination (a.k.a. subgraph sharing, multi-query optimization)

Eliminate redundant computations.

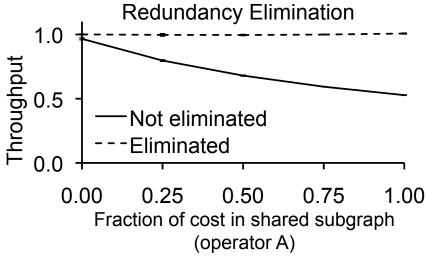


2.2.1 Example

Consider two telecommunications applications, one of which continuously updates billing information, and the other monitors for network problems. Both applications start with an operator A that deduplicates call-data records and enriches them with caller information. The first application consists of operator A followed by an operator B that filters out everything except long-distance calls, and calculates their costs. The second application consists of operator A followed by an operator C that performs quality control based on dropped calls. Since operator A is common to both applications, redundancy elimination can share A, thus saving resources.

2.2.2 Profitability

Redundancy elimination is profitable if resources are limited and the cost of redundant work is significant. The chart shows the performance of running two applications together on a single core, one with operators A and B, the other with operators A and C. The total cost of operators A, B, and C is held constant. However, without redundancy elimination, throughput degrades when a large fraction of the cost belongs to operator A, since this work is duplicated. In fact, when A does all the work, redundancy elimination improves throughput by a factor of two, because it runs A only once instead of twice.



2.2.3 Safety

Redundancy elimination is safe if the following conditions hold:

- *Ensure same algorithm.* The redundant operators must, indeed, perform an equivalent computation. For example, if both of them compute an average, but one uses the arithmetic mean and the other the geometric mean, they cannot be shared.

- *Ensure combinable state.* Redundant operators are easy to combine if they are stateless. If they are stateful and work on different data, more care is needed. For instance, a simple counter on a combined stream would differ from separate counters on subsets of the stream.

2.2.4 Variations

Multi-tenancy

Redundant subgraphs as described above often occur in streaming systems that are shared by many different streaming applications. Redundancies are likely when many users launch applications composed from a small set of data sources and built-in operators. While redundancy elimination could be viewed as just a special case of operator reordering (Section 2.1), in fact, the literature has taken it up as a domain in its own right. This separate treatment has been fruitful, leading to more comprehensive approaches. The RETE algorithm is a seminal technique for sharing computation between a large number of continuous applications [37]. NiagaraCQ implements sharing even when operators differ in certain constants, by implementing the operators using relational joins against the table of constants [25]. YFilter implements sharing between applications written in a subset of XPath, by compiling them all into a combined NFA (non-deterministic finite automaton) [32].

Other approaches for eliminating operators

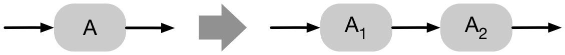
Besides the sophisticated techniques for collapsing similar or identical subgraphs, there are other, more mundane ways to remove an operator from a stream graph. An optimizer can remove a no-op, i.e., an operator that has no effect, such as a projection that keeps all attributes unmodified; for example, no-op operators can arise from simple template-based compilers. An optimizer can remove an idempotent operator, i.e., an operator that repeats the same effect as another operator next to it, such as two selections in a row based on the same predicate; for example, idempotent operators can end up next to each other after operator reordering. Finally, an optimizer can remove a dead subgraph, i.e., a subgraph that never produces any output; for example, a developer may choose to disable a subgraph for debugging purposes, or a library may produce multiple outputs, some of which are unused by a particular application.

2.2.5 Dynamism

A static compiler can detect and eliminate redundancies, no-ops, idempotent operators, and dead subgraphs in an application. However, the biggest gains come in the multi-tenancy case, where the system eliminates redundancies between large numbers of separate applications. In that case, applications are started and stopped independently. When a new application starts, it should share any subgraphs belonging to applications that are already running on the system. Likewise, when an existing application stops, the system should purge any subgraphs that were only used by this one application. These separate starts and stops necessitate dynamic shared sub-graph detection, as done for instance in [92]. Some systems take this approach to its extreme, by treating the addition or removal of applications as a first-class operation just like the addition or removal of regular data items, e.g., in Rete [37].

2.3 Operator Separation (a.k.a. decoupled software pipelining)

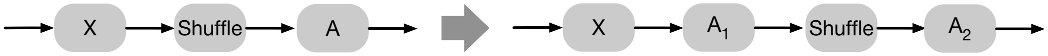
Separate operators into smaller computational steps.



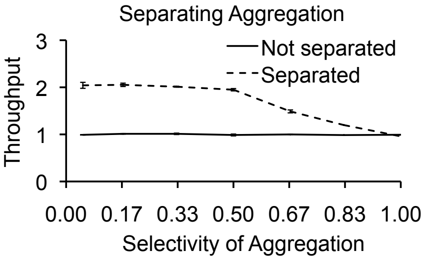
2.3.1 Example

Consider a retail application that continuously watches public discussion forums to discover when users express negative sentiments about a company's products. Assume that the input stream already contains a sentiment score, obtained by a sentiment-extraction operator that analyzes natural-language text to measure how positive or negative it sounds (not shown). Operator A filters data items by sentiment and by product. Since operator A has two filter conditions, it can be separated into two operators A_1 and A_2. This is an enabling optimization: after separation, a reordering optimization (Section 2.1) can hoist the product-selection A_1 before the sentiment analysis, thus reducing the number of data items that the sentiment analysis operator needs to process.

2.3.2 Profitability



Operator separation is profitable if it enables other optimizations such as operator reordering or fission, or if the pipeline parallelism it creates pays off. We report experiments for operator reordering and pipeline parallelism elsewhere, in Sections 2.1.2 and 2.4.2, respectively. Therefore, here, we measure an interaction of operator separation not just with reordering but also with fission. Consider an application that consists of a first parallel segment X, a Shuffle operator, and a second parallel segment with an aggregation operator A. Each segment would be replicated 3-ways, and the shuffle forms a complete bipartite graph. Assume that the cost of the first segment is negligible, and the cost of the second segment consists of a cost of 0.5 for Shuffle plus a cost of 0.5 for the aggregation A. Therefore, throughput is limited by the second segment. With operator separation and reordering, the end of the first parallel segment performs a pre-aggregation A_1 of cost 0.5 before the Shuffle. At selectivity ≤ 0.5, at most half of the data reaches the second segment, and thus, the cost of first segment dominates. Since the cost is 0.5, the throughput is double of that without optimization. At selectivity 1, all data reaches the second segment, and thus, the throughput is the same as without operator separation.



2.3.3 Safety

Operator separation is safe if the following condition holds:

- *Ensure that the combination of the separated operators is equivalent to the original operator.* Given an input stream s, an operator B can be safely separated into operators B_1 and B_2

only if $\mathsf{B}_2(\mathsf{B}_1(s)) = \mathsf{B}(s)$. As discussed in Section 2.3.4 below, establishing this equivalence in the general case is tricky. Fortunately, there are several special cases, particularly in the relational domain, where it is easier. If B is a selection operator, and the selection predicate uses logical conjunction, then B_1 and B_2 can be selections on the conjuncts. If B is a projection that assigns multiple attributes, then B_1 and B_2 can be projections that assign the attributes separately. If B is an idempotent aggregation, then B_1 and B_2 can simply be the same as B itself.

2.3.4 Variations

Separability by construction

The safety of separation can be established by algebraic equivalences. Database textbooks list such equivalences for relational algebra [41], and some streaming systems optimize based on these algebraic equivalences [8]. Beyond the algebraic approach, MapReduce can separate the Reduce operator into a preliminary Combine operator and a final Reduce operator if it is associative [29]. This is useful, because subsequently, Combine can be reordered with the shuffle and fused with the Map operator. Similarly, Yu et al. [122] describe how to automatically separate operators in DryadLINQ [123] based on a notion of decomposable functions: the programmer can explicitly provide decomposable aggregation functions (such as Sum or Count), and the compiler can infer decomposability for certain expressions that call them (such as new T(x.Key, x.Sum(), x.Count())).

Separation by analysis

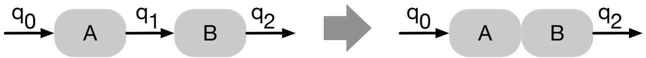
Separating arbitrary imperative code is a difficult analysis problem. In the compiler community, this has become known as DSWP (decoupled software pipelining [89]). In contrast to traditional SWP (software pipelining [67]), which increases instruction-level parallelism in single-threaded code, DSWP introduces separate threads for the pipeline stages. Ottoni et al. propose a static compiler analysis for fine-grained DSWP [89]. Thies et al. propose a dynamic analysis for discovering coarse-grained pipelining, which guides users in manually separating operators [111].

2.3.5 Dynamism

We are not aware of a dynamic version of this optimization. Separating a single operator into two requires sophisticated analysis and transformation of the code comprising the operator. However, the dependent optimizations enabled by operator separation, such as operator reordering, are often done dynamically, as discussed in the corresponding sections.

2.4 Fusion (a.k.a. superbox scheduling)

Avoid the overhead of data serialization and transport.

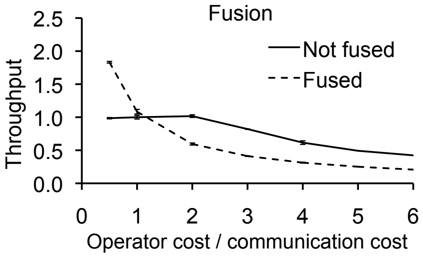


2.4.1 Example

Consider a security application that continuously scrutinizes system logs to detect security breaches. The application contains an operator A that parses the log messages, followed by a selection operator B that uses a simple heuristic to filter out log messages that are irrelevant for the security breach detection. The selection operator B is light-weight compared to the cost of transferring a data item from A to B and firing B. Fusing A and B prevents the unnecessary data transfer and operator firing. The fusion removes the pipeline parallelism between A and B, but since B is light-weight, the savings outweigh the lost benefits from pipeline parallelism.

2.4.2 Profitability

Fusion trades communication cost against pipeline parallelism. When two operators are fused, the communication between them is cheaper. But without fusion, they have pipeline parallelism: the upstream operator can already work on the next data item, while, simultaneously, the downstream operator is still working on the previous data item. The chart shows throughput given two operators of equal cost. The cost of the operators is normalized to a communication cost of 1 for sending a data item between non-fused operators. When the operators are not fused, there are two cases: if operator cost is lower than communication cost, throughput is bounded by communication cost; otherwise, it is determined by operator cost. When the operators are fused, performance is determined by operator cost alone. The break-even point is when the cost per operator equals the communication cost, because the fused operator is 2× as expensive as each individual operator.



2.4.3 Safety

Fusion is safe if the following conditions hold:

- *Ensure resource kinds.* The fused operators must only rely on resources, including logical resources such as local files and physical resources such as GPUs, that are all available on a single host.
- *Ensure resource amounts.* The total amount of resources required by the fused operators, such as disk space, must not exceed the resources of a single host.
- *Avoid infinite recursion.* If there is a cycle in the stream graph, for example for a feedback-loop, data may flow around that cycle indefinitely. If the operators are fused and implemented by function calls, this can cause a stack overflow.

2.4.4 Variations

Single-threaded fusion

A few systems use a single thread for all operators, with or without fusion [21]. But in most systems, fused operators use the same thread, whereas non-fused operators use different threads and can therefore run in parallel. That is the case we refer to as single-threaded fusion. There are different heuristics for deciding its profitability. StreamIt uses fusion to coarsen the granularity of the graph to the target number of cores, based on static cost estimates [46]. Aurora uses fusion to avoid scheduling overhead, picking a fixed schedule that optimizes for throughput, latency, or memory overhead [23]. Spade and Cola fuse operators as much as possible, but only as long as the fused operator performs less work per time unit than the capacity of its host, based on profiling information from a training run [42, 63].

Optimizations enabled by fusion

Fusion often opens up opportunities for traditional compiler optimizations to speed up the code. For instance, in StreamIt, fusion is followed by constant propagation, scalar replacement, register allocation, and instruction scheduling across operator boundaries [46]. In relational systems, fusing two projections into a single projection means that the fused operator needs to allocate only one data item, not two, per input item. Fusion can also open up opportunities for algorithm selection (see Section 2.10). For instance, when Sase fuses a source operator that reads input data with a down-stream operator, it combines them such that the down-stream operator is piggy-backed incrementally on the source operator, producing fewer intermediate results [118].

Multi-threaded fusion

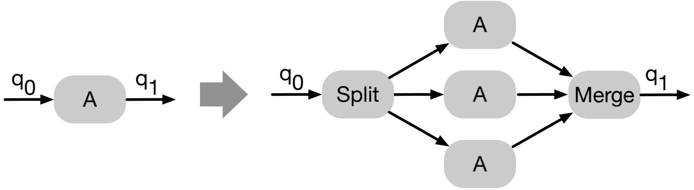
Instead of combining the fused operators in the same thread of control, fusion may just combine them in the same address space, but separate threads of control. That yields the benefits of reduced communication cost, without giving up pipeline parallelism. The fused operators communicate data items through a shared buffer. This causes some overhead for locking or copying data items, except when the operators do not mutate their data items.

2.4.5 Dynamism

Fusion is most commonly done statically. However, the Flextream system performs dynamic fusion by halting the application, re-compiling the code with the new fusion decisions, and then resuming the application [57]. This enables Flextream to adapt to changes in available resources, for instance, when the same host is shared with a different application. However, pausing the application for recompilation causes a latency glitch. Selo et al. mention an even more dynamic fusion scheme as future work in their paper on transport operators [100]. The idea is to decide at runtime whether to route a data item to a fused operator in the same process, or to a version of that same operator in a different process.

2.5 Fission (a.k.a. partitioning, data parallelism, replication)

Parallelize computations.

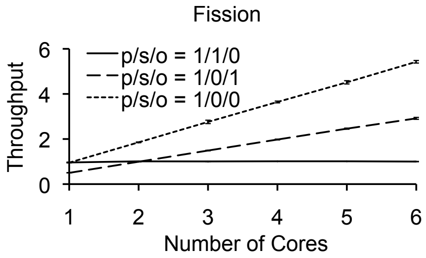


2.5.1 Example

Consider a scientific application that continuously extracts astronomical information from the raw data produced by radio telescopes. Each input data item contains a matrix, and the central operator in the application is a convolution operator A that performs an expensive, but stateless, computation on each matrix. The fission optimization replicates operator A to parallelize it over multiple cores, and brackets the parallel segment by Split and Merge operators to scatter and gather the streams.

2.5.2 Profitability

Fission is profitable if the replicated operator is costly enough to be a bottleneck for the application, and if the benefits of parallelization outweigh the overheads introduced by fission. Split incurs overhead, because it must decide which replica of operator A to send each data item to. Merge may also incur overhead if it must put the streams back in the correct order. These overheads must be lower than the cost of the replicated operator A itself in order for fission to be profitable. The chart shows throughput for fission. Each curve is specified by its p/s/o ratio, which stands for parallel/sequential/overhead. In other words, p is the cost of A itself, s is the cost of any sequential part of the graph that is not replicated, and o is the overhead of Split and Merge. When p/s/o is 1/1/0, the parallel part and the sequential part have the same cost, so no matter how much fission speeds up the parallel part, the overall time remains the same due to pipeline parallelism. When p/s/o is 1/0/1, then fission has to overcome an initial overhead equal to the cost of A, and therefore only turns a profit above two cores. Finally, a p/s/o of 1/0/0 enables fission to turn a profit right away.



2.5.3 Safety

Fission is safe if the following conditions hold:

- *If there is state, keep it disjoint, or synchronize it.* Stateless operators are trivially safe; they can be replicated much in the same way that SIMD instructions can operate on multiple data items at once. Operators with partitioned state can benefit from fission, if the operator is replicated strictly on partitioning boundaries. An operator with *partitioned state* is one that maintains disjoint state based on a particular key attribute of each data item, for example, a separate average stock price based on the value of the stock-ticker attribute. Such operators are, in effect, multiple operators already. Applying fission to such operators makes them separate in actuality as well. Finally, if operators share the same address space after fission, they can share state as long as they perform proper synchronization to avoid race conditions.

- *If ordering is required, merge in order.* Ordering is a subtle constraint, because it is not the operator itself that determines whether ordering matters. Rather, it is the downstream operators that consume the operator's data items. If an operation is commutative across data items, then the order in which the data items are processed is irrelevant. If downstream operators must see data items in a particular order but the operator itself is commutative, then the transformation must ensure that the output data is combined in the same order that the input data was partitioned. There are various approaches for re-establishing the right order, if required. CQL uses logical timestamps [8]. StreamIt uses round-robin or duplication [45]. And MapReduce, instead of re-establishing the old order, uses a distributed "sort" stage [29].

2.5.4 Variations

Fission for large batch jobs

Large batch jobs can be viewed as a special case of stream processing where the computation is arranged as a data-flow graph, streams are finite, and operators process data in a single pass. Distributed datases, such as Volcano [47] and Gamma [31], use fission to process large batch jobs. Both support fission for stateful operators, as long as the state is grouped by keys. More recently, distributed data processing frameworks such as MapReduce [29] and Dryad [60] use fission to scale computation accross large clusters. As discussed in Section 2.1, fission is commonly combined with a reordering of split and merge operators at the boundaries between parallel segments.

Fission for infinite streams

In contrast to batch processing, streaming applications process conceptually infinite amounts of data. A good example for fission of infinite streams is StreamIt [45]. StreamIt addresses the safety question of fission by only replicating operators that are either stateless, or whose operator state is a read-only sliding window, which can be replicated along with the operator itself. In terms of profitability, the StreamIt experience shows that fission is preferable to pipeline and task parallelism, because it balances load more evenly. Besides StreamIt, there is other work on fission for infinite streams, which is discussed below under dynamism. In most systems, the streaming language is designed explicitly for fission, making it easy for the compiler to establish safety. When the language is not designed for fission, safety must be established either by static or by dynamic dependence analysis. An example for a static analysis that discovers fission opportunities is parallel-stage decoupled software pipelining [94]. And Thies et al. explore using dynamic analysis to discover fission opportunities [111].

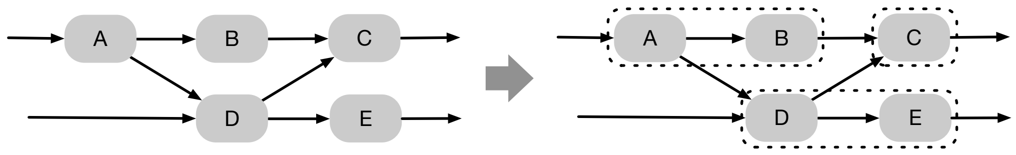
2.5.5 Dynamism

To make the *profitability* decision for fission dynamic, we need to dynamically adjust the width of the parallel segment, in other words, the number of replicated parallel operators. SEDA does that by using a thread-pool controller, which keeps the size of the thread pool below a maximum, but may adjust to a smaller number of threads to improve locality [114]. MapReduce dynamically adjusts the number of workers dedicated to the map task [29]. And "elastic operators" adjust the number of parallel threads based on trial-and-error with observed profitability [99].

To make the *safety* decision for fission dynamic, we need to dynamically resolve conflicts on state and ordering. Brito et al. use software transactional memory, where simultaneous updates to the same state are allowed speculatively, with roll-back if needed [18]. The ordering is guaranteed by ensuring that transactions are only allowed to commit in the same order in which the input data arrived.

2.6 Placement (a.k.a. layout)

Assign operators to hosts and cores.

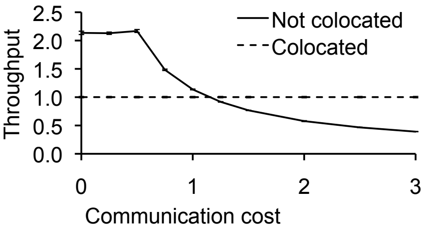


2.6.1 Example

Consider a telecommunications application that continuously computes usage information for long-distance calls. The input stream consists of call-data records. The example has three operators: operator A preprocesses incoming data items, operator B selects long-distance calls, and operator C computes and records billing information for the selected calls. In general, the stream graph might contain more operators, such as D and E, which perform additional functions, such as classifying customers based on their calling profile and determining targeted promotions. If we assume that preprocessing (operator A) and billing (operator C) are both expensive, it makes sense to place them on different hosts. On the other hand, selection (operator B) is cheap, but it reduces the data volume substantially. Therefore, it should be placed on the same host as A, because that reduces the communication cost, by eliminating data that would otherwise have to be sent between hosts.

2.6.2 Profitability

Placement trades communication cost against resource utilization. When multiple operators are placed on the same host, they compete for common resources, such as disk, memory, or CPU. The chart is based on a scenario where two operators compete for disk only. In other words, each operator accesses a file each time it fires. The two operators access different files, but since there is only one disk, they compete for the I/O subsystem. The host is a multi-core machine, so the operators do not compete for CPU. When communication cost is low, the throughput is roughly twice as high when the operators are on separate hosts because they can each access separate disks and the cost of communicating across hosts is marginal. When communication costs are high, the benefit of accessing separate disks is overcome by the expense of communicating across hosts, and it becomes more profitable to share the same disk even with contention.



2.6.3 Safety

Placement is safe if the following conditions hold:

- *Ensure resource kinds.* Placement is safe if each host has the right resources for all the operators placed on it. For example, source operators in financial stream applications often run on FPGAs, and the Lime streaming language supports operators on both CPUs and FPGAs [11]. Operators compiled for an FPGA must be placed on hosts with FPGAs.

- *Ensure resource amounts.* The total amount of resources required by the fused operators, such as FPGA capacity, must not exceed the resources of a single host.

- *Obey security and licensing restrictions.* Besides resource constraints, placement can also be restricted by security, where certain operators can only run on trusted hosts. In addition to these technical restrictions, legal issues may also apply. For example, licensing may restrict a software package to be installed on only a certain number of hosts.

- *If placement is dynamic, move only relocatable operators.* Dynamic placement requires operator migration, i.e., moving an operator from one host to another. Doing this safely requires moving the operator's state, and ensuring that no in-flight data items are lost in the switch-over. Depending on the system, this may only be possible for certain operators, for instance, operators without state, or without OS resources such as sockets or file descriptors.

2.6.4 Variations

Placement for load balancing

Section 2.7 discussed placement algorithms that focus primarily on load balancing [119, 7].

Placement for other constraints

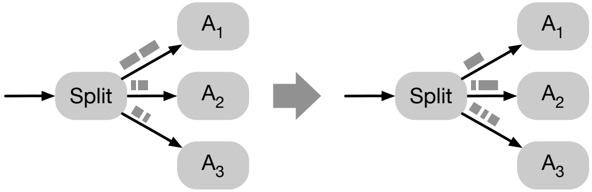
While load balancing is usually at least part of the consideration for placement, often other constraints complicate the problem. Pietzuch et al. present a decentralized placement algorithm for a geographically distributed streaming system, where some operators are geographically pinned [92]. SODA performs placement for load balancing while also taking into account constraints arising from resource matching, licensing, and security [116]. SPADE allows the programmer to guide placement by specifying host pools [42]. When StreamIt is compiled to a multi-core with a software-programmable communcation substrate, placement considers not just load balancing, but also communication hops in the grid of cores, and the compiler generates custom communication code [46].

2.6.5 Dynamism

The majority of the placement decisions are usually made statically, either during compilation or at job submission time. However, some placement algorithms continue to be active after the job starts, to adapt to changes in load or resource availability. As discussed in Section 2.6.3, this poses additional safety requirements. Published algorithms assume that the safety requirements are satisfied by a system mechanism for migrating operators between hosts [119, 92].

2.7 Load Balancing

Distribute workload evenly across resources.

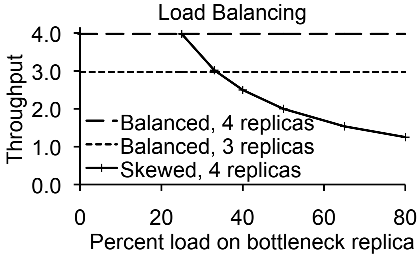


2.7.1 Example

Consider a security application that continuously checks that outgoing messages from a hospital do not reveal confidential patient information. The application uses a natural-language processing operator A to check whether outgoing messages contain text that could reveal confidential information, such as social security numbers or medical conditions, to unauthorized people. Operator A is expensive, and furthermore, its cost varies based on the size and contents of the data items. Since A is expensive, the fission optimization (see Section 2.5) has been applied to create parallel replicas A_1, A_2, and A_3. When one of the replicas is busy with a message that takes a long time to process, but another replica is idle, this optimization sends the next message to the idle replica so it gets processed quickly. In other words, when the load is unevenly distributed, the optimization balances it to improve overall performance.

2.7.2 Profitability

Load balancing is profitable if it compensates for skew. The chart shows the impact of load balancing in an experiment consisting of a Split operator that streams data to 3 or 4 replicated operators. With perfect load balancing, throughput is close to 4 with 4 replicas, and close to 3 with 3 replicas. Without load balancing, there is skew, and throughput is bounded by whichever replica receives the most load. For example, with keyed partitions, this replica might be responsible for data items corresponding to a popular key. If the bottleneck replica receives 33% of the load, then even with a total of 4 replicas, the throughput is only 3.



2.7.3 Safety

Load balancing is safe if the following conditions hold:

- *Avoid starvation.* The work assignment must ensure that every data item eventually gets processed.

- *Ensure each worker is qualified.* If load balancing is done after fission, each replica must be capable of processing each data item. That means replicas must be either stateless or have access to a common shared state.

- *Establish placement safety.* If load balancing is done while placing operators, the safety conditions from Section 2.6 must be met.

2.7.4 Variations

Balancing load while placing operators

StreamIt uses fusion (Section 2.4) and fission (Section 2.5) to balance load at compile-time, by adjusting the granularity of the stream graph to match the target number and capacity of cores [46]. Xing et al. use operator migration to balance load at runtime, by placing operators on different hosts if they tend to experience load spikes at the same time, and vice versa [119]. While Xing et al. focus only on computation cost, Wolf et al. use operator placement at job-submission time to balance both computation cost and communication cost [116]. After placing operators on hosts, their load can be further balanced via priorities [7].

Balancing load while assigning work to operators

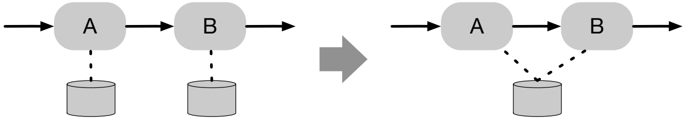
Instead of balancing load by deciding how to arrange the operators, an alternative approach is to first use fission (Section 2.5) to replicate operators, and then balance load by deciding how much streaming data each replica gets to process. The distributed queue component in River [10] offers two approaches for this: in the push-based approach, the producer keeps track of consumer queue lengths, and uses a randomized credit-based scheme for routing decisions, whereas in the pull-based approach, consumers request data when they are ready. Another example for the push-based approach is the use of back-pressure for load balancing in System S [7]. MapReduce [29] uses the pull-based approach. However, Condie et al. argue that the push-based approach is more appropriate when adapting MapReduce to streaming [26]. In MapReduce, as in other systems with fission by keys, the load balance depends on the how evenly the system partitions the data and the skew in the data. Work stealing is an approach for re-arranging work even after it has been pushed or pulled to operators [16].

2.7.5 Dynamism

As the discussion of variations above shows, there are two main techniques for load balancing: based on placement, or based on tuple routing. Roughly speaking, the placement-based variants tend to be static, whereas the routing-based variants are dynamic. Placement has the advantage that it does not necessarily require fission. Placement can be made dynamic too, but that has issues: operator migration causes freeze times; if load spikes are sudden, changing the placement may take too long; and migrating a stateful operator is an engineering challenge [33]. Routing incurs a frequent small overhead for each data item instead of an occasional large overhead for each reconfiguration.

2.8 State Sharing **(a.k.a. synopsis sharing, double-buffering)**

Optimize for space by avoiding unnecessary copies of data.

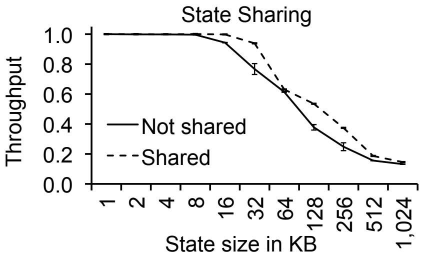


2.8.1 Example

Consider a financial application that continuously computes the volume-weighted average price and other statistics of stocks for both one hour and one day. Assume that the application maintains large windows for each aggregation—enough so that their memory requirements may be substantial fractions of a single host. However, if the only difference between the aggregations is their time granularity, then they can share the same aggregation window, thereby reducing the total amount of memory required for both operators.

2.8.2 Profitability

State sharing is profitable if it reduces stalls due to cache misses or disk I/O, by decreasing the memory footprint. The chart shows the results of an experiment with two operators, both acting on the same stream of data. To provide measurably bad locality, each operator walks a fixed number of randomly selected locations in the state each time it fires. At low state sizes, all state fits in the 32KB L1 cache, and throughput for both versions is high. As the state size increases, the not-shared version does not fit in L1 cache anymore, and its throughput degrades. Eventually, the shared version does not fit in L1 cache anymore either, but both still fit in L2 cache, so the throughput becomes the same again. This phenomenon is repeated at the L2 cache size: the throughput of the not-shared version degrades first, and the throughput of the shared version follows later when it does not fit in L2 cache anymore either.



2.8.3 Safety

State sharing is safe if the following conditions hold:

- *Ensure state is visible to both operators.* The operators that share the state must have common access to it. Typically, this is accomplished by fusion, putting them in the same operating-system process.
- *Avoid race conditions.* State sharing must prevent race conditions, either by ensuring that the data is immutable, or by properly synchronizing accesses.
- *Manage memory safely.* The memory for the shared state is managed properly. It is neither reclaimed too early, nor is it allowed to grow without bounds, i.e., leak.

2.8.4 Variations

State-sharing techniques vary by what kind of state is being shared. We discuss the prominent variations from the literature in order from most general to least general.

Shared operator state

The most general variant deals with operators that have arbitrary non-trivial state. It imposes the most challenging requirements on synchronization and memory management. The straightforward approach is to use shared memory and mutual-exclusion locks. But when conflicts are rare, this may unnecessarily restrict concurrency. Therefore, another approach uses STM (software transactional memory) to manage shared data representing a table or a graph [18].

Shared window

In this variant, multiple consumers can peek into the same window. Even though operators with windows are technically stateful, this is a simple case of state that is easier to share [45]. CQL implements windows by non-shared arrays of pointers to shared data items, such that a single data item might be pointed to from multiple windows and event queues [8].

Shared queue

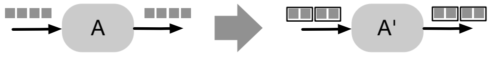
In this variant, the producer can write a new item into a queue at the same time that the consumer reads an old item. To ensure proper synchronization without sacrificing actual concurrency or requiring extra data copies, the queue must have a capacity of at least two data items; therefore, this variant is sometimes called double-buffering. Sermulins et al. show how to further optimize a shared queue, by making it local and computing all offsets at compile-time, so that it can be implemented by scalar variables instead of an array [101]. Once this is done, traditional compiler optimizations can improve the code even further, by allocating queue entries to registers.

2.8.5 Dynamism

We are not aware of a dynamic version of this optimization: the decision whether or not state can be shared is made statically. However, once that decision is made, the implementation techniques can be more or less dynamic. StreamIt uses a fully-static approach, where a static schedule prescribes exactly what data can be accessed by which operator at a what time [101]. Brito et al.'s work is more dynamic, where access to shared state is reconciled by software transactional memory [18].

2.9 Batching (a.k.a. train scheduling, execution scaling)

Process multiple data items in a single batch.

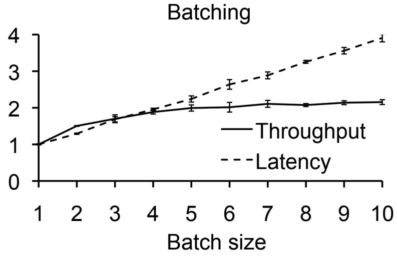


2.9.1 Example

Consider a healthcare application that repeatedly fires an FFT (Fast Fourier Transform) operator for medical imaging. Efficient FFT implementations contain enough code such that instruction cache locality becomes an issue. If the FFT is used as an operator in a larger application together with other operators, batching can amortize the cost of bringing the FFT in cache over multiple data items. In other words, each time the FFT operator fires, it processes a batch of data items in a loop. This will increase latency, because data items are held until the batch fills up. But depending on the application, this latency can be tolerated if it leads to higher fidelity otherwise.

2.9.2 Profitability

Batching trades throughput for latency. Batching can improve throughput by amortizing operator-firing costs over more data items. Such amortizable costs include calls that might be deeply nested; warm-up costs, in particular, for the instruction cache; and scheduling costs, possibly involving a context switch. On the other hand, batching leads to worse latency, because a data item will not be processed as soon as it is available, but only later, when its entire batch is available. The figure shows this trade-off for batch sizes from 1 to 10 data items. For throughput, higher is better; initially, there is a large improvement in throughput, but the throughput curve levels out when the per-batch cost has been amortized. For latency, lower is better; latency increases linearly with batch size, getting worse the larger the batch is.



2.9.3 Safety

Batching is safe if the following conditions hold:

- *Avoid deadlocks.* Batching is only safe if it does not cause deadlocks. Batching can cause deadlock if the operator graph is cyclic. This happens if an operator waits for a number of data items to form a batch, but some of those data items must go around a feedback loop, and the feedback loop is depleted because the operator is waiting. Batching can also cause deadlock if the batched operator shares a lock with an upstream operator. An example is if the batched operator waits for a number of data items to form a batch while holding the lock, thus preventing the upstream operator from sending data items to complete the batch.

- *Satisfy deadlines.* Certain applications have hard real-time constraints, others have quality-of-service (QoS) constraints involving latency. In either case, batching must take care to keep latency within acceptable levels. For instance, video processing must keep up a frame rate to avoid jitter.

2.9.4 Variations

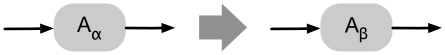
Batching is a streaming optimization that plays well into the hands of more traditional (not necessarily streaming) compiler optimizations. In particular, batching gives rise to loops, and the compiler may optimize these loops with unrolling or with software pipelining [67]. The compiler for a streaming language may even combine the techniques directly [101].

2.9.5 Dynamism

The main control variable in batching is the batch size, i.e., the number of data items per batch. The batch size can be controlled either statically or dynamically. On the static side, *execution scaling* [101] is a batching algorithm for StreamIt that trades the instruction-cache benefits of batching against the data-cache cost of requiring larger buffers. On the dynamic side, *train scheduling* [23] is a batching algorithm for Aurora that amortizes context-switching costs when sharing few cores among many operators, leaving the batch size open. And SEDA [114] uses a *batching controller* that dynamically finds the largest batch size that still exhibits acceptable latency, making the system react to changing load conditions.

2.10 Algorithm Selection (a.k.a. translation to physical query plan)

Use a faster algorithm for implementing an operator.

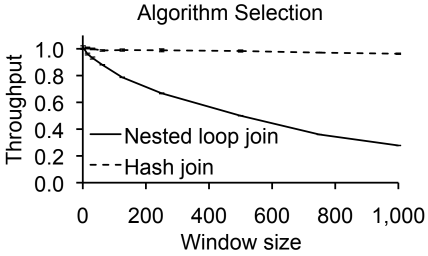


2.10.1 Example

Consider a transportation application that, for tolling purposes, continuously monitors which vehicles are currently on congested road segments (this example is inspired by the Linear Road benchmark [8]). The application joins two input streams: one stream sends, at regular intervals, a table of all congested road segments, and the other stream sends location updates that map vehicles to road segments. A too-obvious implementation would implement every relational join as a nested-loop join A_α. However, in this case, the join checks the equality of road segment identifiers. Therefore, a better join algorithm, such as a hash join A_β, can be chosen.

2.10.2 Profitability

Algorithm selection is profitable if it replaces a costly operator with a cheaper operator. In some cases, neither algorithm is better in all circumstances. For example, algorithm A_α may be faster for small inputs and A_β may be faster for large inputs. In other cases, the algorithms optimize for different metrics. For example, algorithm A_α may be faster but algorithm A_β may use less memory. Finally, there are cases with trade-offs between performance and generality: algorithm A_α may be faster, but algorithm A_β may work in a wider set of circumstances. The chart compares throughput of a nested loop join vs. a hash join. At small window sizes, the performance difference is in the noise, whereas at large window sizes, the hash join clearly performs better. On the other hand, hash joins are less general, since their join condition must be an equality, not an arbitrary predicate.



2.10.3 Safety

Algorithm selection is safe if the following condition holds:

- *Ensure same behavior.* Both operators must behave the same for the given inputs. If algorithm A_α is less general than algorithm A_β, then choosing the operator with A_α instead of A_β is only safe if A_α is general enough for the particular usage. The join example from Section 2.10.1 illustrates this.

2.10.4 Variations

Physical query plans

The motivating example for this section, where the choice is between a nested-loop join and a hash join, is common in database systems. Compilers for databases typically first translate an application (or query) into a graph (or plan) of logical operators, and then translate that to a graph (or plan) of physical operators [41]. The algorithm selection happens during the translation from logical to physical operators. Join operators in particular have many implementation choices; for instance, an index lookup join may speed up join conditions like $a > 5$ with a B-tree. When join conditions get more complex, deciding the best strategy becomes more difficult. A related approach is Sase, which can fuse certain operators with the source operator, and then implement these operators by a different algorithm [118].

Auto-tuners

Outside of streaming systems, there are several successful software packages that perform "empirical optimization". In order to tune itself to a specific hardware platform, the software package automatically runs a set of performance experiments during installation to select the best-performing algorithms and parameters. Prominent examples include Fftw [39], Spiral [120], and Atlas [115]. Yotov et al. compare this empirical optimization approach to more traditional, model-based compiler optimizations [121].

Different semantics

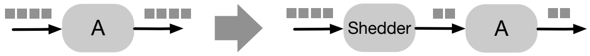
Algorithm selection can be used as a simple form of load shedding. While most approaches to load shedding work by dropping data items (as described in Section 2.11), load shedding by algorithm selection merely switches to a cheaper implementation. Unlike the other variations of algorithm selection, this is, by definition, not safe, because the algorithms are not equivalent. This choice can happen either at job admission time [116], or dynamically, as described below.

2.10.5 Dynamism

When algorithm selection is used to react to runtime conditions, it must be dynamic. In Seda, each operator can decide its own policy for overload, and one alternative is to provide degraded service, i.e., algorithm selection [114]. In Borealis, operators have control inputs, for instance, to select a different algorithm variant for the operator [3]. To implement dynamic algorithm selection, the compiler statically provisions both variants of the algorithm, and the runtime system dynamically picks one or the other as needed. In other words, this approach does for algorithm selection what the Eddy [12] does for operator reordering: it statically inserts a dynamic routing component.

2.11 Load Shedding (a.k.a. admission control, graceful degradation)

Degrade gracefully when overloaded.

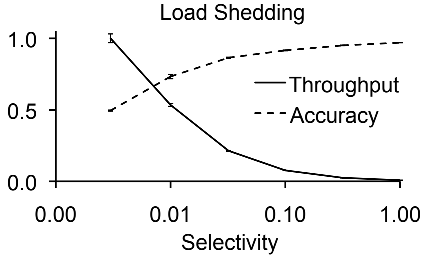


2.11.1 Example

Consider an emergency management application that provides logistics information to police and fire companies as well as to the general public. Under normal conditions, the system can easily keep up with the load, and display information to everyone who asks. However, when disaster strikes, the load can increase by orders of magnitude, and exceed the capacity of the system. Without load shedding, the requests would pile up, and nobody would get timely responses. Instead, it is preferable to shed some of the load by only providing complete and accurate replies to requests from police or fire companies, and degrading accuracy for everyone else.

2.11.2 Profitability

Load shedding improves throughput at the cost of reducing accuracy. Consider an aggregate operator A that constructs a histogram over windows of 1,000 tuples each, for instance, to visualize system state in a graphical dash-board. For each window, it counts each data item as belonging to a "bucket". The selectivity of an operator is the number of output data items per input data item. When there is no load shedding, i.e., when selectivity is 1, the histogram has perfect accuracy, i.e., an accuracy of 1. On the other hand, if the load-shedder only forwards ten out of every thousand data items, i.e., when selectivity is 0.01, the histogram has a lower accuracy. The chart measures accuracy as 1 minus error, where the error is the Pythagorean distance between the actual histogram and the expected histogram.



2.11.3 Safety

Unlike the other optimizations in this paper, load shedding is, by definition, *not* safe. While the other optimizations try to compute the same result as in the unoptimized case, load shedding computes a different, approximate, result; the quality of service of the application will degrade. However, depending on the particular application, this drop in quality may be acceptable. Some applications deal with inherently imprecise data to begin with: for example, sensor readings from the physical world have limited precision. Other applications produce outputs where correctness is not a clear-cut issue: for example, advertisement placement and prioritization. Finally, there are applications that are inherently resilient to imprecision: for example, iterative page-rank computation uses a convergence check [90].

2.11.4 Variations

Load shedding in network applications

Network stacks and web servers are vulnerable to load spikes, and load shedding has been a prime motivator for implementing them as graphs of streams and operators. Receive livelock occurs when input-processing starves downstream processing to the extent that data items must be dropped for the system to make progress [81]. The Scout operating system drops data items early if it can predict that they will miss their deadline [82]. The Click router puts load shedders into their own separate operators to modularize the application [65]. And in the Seda architecture for event-based servers, each operator can elect between different approaches for dealing with overload, by back-pressure, load shedding, or even algorithm selection (see Section 2.10) [114].

Load shedding in relational systems

Papers on load shedders for both Aurora and STREAM observe that in general, shedders should be as close to sources as possible, but in the presence of subgraph sharing (see Section 2.2), shedders may need to be delayed until just after the shared portion [108, 14]. Gedik et al. propose going one step further, by moving the load shedders out of the streaming system entirely and onto the sensors that produce the data in the first place [42].

2.11.5 Dynamism

By definition, load shedding is always applied dynamically.

2.12 Discussion

The previous sections surveyed the major streaming optimizations one by one. A bigger picture emerges when making observations across individual optimizations. This section discusses these observations, puts them in context, and proposes avenues for future research on streaming optimizations.

2.12.1 How to specify streaming applications

Not only is there a large number of streaming languages, there are several language families and other approaches for implementing streaming applications. The programming model is relevant for optimizations, since it influences how and where they apply. The following list of programming models is ordered from low-level to high-level. For conciseness, we only list one representative example for each.

- *Non-streaming language.* This is the lowest-level approach, where the application is written in a traditional language like C or Fortran, and the compiler must do all the work of extracting streams, like in decoupled software pipelining [89].

- *Annotated non-streaming language.* This approach adds pragmas to indicate streams in a traditional language like C or Fortran. An example is Brook [19].

- *Extension to non-streaming language.* This approach adds language features to turn a traditional language like C into a streaming language. An example is Hancock [27].

- *Framework in object-oriented language.* In this approach, an operator is specified as a subclass of a class with abstract event-handling methods. Examples are common in the systems community, e.g., SEDA [114].

- *Graph, specified textually.* Some streaming languages allow the user to specify the stream graph directly in terms of operators and streams. An example is SPADE [42].

- *Graph, specified visually.* Instead of specifying the stream graph textually in a language, some systems, such as Aurora, provide a visual environment for that instead [4].

- *Graph, composed with combinators.* Some streaming languages support graph construction only with a small set of built-in combinators. For example, StreamIt provides three combinators: pipeline, split-join, and feedback loop [45].

- *Queries written in SQL dialect.* The databases community has developed dialects of SQL for streaming, for example, CQL [8].

- *Rules written in Datalog dialect.* There are also dialects of logic languages for streaming, for example, Overlog [74].

- *Patterns compiled to automatons.* The complex event processing community has developed pattern languages, which can be compiled into state machines for detecting events on streams. An example is SASE [118].

- *Tag-based planner.* This is the highest-level approach, where the user merely selects tags, and the system synthesizes an application, as in Mario [96]. The user experience more closely resembles search than programming.

As a rule of thumb, the advantages of low-level approaches are generality (pretty much any application can be expressed) and predictability (the program will perform as the author expects). On the other hand, the advantages of high-level approaches are usability (certain applications can

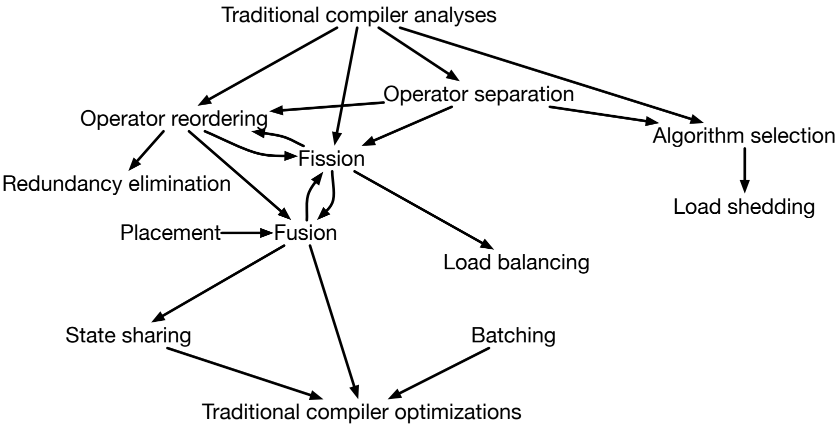


Figure 2.2: Interactions of streaming optimizations with each other and with traditional compilers. An edge from X to Y indicates that X can help to enable Y.

be expressed concisely) and optimizability (the safety conditions are easy to discover). Of course, this rule of thumb is over-simplified, since generality, predictability, usability, and optimizability depend on more factors than whether the programming model is low-level or high-level.

Avenues for future work. For low-level stream programming models, research is needed to make them easier to use and optimize, for example, by providing more powerful analyses. For high-level stream programming models, research is needed to make them more general, and to make it easier for users to understand the performance characteristic of their application after optimization. In Chapters 3 and 4 of this thesis, we explore an intermediate language that allows the same optimization to apply to multiple languages.

2.12.2 How streaming optimizations enable each other

Figure 2.2 sketches the most important ways in which stream processing optimizations enable each other. We defer the discussion of interactions with traditional compiler analyses and optimizations to the next subsection. Among the streaming optimizations, the primary enablers are operator separation and operator reordering. Both also have benefits on their own, but much of their power comes from facilitating other optimizations. There is a circular enablement between operator reordering and fission: operator reordering enables more effective fission by bringing operators together that can be part of the same parallel segment, whereas fission enables reordering of the split and merge operators that fission inserts. In addition, fission makes it easier to balance load, because it introduces data parallelism, which tends to be more homogeneous and malleable than pipeline or task parallelism. A streaming system that implements multiple of these optimizations is well advised to apply them in some order consistent with the direction of the edges in Figure 2.2. It can even make sense to repeatedly attempt optimizations that form an enabling cycle.

Avenues for future work. Finding the right sequence in which to apply optimizations is an interesting problem when there are variants of optimizations with complex interactions. Furthermore, while there is literature with cost models for individual optimizations, extending those to work on multiple optimizations is challenging; in part, that is because the existing cost models are usually sophisticated and custom-tailored for their optimization.

2.12.3 How streaming optimizations interact with traditional compilers

By *traditional compiler*, we refer to compilers for languages such as Fortran, C, C++, or Java. These languages do not have streaming constructs, and rely heavily on functions, loops, arrays, objects, and similar shared-memory control constructs and data structures. Traditional compilers excel at optimizing code written in that style.

The top-most part of Figure 2.2 sketches the most important ways in which traditional compiler analyses can enable streaming optimizations. Specifically:

- *Operator reordering* can be enabled by commutativity analysis [97].
- *Operator separation* can be supported by compiler analysis for decoupled software pipelining (DSWP) [89].
- *Fission* can also be supported by compiler analysis for parallel-stage DSWP [89].
- *Load balancing* can be supported by worst-case execution time (WCET) analysis [71].

That does not mean that without compiler analysis, these optimizations are impossible. To the contrary, many streaming systems apply the optimizations successfully, by using a programming model that is high-level enough to establish certain safety properties by construction.

At the other end, the bottom-most part of Figure 2.2 sketches the most important ways in which streaming optimizations have been used to enable traditional compiler optimizations. Specifically:

- *Fusion* enables function inlining, and that in turn is a core enabler for many other compiler optimizations, such as constant folding and register allocation.
- *State sharing* enables scalar replacement, where an array is implemented by one local variable per array element [101].
- *Batching* enables loop unrolling and/or software pipelining [67].

In each case, the streaming optimization increases the amount of information available to the traditional compiler. Note, however, that this in itself does not automatically lead to improved optimization. Some engineering is usually needed to ensure that the traditional compiler will indeed take advantage of its optimization opportunities [83]. For instance, when the generated code uses pointers or deeply-nested calls, the traditional compiler cannot always establish the safety or profitability of transformations.

Avenues for future work. One fruitful area for research would be new compiler analyses to help enable streaming optimizations in more general cases. Another area of research is in how to communicate source-level information to a low-level compiler for optimization of generated code.

2.12.4 Dynamic optimization for streaming systems

Several streaming optimizations have both static and dynamic variants. Table 2.1 summarizes these variations, and each optimization section has a subsection on dynamism. In general, the advantages of static optimizations are that they can afford to be more expensive; it is easier to make them more comprehensive; and it is easier for them to interact with traditional compilers.

On the other hand, the advantages of dynamic optimizations are that they are more autonomous; they have access to more information to support profitability decisions; they can react to changes in resources or load; and they can even speculate on safety, as long as they have a safe fall-back mechanism. The literature lists some intermediate approaches, which optimize either at application launch time, or periodically re-run a static optimizer at runtime, as in Flextream [57]. This is in contrast to the fully dynamic approach, where the application is transformed for maximum runtime flexibilities, as in Eddies [12].

Avenues for future work. There are several open problems in supporting more dynamic optimizations. One is low-overhead profiling and simple cost models to support profitability trade-offs. Another is the runtime support for dynamic optimization, for instance, efficient and safe migration of stateful operators.

2.12.5 Assumptions, stated or otherwise

Stream processing has become popular in several independent research communities, and these communities have different assumptions that influence the shape and feasibility of streaming optimizations.

Even, predictable, and balanced load. Pretty much all static optimizations make this assumption. On the other hand, other communities, such as the systems community, assume to the contrary that load can fluctuate widely. In fact, that is a primary motivation for two of the optimizations: load balancing and load shedding.

Centralized system. Many optimizations assume shared memory and/or a shared clock, and are thus not directly applicable to distributed streaming systems. This is true for most cases of state sharing, and for dynamic techniques such as changing the number of replicas in fission to adapt to load. Authors of distributed systems tend to emphasize distribution, but it does not always occur to authors of centralized systems to state the centralized assumptions.

Fault tolerance. Many optimizations are orthogonal to whether or not the system is fault tolerant. However, for some optimizations, making them fault tolerant requires significant additional effort. An example is the Flux operator, which makes fission fault tolerant by maintaining hot stand-by operators, and implementing protocols for fault detection, take-over, and catch-up [102].

Avenues for future work. For any optimization that explicitly states or silently makes restrictive assumptions, coming up with a way to overcome the restrictions can be a rewarding research project. Examples include getting a centralized optimization to work (and scale!) in a distributed system, or removing the dependence on fault-tolerance from an optimization.

2.12.6 Metrics for streaming optimization profitability

There are many ways to measure whether a streaming optimization was profitable, including throughput, latency, quality of service (QoS), power, and network utilization. The goals are frequently in-line with each other: many optimizations that improve throughput will also improve the other metrics. For that reason, most of this survey focuses on throughput. Notable exceptions include the trade-off between throughput and latency seen in batching, fission, and operator separation; the trade-off between throughput and QoS or accuracy in load shedding; and the trade-off between throughput and power in fission. As a concrete example for such trade-offs, *slack* refers to the permissible wiggle-room for degrading latency up to a deadline, which can be exploited by a controller to optimize throughput [114].

Avenues for future work. For performance evaluation, standard benchmarks would be a great service to the streaming optimization community. Some examples of existing benchmarking work are: the BiCEP benchmarks [78], the StreamIt benchmarks [110], and the Stanford stream query repository which includes the Linear Road [8] application (§ 4.5). However, more work is needed.

2.13 Requirements for a Streaming IL

In addition to the big picture observations, this catalog helps clarify what information a streaming IL needs to provide in order to support streaming optimizations. Some interesting trends emerge from the discussion of each operator:

- Four of the eleven optimizations (state sharing, fission, load balancing, and placement) have output that depends on the order that the operators execute. Fission and placement are particularly important, since streaming applications must scale across clusters to process large data sets. This indicates that an IL for streaming should be explicit in how deterministic execution is enforced.

- Five of the eleven optimizations (reordering, redundancy elimination, operator separation, fusion, and fission) modify the topology of the data flow graph. Information about operator connectivity (i.e. communication) is, therefore, an important requirement for a streaming IL.

- Eight of the eleven optimizations (redundancy elimination, fission, fusion, separation, reordering, placement, load-balancing, and state sharing) have safety conditions that depend on the state of an operator. This argues that an IL designed for streaming optimizations needs information about operator state.

- Nine of the eleven optimizations (operator separation and state sharing being the exceptions) have a dynamic variation. This fact argues that a streaming IL should provide support for dynamism.

- All eleven of the optimizations have at least one unique safety requirement. This suggests that there is not a finite set of features that an IL can capture to support all optimizations. Rather, the IL needs to be extensible.

In summary, this catalog of optimizations suggests that an intermediate language for stream processing should make it explicit how determinism is enforced, and provide information about operator state and connectivity, as that information is required for a many optimizations. It should also be extensible, so that it can support the unique requirements of each optimization. Chapters 3 and 4 will show how the intermediate language presented in this thesis meets those requirements. However, the catalog also argues that a streaming IL should support dynamic optimizations. Chapter 6 points to this as one of the major limitations in our IL.

2.14 Chapter Summary

This chapter presents a catalog of optimizations for stream processing. It consolidates the extensive prior optimizations work, and also provides a practical guide for users and implementors. The challenge in organizing such a catalog is to provide a framework in which to understand the optimizations. To that end, this chapter is structured in a similar style to catalogs of design patterns or refactoring. This survey establishes a common terminology across the various research communities that have embraced stream processing. This enables members from the different communities to easily understand and apply the optimizations, and lays a foundation for continued research in streaming optimizations. In the overall context of this thesis, this catalog clarifies what information a streaming IL needs to provide in order to support streaming optimizations.

3

The Brooklet Calculus for Stream Processing

The previous chapter identifies some of the key requirements for a streaming IL that emerge from the survey of optimizations: no assumption of deterministic execution; explicit state and communication; extensibility; and support for dynamism. In this chapter, we formalize three of the requirements—non-determinism, state, and communication—in a calculus. We leave support for dynamism to future work. Although there are many formal models for streaming systems [9, 62, 68, 70], none of the prior work incorporates these three requirements.

The Brooklet [1] calculus for stream processing [104] defines a core minimal language that was designed to meet two goals. First, as already mentioned, to enable reasoning about the correctness of optimizations. Second, it was designed to be flexible enough to represent a diverse set of streaming languages. Language designers have developed numerous domain-specific streaming languages [8, 19, 25, 42, 88, 93, 107, 112, 123] that are both tailored to the needs of their particular applications, and optimized for performance on their particular target runtimes. In this chapter, we discuss three prominent examples: CQL, StreamIt, and Sawzall:

- CQL [8] and other StreamSQL dialects [107] are popularly used for algorithmic trading. CQL extends SQL's well studied relational operators with a notion of windows over infinite streams of data, and relies on classic query optimizations [8], such as moving a selection before a join.

- StreamIt [112], a synchronous data-flow language, has been used for MPEG encoding and decoding [34]. The StreamIt compiler enforces static data transfer rates between user-defined operators with fixed topologies, and improves performance through operator fusion, fission, and pipelining [112].

- Sawzall [93], a scripting language for Google's MapReduce [29] platform, is used for web-

[1] Brooklet is so named because it is the essence of a stream, and is unrelated to the Brook language [19].

related analysis. The MapReduce framework streams data items through multiple copies of user-defined *map* operators and then aggregates the results through *reduce* operators on a cluster of workstations. We view Sawzall as a streaming language in the broader sense, and address it in this chapter to showcase the generality of our work.

These three examples by no means comprise an exhaustive list of stream programming languages, but they are representative of the design space.

The challenge in defining a calculus is deciding what parts of a language constitute the core concepts that need to be modeled in the formal semantics, and what details can be abstracted away. The two goals of reasoning optimizations and representing a diverse set of languages guide our design. First, to understand how a language is implemented, we need to understand how the operators communicate on a distributed platform. Therefore, Brooklet makes communication explicit as a core concept. Second, to understand how a language maps to an execution environment, we need to understand how the state embodied in its operational building blocks is implemented on a distributed platform. Therefore, Brooklet makes state explicit as a core concept. Third, to understand how to optimize stream programs, we need to understand how to enable language-level determinism on top of the inherent implementation-level non-determinism of a distributed system. Therefore, Brooklet exposes non-determinism as another core concept.

On the other hand, modeling local deterministic computations is well-understood, so our semantics treat local computations as opaque functions. Since our semantics are small-step, this abstraction loses none of the fine-grained interleaving effects of the distributed computation.

In this chapter we make the following contributions:

- We define a core calculus for stream processing that is general, and facilitates reasoning about program implementation by modeling state, communication, and non-determinism as core concepts.

- We translate CQL, StreamIt, and Sawzall to Brooklet, demonstrating the comprehensiveness of our calculus. This translation also defines the first formal semantics for Sawzall.

- We use our calculus to show the conditions that enable three vital optimizations: operator fission, operator fusion, and operator re-ordering.

This sets a foundation for River (§4), which can serve as a common intermediate language for stream processing with a rigorous formal semantics.

3.1 Notation

Throughout the chapter, an over-bar, as in $\overline{q}$, denotes a finite sequence $q_1, \ldots, q_n$, and the i-th element in that sequence is written q_i, where $1 \leq i \leq n$. The lower-case letter b is reserved for lists, and $\bullet$ is an empty list. A comma indicates *cons* or *append*, depending on the context; for example d, b is a list consed from the first item d and the remaining items b. A bag is a set with duplicates. The notation $\{e : \mathit{condition}\}$ denotes a bag comprehension: it specifies the bag of all e's where the *condition* is true. The symbol $\varnothing$ stands for both an empty set and an empty bag. If E is a store, then the substitution $[v \mapsto d]E$ denotes the store that maps name v to value d and is otherwise identical to E. Angle brackets identify a tuple. For example, $\langle \sigma, \tau \rangle$ is a tuple that contains the elements σ and τ. In inference rules, an expression of the form $d, b = b'$ performs pattern matching; it succeeds if the list b' is non-empty, in which case it binds d to the first element of b' and b to the remainder of b'. Pattern-matching also works on other meta-syntax, such as tuple construction. An underscore character _ indicates a wildcard, and matches anything. Semantics brackets such as $[\![\, P_b \,]\!]^p_z$ indicate translation. The subscripts $_{b,c,s,z}$ stand for Brooklet, CQL, StreamIt, and Sawzall, respectively.

3.2 Brooklet

A stream processing language is a language that hides the mechanics of stream processing; it notably has built-in support for moving data through computations and for composing the computations with each other. Brooklet is a core calculus for such stream processing languages. It is designed to model a diversity of streaming languages, and to facilitate reasoning about language implementation. To achieve these goals, Brooklet models state, communication, and non-determinism as core concepts, and abstracts away local deterministic computations.

Brooklet syntax:

$$
\begin{array}{lcll}
P_b & ::= & out\ in\ \overline{op} & \textit{Brooklet program} \\
out & ::= & \texttt{output}\ \overline{q}\ \texttt{;} & \textit{Output declaration} \\
in & ::= & \texttt{input}\ \overline{q}\ \texttt{;} & \textit{Input declaration} \\
op & ::= & \texttt{(}\ \overline{q}, \overline{v}\ \texttt{)} \leftarrow f\ \texttt{(}\ \overline{q}, \overline{v}\ \texttt{);} & \textit{Operator} \\
q & ::= & id & \textit{Queue identifier} \\
v & ::= & \texttt{\$}\ id & \textit{Variable identifier} \\
f & ::= & id & \textit{Function identifier}
\end{array}
$$

Brooklet example: IBM market maker.

```
output result;
input bids, asks;
(ibmBids) ⟵ SelectIBM(bids);
(ibmAsks) ⟵ SelectIBM(asks);
($lastAsk)⟵ Window(ibmAsks);
(ibmSales)⟵ SaleJoin(ibmBids,$lastAsk);
(result,$cnt) ⟵ Count(ibmSales,$cnt);
```

Brooklet semantics: $F_b \vdash \langle V, Q\rangle \longrightarrow \langle V', Q'\rangle$

$$
\frac{\begin{array}{c} d, b = Q(q_i) \\ op = \texttt{(\_,\_)} \leftarrow f\texttt{(}\overline{q}, \overline{v}\texttt{);} \\ (\overline{b}', \overline{d}') = F_b(f)(d, i, V(\overline{v})) \\ V' = \mathit{updateV}(op, V, \overline{d}') \\ Q' = \mathit{updateQ}(op, Q, q_i, \overline{b}') \end{array}}{F_b \vdash \langle V, Q\rangle \longrightarrow \langle V', Q'\rangle} \quad \text{(E-FireQueue)}
$$

$$
\frac{op = \texttt{(\_,}\overline{v}\texttt{)} \leftarrow f\texttt{(\_,\_);}}{\mathit{updateV}(op, V, \overline{d}) = [\overline{v} \mapsto \overline{d}]V} \quad \text{(E-UpdateV)}
$$

$$
\frac{\begin{array}{c} op = \texttt{(}\overline{q}\texttt{,\_)} \leftarrow f\texttt{(\_,\_);} \\ d_f, b_f = Q(q_f) \\ Q' = [q_f \mapsto b_f]Q \\ Q'' = [\forall q_i \in \overline{q} : q_i \mapsto Q(q_i), b_i]Q' \end{array}}{\mathit{updateQ}(op, Q, q_f, \overline{b}) = Q''} \quad \text{(E-UpdateQ)}
$$

Figure 3.1: Brooklet syntax and semantics.

3.2.1 Brooklet Program Example: IBM Market Maker

As an example of a streaming program, we consider a hypothetical application that trades IBM stock. Data arrives on two input streams, `bids(symbol,price)` and `asks(symbol,price)`, and leaves on the `result(cnt,symbol,price)` output stream. Since the application is only interested in trading IBM stock, it filters out all other stock symbols from the input. The application then matches bid and ask prices from the filtered streams to make trades. To keep the example simple, we assume that each sale is for exactly one share. The Brooklet program in the bottom left corner of Fig. 3.1 produces a stream of trades of IBM stock, along with a count of the number of trades.

3.2.2 Brooklet Syntax

A Brooklet program defines a directed, possibly cyclic, graph of *operators* containing pure *functions* connected by FIFO *queues*. It uses *variables* to explicitly thread state through operators. Data items on a queue model network packets in transit. Data items in variables model stored state; since data items may be lists, a variable may store arbitrary amounts of historical data. The following line from the market maker application defines an operator:

```
(ibmSales) ⟵ SaleJoin(ibmBids, $lastAsk);
```

The operator reads data from input queue `ibmBids` and variable `$lastAsk`. It passes that data as parameters to the pure function `SaleJoin`, and writes the result to the output queue `ibmSales`. Brooklet does not define the semantics of `SaleJoin`. Modeling local deterministic computations is well-understood [85, 91], so Brooklet abstracts them away by encapsulating them in opaque functions. On the other hand, a Brooklet program does define explicit uses of state. In the example, the following line defines a window over the stream `ibmAsks`:

```
($lastAsk) ⟵ Window(ibmAsks);
```

The window contains a single tuple corresponding to the most recent ask for an IBM stock, and the tuple is stored in the variable `$lastAsk`. Both the `Window` and `SaleJoin` operators access `$lastAsk`.

The `Window` operator writes data to `$lastAsk`, but does not use the data stored in the variable in its internal computations. Operators that incrementally update state must both read and write the same variable, such as in the `Count` operator:

```
(result, $cnt) ⟵ Count(ibmSales, $cnt);
```

Queues that appear only as operator input, such as `bids` and `asks`, are program inputs, and queues that appear only as operator output, such as `result`, are program outputs. Brooklet's syntax uses the keywords `input` and `output` to declare a program's input and output queues. We say that a queue is *defined* if it is an operator output or a program input. We say that a queue is *used* if it is an operator input or a program output. Variables may be defined and used in several clauses, since they are intended to thread state through a streaming application. In contrast, each queue must be defined once and used once. This restriction facilitates using our semantics for proofs and optimizations. The complete Brooklet grammar appears in Fig. 3.1.

3.2.3 Brooklet Semantics

A program operates on data items from a domain $\mathcal{D}$, where a data item is a general term for anything that can be stored in queues or variables, including tuples, bags of tuples, lists, or entire relations from persistent storage. Queue contents are represented by lists of data items.

We assume that the transport network is lossless and order-preserving but may have arbitrary delays, so queues support only *push*-to-back and *pop*-from-front operations.

3.2.3.1 Brooklet Execution Configuration.

The function environment F_b maps function names to function implementations. This environment allows us to treat operator functions as opaque. For example, $F_b(\texttt{SelectIBM})$ would return a function that filters out data items whose stock symbol differs from IBM.

At any given time during program execution, the configuration of the Brooklet program is defined as a pair $\langle V, Q\rangle$, where V is a store that maps variable names to data items (in the market maker example, `$cnt` is initialized to zero and `$lastAsk` is initialized to the tuple $\langle\text{'IBM'}, \infty\rangle$), and Q is a store that maps queue names to lists of data items (initially, all queues except the input queues are empty).

3.2.3.2 Brooklet Execution Semantics.

Computation proceeds in small steps. Each step fires Rule E-FireQueue from Fig. 3.1. To explain this rule, we illustrate each line rule one by one, starting with the following intermediate configuration of the market maker example:

$$V = \big[\texttt{\$lastAsk} \mapsto \langle\text{'IBM'}, 119\rangle, \texttt{\$cnt} \mapsto 0\big]$$

$$Q = \begin{bmatrix} \texttt{bids} \mapsto \bullet, & \texttt{ibmBids} \mapsto \big(\langle\text{'IBM'}, 119\rangle, \langle\text{'IBM'}, 124\rangle\big), \\ \texttt{asks} \mapsto \bullet, & \texttt{ibmAsks} \mapsto \bullet, \\ \texttt{ibmSales} \mapsto \bullet, & \texttt{result} \mapsto \bullet \end{bmatrix}$$

$d, b = Q(q_i)$: Non-deterministically select a firing queue q_i. For a queue to be eligible as a firing queue, it must satisfy two conditions: it must be non-empty (because we are binding d, b to its head and tail), and it must appear as an input to some operator (because we are executing that operator's firing function). This step can select any queue satisfying these two conditions.

E.g., $q_i = \texttt{ibmBids}$, $d = \langle\text{'IBM'}, 119\rangle$, $b = \big(\langle\text{'IBM'}, 124\rangle\big)$.

$op = \texttt{(\_,\_)} \leftarrow f\,\texttt{(}\overline{q}, \overline{v}\texttt{);}$: Because of the single-use restriction, q_i uniquely identifies an operator.

E.g., $op =$ `(ibmSales)` $\longleftarrow$ `SaleJoin(ibmBids, $lastAsk);`.

$(\overline{b}', \overline{d}') = F_b(f)(d, i, V(\overline{v}))$: Use the function name to look up the corresponding function from the environment. The function parameters are the data item popped from q_i; the index i relative to the operator's input list; and the current values of the variables in the operator's input list. For each output queue, the function returns a list b'_j of data items to append, and for each output variable, the function returns a single data item d'_j to store.

E.g., $\overline{b}' = \Big((\langle \text{'IBM'}, 119, 119 \rangle) \Big)$, $\overline{d}' = \bullet$,
$d = \langle \text{'IBM'}, 119 \rangle$, $i = 1$, $V(\overline{v}) = \langle \text{'IBM'}, 119 \rangle$.

$V' = \mathit{updateV}(op, V, \overline{d}')$: Update the variables using the output $\overline{d}'$.
E.g., in this example, $\overline{d}' = \bullet$, so $V' = V$.

$Q' = \mathit{updateQ}(op, Q, q_i, \overline{b}')$: Update the queues: remove the popped data item from the firing queue, and for each output queue, push the corresponding list of output data items. The example has only one output queue and datum.

E.g., $Q' = \begin{bmatrix} \texttt{bids} \mapsto \bullet, & \texttt{ibmBids} \mapsto \big(\langle \text{'IBM'}, 124 \rangle \big), \\ \texttt{asks} \mapsto \bullet, & \texttt{ibmAsks} \mapsto \bullet, \\ \texttt{ibmSales} \mapsto \big(\langle \text{'IBM'}, 119, 119 \rangle \big), & \texttt{result} \mapsto \bullet \end{bmatrix}$

3.2.4 Brooklet Execution Function

We denote a program's input $\langle V, Q \rangle$ as I_b and an output $\langle V', Q' \rangle$ as O_b. Given a function environment F_b, program P_b, and input I_b, the function $\rightarrow_b^* (F_b, P_b, I_b)$ yields the set of all final outputs. An execution yields a final output when no queue is eligible to fire. Due to non-determinism, the set may have more than one element. One possible output O_b of our running example is:

$$V = \big[\texttt{\$lastAsk} \mapsto \langle \text{'IBM'}, 119 \rangle, \texttt{\$cnt} \mapsto 1 \big]$$

$$Q = \begin{bmatrix} \texttt{bids} \mapsto \bullet, & \texttt{asks} \mapsto \bullet, & \texttt{ibmSales} \mapsto \bullet, \\ \texttt{ibmBids} \mapsto \bullet, & \texttt{ibmAsks} \mapsto \bullet, & \texttt{result} \mapsto \big(\langle 1, \text{'IBM'}, 119 \rangle \big) \end{bmatrix}$$

The example illustrates the finite case. However, in general, streams are conceptually infinite. To use our semantics in the general case, we use a theoretical result from prior work: if a stream program is computable, then one can generalize from all finite prefixes of an infinite stream to the infinite case [50]. If $\rightarrow_b^*$ yields the same result for all finite inputs to two programs, then we consider these two programs equivalent even on infinite inputs.

3.2.5 Brooklet Summary

Brooklet is a core calculus for stream processing. We designed it to model a diverse set of streaming languages, and to facilitate reasoning about program implementation. Brooklet models communication through explicit queues, thus making it clear where an implementation needs to send data. It models state through explicit variables, thus making it clear where an implementation needs to store data. Finally, Brooklet captures inherent non-determinism by *not* specifying which queue to fire for each step, thus permitting all interleavings possible in a distributed implementation.

3.3 Language Mappings

We demonstrate Brooklet's generality by mapping three streaming languages CQL, StreamIt, and Sawzall to it. Each translation exposes communication as explicit queues; exposes implicit uses of state as explicit variables; exposes a mechanism for implementing global determinism on top of an inherently non-deterministic runtime; and abstracts away local deterministic computations with higher-order wrappers that statically bind the original function and dynamically adapt the runtime arguments (thus preserving small step semantics).

3.3.1 CQL and Stream-Relational Algebra

CQL, the Continuous Query Language, is a member of the StreamSQL family of languages. StreamSQL gives developers who are familiar with SQL's select-from-where syntax an incremental learning path to stream programming. This chapter uses CQL to represent the entire StreamSQL family, because it has a clean design, has made significant impact [8], and has a formal semantics [9].

CQL syntax:

$$
\begin{array}{lcll}
P_c & ::= & P_{cr} \mid P_{cs} & \textit{CQL program} \\
P_{cr} & ::= & & \textit{(Relation query)} \\
 & & \mathit{RName} & \textit{Relation name} \\
 & \mid & \mathit{S2R}(P_{cs}) & \textit{Stream to relation} \\
 & \mid & \mathit{R2R}(\overline{P_{cr}}) & \textit{Relation to relation} \\
P_{cs} & ::= & & \textit{(Stream query)} \\
 & & \mathit{SName} & \textit{Stream name} \\
 & \mid & \mathit{R2S}(P_{cr}) & \textit{Relation to stream} \\
\multicolumn{3}{l}{\mathit{RName} \mid \mathit{SName} ::= \mathit{id}} & \textit{Input name} \\
\multicolumn{3}{l}{\mathit{S2R} \mid \mathit{R2R} \mid \mathit{R2S} ::= \mathit{id}} & \textit{Operator name}
\end{array}
$$

CQL example: Bargain finder.

```
IStream(BargainJoin(Now(quotes), history))
```

CQL program translation: $\llbracket\, F_c, P_c \,\rrbracket^p_c = \langle F_b, P_b \rangle$

$$\llbracket\, F_c, \mathit{SName} \,\rrbracket^p_c = \varnothing, \texttt{output}\,\mathit{SName}\,\texttt{;}\,\texttt{input}\,\mathit{SName}\,\texttt{;}\bullet \qquad (\mathrm{T}^p_c\text{-SName})$$

$$\llbracket\, F_c, \mathit{RName} \,\rrbracket^p_c = \varnothing, \texttt{output}\,\mathit{RName}\,\texttt{;}\,\texttt{input}\,\mathit{RName}\,\texttt{;}\bullet \qquad (\mathrm{T}^p_c\text{-RName})$$

$$\frac{\begin{array}{c} F_b, \texttt{output}\ q_o\texttt{;}\ \texttt{input}\ \overline{q}\texttt{;}\ \overline{op} = \llbracket\, F_c, P_{cs} \,\rrbracket^p_c \\ q'_o = \mathit{freshId}() \qquad v = \mathit{freshId}() \\ F'_b = [\mathit{S2R} \mapsto \mathit{wrapS2R}(F_c(\mathit{S2R}))]F_b \\ \overline{op}' = \overline{op}, (q'_o, v) \leftarrow \mathit{S2R}(q_o, v)\texttt{;} \end{array}}{\llbracket\, F_c, \mathit{S2R}(P_{cs}) \,\rrbracket^p_c = F'_b, \texttt{output}\ q'_o\texttt{;}\ \texttt{input}\ \overline{q}\texttt{;}\ \overline{op}'} \qquad (\mathrm{T}^p_c\text{-S2R})$$

$$\frac{\begin{array}{c} F_b, \texttt{output}\ q_o\texttt{;}\ \texttt{input}\ \overline{q}\texttt{;}\ \overline{op} = \llbracket\, F_c, P_{cr} \,\rrbracket^p_c \\ q'_o = \mathit{freshId}() \qquad v = \mathit{freshId}() \\ F'_b = [\mathit{R2S} \mapsto \mathit{wrapR2S}(F_c(\mathit{R2S}))]F_b \\ \overline{op}' = \overline{op}, (q'_o, v) \leftarrow \mathit{R2S}(q_o, v)\texttt{;} \end{array}}{\llbracket\, F_c, \mathit{R2S}(P_{cr}) \,\rrbracket^p_c = F'_b, \texttt{output}\ q'_o\texttt{;}\ \texttt{input}\ \overline{q}\texttt{;}\ \overline{op}'} \qquad (\mathrm{T}^p_c\text{-R2S})$$

$$\frac{\begin{array}{c} \overline{F_b, \texttt{output}\ q_o\texttt{;}\ \texttt{input}\ \overline{q}\texttt{;}\ \overline{op}} = \overline{\llbracket\, F_c, P_{cr} \,\rrbracket^p_c} \\ n = |\overline{P_{cr}}| \qquad q'_o = \mathit{freshId}() \qquad \overline{q}' = \overline{q}_1, \ldots, \overline{q}_n \\ \forall i \in 1 \ldots n : v_i = \mathit{freshId}() \qquad \overline{op}' = \overline{op}_1, \ldots, \overline{op}_n \\ F'_b = [\mathit{R2R} \mapsto \mathit{wrapR2R}(F_c(\mathit{R2R}))](\cup \overline{F_b}) \\ \overline{op}'' = \overline{op}', (q'_o, \overline{v}) \leftarrow \mathit{R2R}(\overline{q_o}, \overline{v})\texttt{;} \end{array}}{\llbracket\, F_c, \mathit{R2R}(\overline{P_{cr}}) \,\rrbracket^p_c = F'_b, \texttt{output}\ q'_o\texttt{;}\texttt{input}\ \overline{q}'\texttt{;}\overline{op}''} \qquad (\mathrm{T}^p_c\text{-R2R})$$

CQL domains:

$$
\begin{array}{ll}
\tau \in \mathcal{T} & \textit{Time} \\
e \in \mathcal{TP} & \textit{Tuple} \\
\sigma \in \Sigma = \mathrm{bag}(\mathcal{TP}) & \textit{Instantaneous relation} \\
r \in \mathcal{R} = \mathcal{T} \to \Sigma & \textit{Time-varying relation} \\
s \in \mathcal{S} = \mathrm{bag}(\mathcal{TP} \times \mathcal{T}) & \textit{Time-varying stream}
\end{array}
$$

CQL operator signatures:

$$
\begin{array}{ll}
\mathrm{S2R} & : \mathcal{S} \times \mathcal{T} \to \Sigma \\
\mathrm{R2S} & : \Sigma \times \Sigma \to \Sigma \\
\mathrm{R2R} & : \Sigma^n \to \Sigma
\end{array}
$$

CQL operator wrapper signatures:

$$
\begin{array}{ll}
\mathrm{S2R} & : (\Sigma \times \mathcal{T}) \times \{1\} \times \mathcal{S} \to (\Sigma \times \mathcal{T}) \times \mathcal{S} \\
\mathrm{R2S} & : (\Sigma \times \mathcal{T}) \times \{1\} \times \Sigma \to (\Sigma \times \mathcal{T}) \times \Sigma \\
\mathrm{R2R} & : (\Sigma \times \mathcal{T}) \times \{1 \ldots n\} \times (2^{\Sigma \times \mathcal{T}})^n \\
 & \qquad \to (\Sigma \times \mathcal{T}) \times (2^{\Sigma \times \mathcal{T}})^n
\end{array}
$$

CQL operator wrappers:

$$\frac{\begin{array}{c} \sigma, \tau = d_q \qquad s = d_v \\ s' = s \cup \{\langle e, \tau \rangle : e \in \sigma\} \qquad \sigma' = f(s', \tau) \end{array}}{\mathit{wrapS2R}(f)(d_q, \_, d_v) = \langle \sigma', \tau \rangle, s'} \qquad (\mathrm{W}_c\text{-S2R})$$

$$\frac{\sigma, \tau = d_q \qquad \sigma' = d_v \qquad \sigma'' = f(\sigma, \sigma')}{\mathit{wrapR2S}(f)(d_q, \_, d_v) = \langle \sigma'', \tau \rangle, \sigma} \qquad (\mathrm{W}_c\text{-R2S})$$

$$\frac{\begin{array}{c} \sigma, \tau = d_q \qquad d'_i = d_i \cup \{\langle \sigma, \tau \rangle\} \\ \forall j \neq i \in 1 \ldots n : d'_j = d_j \\ \exists j \in 1 \ldots n : \nexists \sigma : \langle \sigma, \tau \rangle \in d_j \end{array}}{\mathit{wrapR2R}(f)(d_q, i, \overline{d}) = \bullet, \overline{d}'} \qquad (\mathrm{W}_c\text{-R2R-Wait})$$

$$\frac{\begin{array}{c} \sigma, \tau = d_q \qquad d'_i = d_i \cup \{\langle \sigma, \tau \rangle\} \\ \forall j \neq i \in 1 \ldots n : d'_j = d_j \\ \forall j \in 1 \ldots n : \sigma_j = \mathit{aux}(d_j, \tau) \end{array}}{\mathit{wrapR2R}(f)(d_q, i, \overline{d}) = \langle f(\overline{\sigma}), \tau \rangle, \overline{d}'} \qquad (\mathrm{W}_c\text{-R2R-Ready})$$

$$\frac{\langle \sigma, \tau \rangle \in d}{\mathit{aux}(d, \tau) = \sigma} \qquad (\mathrm{W}_c\text{-R2R-Aux})$$

Figure 3.2: CQL semantics on Brooklet.

3.3.1.1 CQL Program Example: Bargain Finder.

A CQL program P_c is a query that computes a stream or relation from other streams or relations. The following hypothetical example uses CQL for algorithmic trading:

```
select IStream(*) from quotes[Now], history
  where quotes.ask <= history.low and quotes.ticker == history.ticker
```

This program finds bargain quotes, whose ask price is lower than the historic low. The program has two inputs, a stream `quotes` and a time-varying relation `history`. A *stream* in CQL is a bag of time-tagged tuples. The same information can be more conveniently represented as a mapping from time stamps to bags of tuples. CQL calls such a mapping a *time-varying relation*, and each individual bag of tuples an *instantaneous relation*. In the example, input `history(ticker,low)` is the time-varying relation r_h:

$$r_h = \Big[1 \mapsto \big\{\langle\text{‘IBM’}, 119\rangle, \langle\text{‘XYZ’}, 38\rangle\big\}, 2 \mapsto \big\{\langle\text{‘IBM’}, 119\rangle, \langle\text{‘XYZ’}, 35\rangle\big\}\Big]$$

The instantaneous relation $r_h(1)$ is $\{\langle\text{‘IBM’}, 119\rangle, \langle\text{‘XYZ’}, 38\rangle\}$. The CQL stream s_q represents the input `quotes(ticker,ask)`:

$$s_q = \Big\{\langle\langle\text{‘IBM’}, 119\rangle, 1\rangle, \langle\langle\text{‘IBM’}, 124\rangle, 1\rangle, \langle\langle\text{‘XYZ’}, 35\rangle, 2\rangle, \langle\langle\text{‘IBM’}, 119\rangle, 2\rangle\Big\}$$

The subquery `quotes[Now]` uses the window `[Now]` to turn the `quotes` stream into a time-varying relation r_q:

$$r_q = \Big[1 \mapsto \big\{\langle\text{‘IBM’}, 119\rangle, \langle\text{‘IBM’}, 124\rangle\big\}, 2 \mapsto \big\{\langle\text{‘XYZ’}, 35\rangle, \langle\text{‘IBM’}, 119\rangle\big\}\Big]$$

The next step of the query joins the quote relation r_q with the history relation r_h into a bargains relation r_b:

$$r_b = \Big[1 \mapsto \big\{\langle\text{‘IBM’}, 119, 119\rangle\big\}, 2 \mapsto \{\langle\text{‘XYZ’}, 35, 35\rangle, \langle\text{‘IBM’}, 119, 119\rangle\}\Big]$$

Finally, the `IStream` operator monitors insertions into relation r_b and emits them as output stream s_o of time-tagged tuples:

$$s_o = \Big\{\langle\langle\text{‘IBM’}, 119, 119\rangle, 1\rangle, \langle\langle\text{‘XYZ’}, 35, 35\rangle, 2\rangle\Big\}$$

While CQL uses select-from-where syntax, the CQL semantics use an equivalent stream-relational algebra syntax (similar to relational algebra in databases):

```
IStream(BargainJoin(Now(quotes), history))
```

This algebraic notation makes the operator tree clearer. The leaves are stream name `quotes` and relation name `history`. CQL has three categories of operators. S2R operators turn a stream into a relation; e.g., `Now(quotes)` turns stream `quotes` into relation r_q. R2R operators turn one or more relations into a new relation; e.g., `BargainJoin(`r_q, r_h`)` turns relations r_q and r_h into the bargain relation r_b. Finally, R2S operators turn a relation into a stream; e.g., `IStream(`r_b`)` turns relation r_b into the stream of its insertions. CQL has no S2S operators, because they would be redundant. CQL's R2R operators coincide with traditional database relational algebra.

The CQL grammar is in Fig. 3.2. A CQL program P_c can be either a relation query P_{cr} or a stream query P_{cs}, and queries are either simple identifiers RName or SName, or composed using operators from the categories S2R, R2R, or R2S.

3.3.1.2 CQL Implementation Issues.

Before we translate CQL to Brooklet, let us discuss the three issues of communication, state, and non-determinism in CQL.

CQL communication. In CQL, communication between operators is implicit. The communication becomes explicit by translating a CQL query into its relational algebra equivalent.

CQL state. CQL represents global state explicitly as named relations, such as the `history` relation from our running example. But in addition, all three kinds of CQL operators implicitly maintain local state, referred to as "synopses" in [8]. An S2R operator maintains the state of a window on a stream to produce a relation. An R2S operator stores the previous state of the relation to compute the stream of differences. Finally, an R2R operator uses state to buffer data from whichever relation is available first, so it can be retrieved later to compute an output when data with matching time stamps is available for all relations.

CQL non-determinism. CQL is deterministic in the sense that the output of a program is fully determined by the times and values of its inputs [9]. CQL requires that data arrive in order. That is, an operator never receives a data item with a lower time stamp than previously received item. Time stamps may be assigned to inputs in two ways. First, a CQL implementation might assign time stamps to data using its own system time. In this case, there is no difficulty in satisfying the ordering requirement. Second, an application might include time stamps as part of its data, such as quotes arriving from a stock exchange. Time stamps from an application may have ambiguities, such as result from unsynchronized application clocks or non-order preserving data transmissions. In this case, a CQL implementation must resolve the ambiguities and ensure the proper ordering of data before it is input to the streaming system [105]. However, a CQL implementation does not fully determine the order in which operators execute. Thus, a CQL implementations can permit non-determinism to exploit parallelism. For example, in the query `BargainJoin(Now(quotes), history)`, the operators `Now` and `BargainJoin` can run in parallel in separate processes, as long as `BargainJoin` always waits for its two inputs to have the same time stamp.

Translation to Brooklet will make all communication and state explicit, and will clarify how the implementation enforces determinism.

3.3.1.3 CQL Translation Example.

Given the CQL example program from Fig. 3.2, the translation to Brooklet is the program P_b:

```
output q_o;
input quotes, history;
(q_q, $v_n)         ⟵ wrapNow(quotes, $v_n);
(q_b, $v_q, $v_h) ⟵ wrapBargainJoin(q_q, history, $v_q, $v_h);
(q_o, $v_o)         ⟵ wrapIStream(q_b, $v_o)
```

The leaves of the query tree serve as input queues; each subquery produces an intermediate queue, which the enclosing operator consumes; and the outermost query operator produces the program output queue. The translation to Brooklet makes the state of the operators explicit. The most interesting state is that of the `wrapBargainJoin` operator. Like each R2R operator, it has a function F_c(`BargainJoin`) that transforms one or more input instantaneous relations of the same time stamp to one output instantaneous relation. Brooklet models the choice of interleavings by allowing either queue q_q or `history` to fire independently. Hence, the Brooklet operator processes one data item each time either queue fires. Assume a data item arrives on the first queue $\mathtt{q_q}$. If there is already a data item with the same time stamp in the variable $\mathtt{v_h}$ associated with the second queue, Brooklet performs the join, which may yield data items for the output queue $\mathtt{q_b}$. Otherwise, it simply stores the data item in $\mathtt{v_q}$ for later.

3.3.1.4 CQL Translation.

Fig. 3.2 shows the translation from CQL to Brooklet by recursion over the input program. Besides building up a program, the translation also builds up a function environment, which it populates with wrappers for the original functions. The translation introduces state, which the Brooklet wrappers maintain and consult to hand the right input to the wrapped CQL functions. Working in concert, the rules enforce a global convention: the execution sends exactly one instantaneous relation on every queue at every time stamp. Operators retain historical data in variables, e.g., to implement windows.

3.3.1.5 CQL Discussion.

CQL is an SQL dialect for streaming [8]. Arasu and Widom specify big-step denotational semantics for CQL [9]. We show how to translate CQL to Brooklet, thus giving an alternative semantics. As we will show below, both semantics define equivalent input/output behavior for CQL programs. Translations from other languages can use similar techniques, i.e., make state explicit as variables; wrap computation in small-step firing functions; and define a global convention for how to achieve determinism.

3.3.2 StreamIt and Synchronous Data Flow

StreamIt [113, 112] is a streaming language tailored for parallel implementations of applications such as MPEG decoding [34]. At its core, StreamIt is a synchronous data flow (SDF) language [70], which means that each time an operator fires, it consumes a fixed number of data items and produces a fixed number of data items. In the MPEG example, data items are pictures. StreamIt distinguishes between primitive and composite operators. A primitive operator (*filter* in StreamIt terminology) has optional local state. A composite operator is either a pipeline, a split-join, or a feedback loop. A pipeline puts operators in sequence, a split-join puts them in parallel, and a feedback loop puts them in a cycle. The topology of a StreamIt program is restricted to well-nested compositions of these. All StreamIt operators and programs have exactly one input and one output. We only focus on StreamIt's SDF core here, and encapsulate the local deterministic part of the computation in opaque pure functions, while keeping the parts of the computation that are relevant to streaming. We omit non-core features such as teleport messaging [34], which delivers control messages between operators and which could be modeled in Brooklet through shared variables.

3.3.2.1 StreamIt Program Example: MPEG Decoder.

The following example StreamIt program P_s is based on a similar example by Drake et al. [34].

```
pipeline {
  splitjoin {
    split roundrobin;
    filter { work { tf ← FrequencyDecode(peek(1)); push(tf); pop(); }}
    filter { work { tm ← MotionVecDecode(peek(1)); push(tm); pop(); }}
    join roundrobin;
  }
  filter { s; work { s,tc ← MotionComp(s,peek(1)); push(tc); pop(); }}
}
```

It illustrates how the StreamIt language can be used to decode MPEG video. The example uses a pipeline and a split-join to compose three filters. Each filter has a work function, which peeks and pops from its predecessor stream, computes a temporary value, and pushes to its successor stream. In addition, the `MotionComp` filter also has an explicit state variable `s` for storing a reference picture between iterations. The full syntax of Streamit is in Appendix B.

3.3.2.2 StreamIt Implementation Issues.

As before, we first discuss the intuition for the implementation before giving the details of the translation.

StreamIt communication. Communication between primitive operators in StreamIt is determined by how they are placed in composite operators. All operators are arranged hierarchally in either a pipeline, a split-join, or a feedback loop. Translation to Brooklet flattens the hierarchy.

StreamIt state. Filters can have explicit state, such as `s` in the example. Furthermore, since Brooklet queues support only push and pop but not peek, the translation of StreamIt will have to buffer data items in a state variable until enough are available to satisfy the maximum `peek()` argument in the work function. Round-robin splitters also need a state variable with a *cursor* that determines where to send the next data item. A cursor is simply an index relative to the splitter. It keeps track of which queue is next in round-robin order. Round-robin joiners also need a cursor, plus a buffer for any data items that arrive out of turn.

StreamIt non-determinism. StreamIt, at the language level, is deterministic. Furthermore, since it is an SDF language, the number of data items peeked, popped, and pushed by each operator is constant. At the same time, StreamIt permits pipeline-, task-, and data-parallelism. This gives an implementation different scheduling choices, which Brooklet models by non-deterministically selecting a firing queue. Despite these non-deterministic choices, an implementation must ensure deterministic end-to-end behavior, which our translation makes explicit with buffering and synchronization.

3.3.2.3 StreamIt Translation Example.

StreamIt program translation turns the StreamIt MPEG decoder P_s from earlier into a Brooklet program P_b:

```
output q_out;
input q_in;
(q_f, q_m, $sc)         ⟵ wrapRRSplit-2(q_in, $sc);
(q_fd, $f)              ⟵ wrapFilter-FrequencyDecode(q_f, $f);
(q_md, $m)              ⟵ wrapFilter-MotionVecDecode(q_m, $m);
(q_d, $fd, $md, $jc)    ⟵ wrapRRJoin-2(q_fd, q_md, $fd, $md, $jc);
(q_out, $s, $mc)        ⟵ wrapFilter-MotionComp(q_d, $s, $mc);
```

Each StreamIt filter becomes a Brooklet operator. StreamIt composite operators are reflected in Brooklet's operator topology. StreamIt's SplitJoin yields separate Brooklet split and join operators. The stateful filter `MotionComp` has two variables: `$s` models its explicit state `s`, and `$mc` models its implicit buffer.

3.3.2.4 StreamIt Translation.

StreamIt program xlation excerpt:

$$\frac{\begin{array}{c} f = \mathit{freshId}() \\ v = \mathit{freshId}() \\ F_b = [f \mapsto \mathit{wrapRRSplit}(|\overline{q}|)] \\ \mathit{op} = (\overline{q}, v) \leftarrow f(q_a, v)\,; \end{array}}{[\![\, F_s, \texttt{split roundrobin;}, \overline{q}, q_a \,]\!]^p_s = F_b, \mathit{op}} \quad (\mathrm{T}^p_s\text{-RR-S}\textsc{plit})$$

$$\frac{\begin{array}{c} f = \mathit{freshId}() \\ \forall i \in 0 \ldots |\overline{q}'| : v_i = \mathit{freshId}() \\ F_b = [f \mapsto \mathit{wrapRRJoin}(|\overline{q}'|)] \\ \mathit{op} = (q_z, \overline{v}) \leftarrow f(\overline{q}', \overline{v})\,; \end{array}}{[\![\, F_s, \texttt{join roundrobin;}, q_z, \overline{q}' \,]\!]^p_s = F_b, \mathit{op}} \quad (\mathrm{T}^p_s\text{-RR-J}\textsc{oin})$$

StreamIt operator wrappers excerpt:

$$\frac{\begin{array}{c} c' = c + 1 \bmod N \qquad b_v = d_{in} \\ \forall i \in 1 \ldots N, i \neq c : b_i = \bullet \end{array}}{\mathit{wrapRRSplit}(N)(d_{in}, \_, c) = \overline{b}, c'} \quad (\mathrm{W}_s\text{-RR-S}\textsc{plit})$$

$$\frac{\begin{array}{c} d'_i = d_{in}, d_i \qquad \forall j \neq i \in 1 \ldots N : d'_j = d_j \\ d''_c, d_{out} = d'_c \qquad \forall j \neq c \in 1 \ldots N : d''_j = d'_j \\ b_{out}, c', \overline{d}''' = \mathit{wrapRRJoin}(N)(\bullet, i, c + 1 \bmod N, \overline{d}'') \end{array}}{\mathit{wrapRRJoin}(N)(d_{in}, i, c, \overline{d}) = (b_{out}, d_{out}), c', \overline{d}'''} \quad (\mathrm{W}_s\text{-RR-J}\textsc{oin}\text{-R}\textsc{eady})$$

$$\frac{\forall j \neq i \in 1 \ldots N : d'_j = d_j \qquad d'_i = d_{in}, d_i \qquad d_c = \bullet}{\mathit{wrapRRJoin}(N)(d_{in}, i, c, \overline{d}) = \bullet, c, \overline{d}'} \quad (\mathrm{W}_s\text{-RR-J}\textsc{oin}\text{-W}\textsc{ait})$$

Figure 3.3: StreamIt round-robin split and join semantics on Brooklet.

We give only a high-level overview of the StreamIt translation here (the details are in Appendix B). Similarly to CQL, there are recursive translation rules, one for each language construct. The base case is the translation of filters, and the recursive cases compose larger topologies for pipelines, split-joins, and feedback loops. Feedback loops turn into cyclic Brooklet topologies. The most interesting aspect are the helper rules for split and join, because they use explicit Brooklet state to achieve StreamIt determinism. Fig. 3.3 shows the rules. The input to the splitter is a queue q_a, and the output is a list of queues $\overline{q}$; conversely, the input to the joiner is a list of queues $\overline{q}'$, and the output is a single queue q_z. Both the splitter and the joiner maintain a cursor to keep track of the next queue in round-robin order. The joiner also stores one variable for each queue, to buffer data that arrives out-of-turn.

3.3.2.5 StreamIt Discussion.

Our translation from StreamIt to Brooklet yields a program with maximum scheduling flexibility, allowing any interleavings as long as the end-to-end behavior matches the language semantics.

This makes it amenable to distributed implementation. In contrast, StreamIt compilers [112] statically fix one schedule, which also determines where intermediate results are buffered. The buffering is implicit state, and StreamIt also has explicit state in filters. As we will see in Section 3.4, state affects the applicability of optimizations. Prior work on formal semantics for StreamIt does not model state [113]. By modeling state, our Brooklet translation facilitates reasoning about optimizations.

3.3.3 Sawzall and MapReduce

Sawzall syntax:

P_z	$::=$	$\overline{out}\ in\ \overline{emit}$	*Sawzall program*
out	$::=$	t `: table` f`;`	*Output aggregator*
in	$::=$	q `: input;`	*Input declaration*
$emit$	$::=$	`emit` t`[`f`(`q`)]` $\leftarrow$ f`(`q`);`	*Emit statement*
q	$::=$	id	*Queue name*
f	$::=$	id	*Function name*
t	$::=$	id	*Table name*

Sawzall example: Query log analyzer.

```
queryOrigins :  table sum;
queryTargets :  table sum;
logRecord :  input;
emit queryOrigins[getOrigin(logRecord)]⟵1;
emit queryTargets[getTarget(logRecord)]⟵1;
```

Sawzall program xlation: $[\![\, F_z, P_z, R \,]\!]^p_z = \langle F_b, P_b \rangle$

$$\frac{\begin{array}{c} \overline{out}, q_{in}\texttt{: input;}, \overline{emit} = P_z \\ \forall i \in 1\ldots R : q_i = \mathit{freshId}() \\ \forall i \in 1\ldots R : v_i = \mathit{freshId}() \\ f_{\texttt{Map}} = \mathit{wrapMap}(F_z, \overline{emit}, R) \\ f_{\texttt{Reduce}} = \mathit{wrapReduce}(F_z, \overline{out}) \\ F_b = [\texttt{Map} \mapsto f_{\texttt{Map}},\ \texttt{Reduce} \mapsto f_{\texttt{Reduce}}] \\ op_m = \texttt{(}\overline{q}\texttt{)} \leftarrow \texttt{Map(}q_{in}\texttt{);} \\ \forall i \in 1\ldots R : op_i = \texttt{(}v_i\texttt{)} \leftarrow \texttt{Reduce(}q_i\texttt{,}v_i\texttt{);} \\ \overline{op}' = op_m, \overline{op} \end{array}}{[\![\, F_z, P_z, R \,]\!]^p_z = F_b, \texttt{output }\ \bullet\ \texttt{;input }\ q_{in}\texttt{;}\overline{op}'} \quad (\mathrm{T}^p_z)$$

Sawzall domains:

$k_1 \in \mathcal{K}_1$	*Input key*	$k_2 \in \mathcal{K}_2$	*Output key*
$x_1 \in \mathcal{X}_1$	*Input value*	$x_2 \in \mathcal{X}_2$	*Output value*
$t \in \mathcal{T}$	*Aggregate name*	$O_z \in \mathcal{K}_2 \to \mathcal{X}_2$	*Output table*

Sawzall operator signatures:

$f_k : \mathcal{K}_1 \times \mathcal{X}_1 \to \mathcal{K}_2$	$f_x : \mathcal{K}_1 \times \mathcal{X}_1 \to \mathcal{X}_2^*$
$f_a : \mathcal{X}_2 \times \mathcal{X}_2 \to \mathcal{X}_2$	

Sawzall operator wrapper signatures:

$$\begin{array}{ll} \text{Map} & : (\mathcal{K}_1 \times \mathcal{X}_1) \times \{1\} \to (\mathcal{T} \times \mathcal{K}_2 \times \mathcal{X}_2)^* \\ \text{Reduce} & : (\mathcal{T} \times \mathcal{K}_2 \times \mathcal{X}_2) \times \{1\} \times O_z \to O_z \end{array}$$

Sawzall operator wrappers:

$$\frac{\begin{array}{c} \texttt{emit }\ t\texttt{[}f_k\texttt{(\_)]} \leftarrow f_x\texttt{(\_);} = emit \\ \overline{b} = \mathit{wrapMap}(F_z, \overline{emit}, R)(d, 1) \\ k_1, x_1 = d \qquad k_2 = F_z(f_k)(k_1, x_1) \\ \overline{x_2} = F_z(f_x)(k_1, x_1) \qquad i = \mathit{hash}(k_2) \bmod R \\ b'_i = b_i, \langle t, k_2, x_{2_1} \rangle, \ldots, \langle t, k_2, x_{2_n} \rangle \\ \forall j \neq i \in 1\ldots R : b'_j = b_j \end{array}}{\mathit{wrapMap}(F_z, (emit, \overline{emit}), R)(d, \_) = \overline{b}'} \quad (\mathrm{W}_z\text{-}\textsc{Map})$$

$$\frac{\forall i \in 1\ldots R : b_i = \bullet}{\mathit{wrapMap}(F_z, \bullet, R)(\_, \_) = \overline{b}} \quad (\mathrm{W}_z\text{-}\textsc{Map}\text{-}\bullet)$$

$$\frac{\begin{array}{c} t, k_2, x_2 = d_q \qquad t\ \texttt{: table }\ f_a\texttt{[];} \in \overline{out} \\ k_2 \in d_v \qquad x'_2 = F_z(f_a)(x_2, d_v(k_2)) \\ d'_v = [k_2 \mapsto x'_2]d_v \end{array}}{\mathit{wrapReduce}(F_z, \overline{out})(d_q, \_, d_v) = d'_v} \quad (\mathrm{W}_z\text{-}\textsc{Reduce})$$

$$\frac{\begin{array}{c} t, k_2, x_2 = d_q \qquad t\ \texttt{: table }\ f_a\texttt{[];} \in \overline{out} \\ k_2 \notin d_v \qquad d'_v = [k_2 \mapsto x_2]d_v \end{array}}{\mathit{wrapReduce}(F_z, \overline{out})(d_q, \_, d_v) = d'_v} \quad (\mathrm{W}_z\text{-}\textsc{Reduce}\text{-}\varnothing)$$

Figure 3.4: Sawzall semantics on Brooklet.

Sawzall [93] is a scripting language for MapReduce [29], which exploits cluster of workstations to analyze a massive but finite sequence of key/value pairs streamed from disk. In Sawzall, a stateless *map* operator transforms data one key/value pair at a time, feeding into a stateful *reduce* operator. The reduce operator works on separate keys separately, incrementally aggregating all values for a key into a single value. Although Sawzall programs are batch jobs, they use incremental operators to process large quantities of data in a single pass, and we therefore consider it a streaming language. Our translation provides the first formal semantics for Sawzall.

In MapReduce, computation proceeds in four stages. First, each map operator writes its output to its own local disk as a set of key/value pairs. Second, a *shuffle* stage copies all the data for a particular key from all the mappers to the local disk of the reducer responsible for that key. Third, a *sort* stage sorts the data for that key. Lastly, the reducer aggregates the sorted data.

Our translation of Sawzall produces a version of the MapReduce paradigm that is more consistent with a streaming model, and is closer to MapReduce online [26] than to traditional MapReduce. There is no shuffle stage. Instead, the mapper sends data directly to the appropriate reducer. There is no sort stage. Data is processed as it arrives, not in sorted order. Consequently, the reduce function must be associative and commutative. Finally, the reducer outputs results incrementally, rather than producing a single result based on the entire data set. We assume that the number of reducers is set ahead of time to a number R, which is consistent with the description in Pike et al.'s paper [93]. A streaming model allows for increased pipelining and resource utilization.

3.3.3.1 Sawzall Program Example: Query Log Analyzer.

The example Sawzall program in Fig. 3.4 is based on a similar example in [93]. The program analyzes a query log to count queries per latitude and longitude, which can then be plotted on a world map. This program specifies one invocation of the map operator, and uses `table` clauses to specify `sum` as the reduce operator. The map operator transforms its input `logRecord` into two key/value pairs:

$$\begin{aligned}\langle k, x\rangle\ &= \langle \texttt{getOrigin(logRecord)}, 1\rangle \\ \langle k', x'\rangle &= \langle \texttt{getTarget(logRecord)}, 1\rangle\end{aligned}$$

Here, `getOrigin` and `getTarget` are pure functions that compute the latitude and longitude of the host issuing the query and the host serving the result, respectively. The latitude and longitude together serve as the key into the tables. Since the number 1 serves as the value associated with the key, the `sum` aggregators end up counting query log entries by key. Fig. 3.4 shows the Sawzall grammar.

3.3.3.2 Sawzall Implementation Issues.

All Sawzall programs have the same topology, and have stateful and non-deterministic implementations.

Sawzall communication. The Sawzall translation is simpler than that of CQL or StreamIt, because each translated program uses the same simple topology. The implementation in Pike et al.'s paper [93] partitions the reducer key space into R parts, where R is a command-line argument upon job submission. There are multiple instances of the reduce operator, one per partition. Thus, the translation hard-codes the data parallelism for the reducers, but generates only one mapper, deferring data parallelism for mappers to a separate optimization step. Our translation does not have a shuffle step. Rather, the mapper sends data to the appropriate reducer directly.

Sawzall state. The map operator is stateless, whereas the reduce operator is stateful, using state to incrementalize its aggregation. Because reduction works independently per key, each instance of the reduce operator can maintain the state for its assigned part of the key space independently.

Sawzall non-determinism. At the language level, Sawzall is deterministic. Sawzall is designed for MapReduce, and the strength of MapReduce is that at the implementation level, it runs on a cluster of workstations for scalability. To exploit the parallelism of the cluster, at the implementation level, MapReduce makes non-deterministic dynamic scheduling decisions. Reducers can start while map is still in process, and different reducers can work in parallel with each other. Different mappers can also work in parallel; we will use Brooklet to address this optimization later in the chapter, and describe a translation with a single map operator for now.

3.3.3.3 Sawzall Translation Example.

Given the Sawzall program P_z from earlier, assuming $R = 4$ partitions, the Brooklet version P_b is:

```
output; /*no output queue, outputs are in variables*/
input qlog;
(q1, q2, q3, q4) ⟵ Map(qlog); /*getOrigin/getTarget*/
($v1) ⟵ Reduce(q1, $v1);
($v2) ⟵ Reduce(q2, $v2);
($v3) ⟵ Reduce(q3, $v3);
($v4) ⟵ Reduce(q4, $v4);
```

There is one reduce operator for each of the R partitions. Each reducer performs the work for both aggregators (`queryOrigins` and `queryTargets`) from the original Sawzall program. The final reduction results are in variables $\$\mathtt{v}_1 \ldots \$\mathtt{v}_4$.

3.3.3.4 Sawzall Translation.

Fig. 3.4 specifies the program translation, domains, and operator wrappers. There is only one program translation rule T_z^p. The translation $[\![F_z, P_z, R]\!]_z^p$ takes the Sawzall function environment, the Sawzall program, and the number of reducer partitions as arguments. All the $\overline{emit}$ statements become part of the single map operator. The map operator wrapper uses a hash function to scatter its output over the reducer key space for load balancing. All the $\overline{out}$ declarations become part of each of the reduce operators. Each reducer's variable stores the mapping from each key in that reducer's partition to the latest reduction result for that key. If the key is new, rule W$_z$-Reduce-∅ fires and registers x_2 as the initial value. At the end of the run, the results in the variables are deterministic, because aggregators are associative and reducers work on disjoint parts of the key space.

3.3.3.5 Sawzall Discussion.

There was no prior formal semantics for Sawzall, but Lämmel studies MapReduce and Sawzall by implementing an emulation in Haskell [68]. Now that we have seen how to translate three languages, it is clear that it is possible to model additional streaming languages or language features on Brooklet. For example, Brooklet can serve as a basis for modeling teleport messaging [34] by using shared variables for out-of-stream communication between operators.

3.3.4 Translation Correctness

We formulate correctness theorems for CQL and StreamIt with respect to their formal semantics [9, 113]. The proofs are in Appendix A and C. We do not formulate a theorem for Sawzall, because it lacks formal semantics; our mapping to Brooklet provides the first formal semantics for Sawzall.

Theorem 3.1 (CQL translation correctness)**.** *For all CQL function environments F_c, programs P_c, and inputs I_c, the results under CQL semantics are the same as the results under Brooklet semantics after translation* $[\![F_c, P_c]\!]^p_c$.

Theorem 3.2 (StreamIt translation correctness)**.** *For all StreamIt function environments F_s, programs P_s, and inputs I_s, the results under StreamIt semantics are the same as the results under Brooklet semantics after translation* $[\![F_s, P_s]\!]^p_s$.

3.4 Optimizations

The previous section used our calculus to understand how a language maps to an execution platform. This section uses our calculus to specify how to use three vital optimizations: operator fission, operator fusion, and operator re-ordering. Each optimization comes with a correctness theorem. The proofs are in the Appendix. The correctness of the optimizations dependes on reasoning about state and communication, requirements identified in by the optimizations catalog in Section 2.13.

3.4.1 Operator Fission

If an operation is commutative across data items, then the order in which the data items are processed is irrelevant. MapReduce uses this observation to exploit the collective computing power of a cluster for analyzing extremely large data sets [29]. The input data set is partitioned, and copies of the map operator process the partitions in parallel. In general, the challenge in exploiting such *data parallelism* is determining if an operator commutes. Sawzall and StreamIt solve this challenge by restricting the programming model. In Brooklet, commutativity analysis can be performed with a simple code inspection. Since a pure function always commutes[2], and all state in Brooklet is explicit in an operator's signature, a sufficient condition for introducing fission is that an operator does not access variables. The transformation must ensure that the output data is combined in the same order that the input data was partitioned. Brooklet can use the round-robin splitter and joiner described in the StreamIt translation for this purpose. Thus, the operator `(out)⟵wrapMap-LatLong(q);` can be parallelized with $N = 3$ copies like this:

```
(q1, q2, q3, $sc)          ⟵ Split(q, $sc);
(q4)                       ⟵ wrapMap-LatLong(q1);
(q5)                       ⟵ wrapMap-LatLong(q2);
(q6)                       ⟵ wrapMap-LatLong(q3);
(out, $v4, $v5, $v6, $jc) ⟵ Join(q4, q5, q6, $v4, $v5, $v6, $jc);
```

The following rule describes how to create the new program with N duplicates of the parallelized operator.

$$\frac{\begin{array}{c} op = \texttt{(}q_{out}\texttt{)} \leftarrow f\texttt{(}q_{in}\texttt{);} \\ \forall i \in 1\dots n : q_i = \mathit{freshId}() \qquad \forall i \in 1\dots n : q'_i = \mathit{freshId}() \\ F'_b, op_s = [\![\, \varnothing, \texttt{split roundrobin}, \overline{q}, q_{in} \,]\!]^p_s \\ \forall i \in 1\dots n : op_i = \texttt{(}q'_i\texttt{)} \leftarrow f\texttt{(}q_i\texttt{);} \\ F''_b, op_j = [\![\, \varnothing, \texttt{join roundrobin}, q_{out}, \overline{q'} \,]\!]^p_s \end{array}}{\langle F_b, op\rangle \longrightarrow^{N}_{split} \langle F_b \cup F'_b \cup F''_b, op_s\ \overline{op}\ op_j\rangle} \qquad (\text{O}_b\text{-S}\small{\text{PLIT}})$$

The precondition is that *op* does not refer to any state variables. The fission optimization illustrates that Brooklet facilitates reasoning over shared state. The rules for round-robin split and join are in Fig. 3.3.

Making multiplexers explicit and fixing the degree of parallelism are important to faithfully model and reason about real-world systems. Possible implementation strategies for avoiding the limitation of a fixed degree of parallelism include using just-in-time compilation to do splitting

[2] At least in the mathematical sense; in systems, floating point operations do not always commute.

online, or putting code on a larger number of machines and then in practice using only a subset as needed.

Theorem 3.3 (Correctness of O$_b$-Split)**.** *For all function environments F_b, Brooklet programs P_b, and degrees of parallelism N, if rule* O$_b$-Split *yields $\langle F_b, P_b\rangle \longrightarrow^N_{split} \langle F'_b, P'_b\rangle$, then $\rightarrow^*_b (F_b, P_b, I_b) = \rightarrow^*_b (F'_b, P'_b, I_b)$ for all Brooklet inputs I_b.*

The proof sketch is in Appendix D.

3.4.2 Operator Fusion

In practice, transmitting data between two operators can incur significant overhead. Data needs to be marshalled/unmarshalled, transferred over a network or written to a mutually accessible location, and buffered by the receiver, not to mention the expense of context switching. This overhead can be offset by *fusing* two operators into one. StreamIt applies this optimization to operators in a pipelined topology [112]. Operators may be fused if they meet two conditions. First, they appear in a simple pipeline. Brooklet makes this topology easy to validate because queues are defined and used exactly once. Second, the state used by the operators must not be modifiable anywhere else in the program. Again, because Brooklet requires an explicit declaration of all state, this condition can be verified with a simple code inspection. The following Brooklet program shows two steps in an MPEG decoder:

$$\begin{array}{l} \texttt{(q}_1\texttt{,\$v1)} \;\longleftarrow\; \texttt{ZigZag(q}_{\texttt{in}}\texttt{,\$v1);} \\ \texttt{(q}_{\texttt{out}}\texttt{,\$v2)} \;\longleftarrow\; \texttt{IQuantization(q}_1\texttt{,\$v2);} \end{array}$$

The fused equivalent of the program is:

$$\texttt{(q}_{\texttt{out}}\texttt{,\$v1,\$v2)} \;\longleftarrow\; \texttt{Fused-ZigZag-IQuant(q}_{\texttt{in}}\texttt{,\$v1,\$v2);}$$

The following rule formalizes this optimization:

$$\frac{\begin{array}{cc} op_1 = \texttt{(}q_1, v_1\texttt{)} \leftarrow f_1\texttt{(}q_{in}, v_1\texttt{);} & (\exists op' = \texttt{(\_,}v_1\texttt{)} \leftarrow f'\texttt{(\_,\_)}) \Rightarrow op' = op_1 \\ op_2 = \texttt{(}q_{out}, v_2\texttt{)} \leftarrow f_2\texttt{(}q_1, v_2\texttt{);} & (\exists op' = \texttt{(\_,}v_2\texttt{)} \leftarrow f'\texttt{(\_,\_)}) \Rightarrow op' = op_2 \\ \multicolumn{2}{c}{f = \mathit{freshId}() \qquad F'_b = [f \mapsto \mathit{fusedOperator}(F_b, f_1, f_2)]F_b} \end{array}}{F_b, op_1\ op_2 \longrightarrow F'_b, \texttt{(}q_{out}, v_1, v_2\texttt{)} \leftarrow f\texttt{(}q_{in}, v_1, v_2\texttt{);}} \qquad (\mathrm{O}_b\text{-}\textsc{Fuse})$$

The preconditions guard against other operators writing variables v_1 or v_2. The following rule defines the new internal function:

$$\frac{(d_{temp}, d'_1) = F_b(f_1)(d_{in}, 1, d_1) \qquad (d_{out}, d'_2) = F_b(f_2)(d_{temp}, 1, d_2)}{\mathit{fusedOperator}(F_b, f_1, f_2)(d_{in}, \_, d_1, d_2) = (d_{out}, d'_1, d'_2)} \qquad (\text{W}_b\text{-}\textsc{Fuse})$$

In our example, this combines F_b(`ZigZag`) and F_b(`IQuantization`) into function F'_b(`Fused-ZigZag-IQuant`). The fusion optimization illustrates that Brooklet facilitates reasoning over topologies.

Theorem 3.4 (Correctness of O$_b$-Fuse)**.** *For all function environments F_b and Brooklet programs P_b, if rule* O$_b$-Fuse *yields $\langle F_b, P_b\rangle \longrightarrow_{Fuse} \langle F'_b, P'_b\rangle$, then $\rightarrow_b^* (F_b, P_b, I_b) = \rightarrow_b^* (F'_b, P'_b, I_b)$ for all Brooklet inputs I_b.*

The proof sketch is in Appendix E.

3.4.3 Reordering of Operators

A general rule of thumb for database query optimizations is that it is better to remove more tuples early in order to reduce downstream computations. The most popular example for this is hoisting a select operator, because a select reduces the tuple volume for operators it feeds into [8]. A select is said to *commute* with another operator if their output result is the same regardless of their execution order. The following program computes the commission on sales of IBM stock. The input is `sale(ticker, price)` and the output is `commission(ticker, cost)`. The commission is 2%.

```
output commission;
input sale;
(qt) ⟵ BrokerCommission(sale);
(commission) ⟵ Select-IBM(qt);
```

The functions for the two operators are:

$$F_b(\texttt{BrokerCommission})(d, \_) = \text{let } \langle \texttt{ticker}, \texttt{price}\rangle = d \text{ in } \langle \texttt{ticker}, 0.02 \cdot \texttt{price}\rangle$$

$$F_b(\texttt{Select-IBM})(d, \_) = \text{let } \langle \texttt{ticker}, \texttt{cost}\rangle = d \text{ in if } \texttt{ticker} = \text{`IBM' then } d \text{ else } \bullet$$

We can reorder the two operators for two reasons. First, the `BrokerCommission` operator is stateless, and therefore operates on each data item independently, so its semantics do not change when it sees a filtered stream of data item. Second, the `Select-IBM` operator only reads the `ticker`, and `BrokerCommission` forwards the `ticker` unmodified. In other words, `Select-IBM` does not rely on any data modified by `BrokerCommission` and vice versa. The optimized program is:

```
output commission;
input sale;
(qt) ⟵ Select-IBM(sale);
(commission) ⟵ BrokerCommission(qt);
```

The following rule encodes the optimization:

$$\frac{\begin{array}{c} op_1 = \texttt{(}q_t\texttt{)} \leftarrow f_1\texttt{(}\overline{q}\texttt{);} \qquad op_2 = \texttt{(}q_{out}\texttt{)} \leftarrow f_2\texttt{(}q_t\texttt{);} \\ F_b(f_1)(d,i) = \text{let } \langle r,w\rangle = d \text{ in } \langle r, f_1(w,i)\rangle \\ F_b(f_2)(d,\_) = \text{let } \langle r,\_\rangle = d \text{ in if } f_2(r) \text{ then } d \text{ else } \bullet \\ \forall i \in 1\ldots|\overline{q}| : q'_i = \mathit{freshId}() \\ op'_1 = \texttt{(}q_{out}\texttt{)} \leftarrow f_1\texttt{(}\overline{q'}\texttt{);} \qquad \forall i \in 1\ldots|\overline{q}| : op_i = \texttt{(}q'_i\texttt{)} \leftarrow f_2\texttt{(}q_i\texttt{);} \end{array}}{F_b, op_1\ op_2 \longrightarrow F_b, \overline{op}\ op'_1} \qquad (\text{O}_b\text{-}\textsc{HoistSelect})$$

The first two preconditions restrict op_1 and op_2 to be stateless operators. The third precondition specifies that f_1 forwards a part r of the data item unmodified, and the fourth precondition specifies that f_2 is a select that only reads r, and forwards the entire data item unmodified. We have chosen in Brooklet to abstract away local deterministic computations into opaque functions, because their semantics are well-studied (e.g., [36, 43, 97]). We leverage this prior work by assuming that a static program analysis can determine the restrictions on the read and write sets of operator functions used for select hoisting.

Theorem 3.5 (Correctness of O$_b$-HoistSelect)**.** *For all function environments F_b and Brooklet programs P_b, if $\langle F_b, P_b\rangle \longrightarrow_{HoistSelect} \langle F'_b, P'_b\rangle$ by rule* O$_b$-HoistSelect*, then* $\rightarrow^*_b (F_b, P_b, I_b) = \rightarrow^*_b (F'_b, P'_b, I_b)$ *for all Brooklet inputs I_b.*

The proof sketch is in Appendix F.

3.4.4 Optimizations Summary

We have used our calculus to understand how a language can apply three vital optimizations. The concise and straightforward formalization of the optimizations validates the design of Brooklet. As shown in Table 2.1, there are many other streaming optimizations. Furthermore, there are stronger variants of the optimizations we sketched; for example, it is sometimes possible to introduce data parallelism even for stateful operators. We believe that the examples in this section are a useful first step towards formalizing optimizations for stream processing languages.

3.5 Chapter Summary

This chapter presents Brooklet, a core calculus for stream processing. It represents stream processing applications as a graph of operators. Operators contain pure functions, communicate through explicit queues, thread all state through explicit variables, and trigger non-deterministically. Explicit state, communication, and non-deterministic execution are central concepts, capturing the reality of distributed implementations. We translate three representative languages, CQL, Sawzall, and StreamIt, to Brooklet, thus demonstrating its generality for language designers. We formalize three vital optimizations, operator fission, operator fusion, and operator reordering, in Brooklet, thus demonstrating its usefulness for language implementors. As we discuss in Chapter 4, Brooklet provides the foundation for River, a common intermediate language for stream processing.

4 From a Calculus to an Intermediate Language for Stream Processing

It is widely accepted that intermediate languages help programming languages by decoupling them from the target platform and vice versa. At its core, an intermediate language provides a small interface with well-defined behavior, facilitating robust, portable, and economic language implementations. Similarly, a calculus is a formal system that mathematically defines the behavior of the essential features of a domain. This chapter demonstrates how to use a calculus as the foundation for an intermediate language for stream processing. Stream processing makes it easy to exploit parallelism on multicores or even clusters. Streaming languages are diverse [8, 11, 42, 93, 112], because they address many real-world domains, including transportation, audio and video processing, network monitoring, telecommunications, healthcare, and finance.

The starting point for this chapter is the Brooklet calculus. A Brooklet application is a stream graph, where each edge is a conceptually infinite stream of data items, and each vertex is an operator. Each time a data item arrives on an input stream of an operator, it fires, executing a pure function to compute data items for its output streams. Optionally, an operator may also maintain state, consisting of variables to remember across operator firings. Chapter 3 demonstrated that Brooklet is general enough to model three different streaming languages.

The finishing point for this chapter is the River intermediate language. Extending a calculus into an intermediate language is challenging. A calculus deliberately abstracts away features that are not relevant in theory, whereas an intermediate language must add them back in to be practical. The question is how to do that while (1) maintaining the desirable properties of the calculus, (2) making the source language development effort economic, and (3) safely supporting common optimizations and reaching reasonable target-platform performance. The answers to these questions are the research contributions of this chapter.

On the implementation side, we wrote front-ends for dialects of three very different streaming languages (CQL [8], Sawzall [93], and StreamIt [112]) on River. We wrote a back-end for River on System S [6], a high-performance distributed streaming runtime. And we wrote three high-level optimizations (placement, fusion, and fission) that work at the River level, decoupled from and thus reusable across front-ends. This is a significant advance over prior work, where source languages, optimizations, and target platforms are tightly coupled. For instance, since River's target platform, System S, runs on a cluster of commodity machines, this chapter reports the first distributed CQL implementation, making CQL more scalable.

Overall, this chapter shows how to get the best of both theory and practice for stream processing. Starting from a calculus supports formal proofs showing that front-ends realize the semantics of their source languages, and that optimizations are safe. And finishing in an intermediate language lowers the barrier to entry for new streaming language implementations, and thus grows the ecosystem of this crucial style of programming.

4.1 Maintaining Properties of the Calculus

Being a calculus, Brooklet makes abstractions. In other words, it removes irrelevant details to reduce stream processing to the features that are essential for formal reasoning. On the other hand, River, being a practical intermediate language, has to take a stand on each of the abstracted-away details. This section describes these decisions, and explains how the intermediate language retains the benefits of the calculus.

4.1.1 Brooklet Abstractions and their Rationale

The following is a list of simplifications in the Brooklet semantics, along with the insights behind them.

Atomic steps. Brooklet defines execution as a sequence of atomic steps. Being a small-step operational semantics makes it amenable to proofs. Each atomic step contains an entire operator firing. By not sub-dividing firings further, it avoids interleavings that unduly complicate the behavior. In particular, Brooklet does not require complex memory models.

Pure functions. Functions in Brooklet are pure, without side effects and with repeatable results. This is possible because state is explicit and separate from operators. Keeping state separate also makes it possible to see right away which operators in an application are stateless or stateful, use local or shared state, and read or write state.

Opaque functions. Brooklet elides the definition of the functions for operator firings, because semantics for local sequential computation are well-understood.

Non-determinism. Each step in a Brooklet execution non-deterministically picks a queue to fire. This non-deterministic choice abstracts away from concrete schedules. In fact, it even avoids the need for any centralized scheduler, thus enabling a distributed system without costly coordination. As discussed in Section 3.3, source languages can implement determinism on top of Brooklet by adding the appropriate protocols. For example, CQL uses time stamps and StreamIt uses static data transfer rates with order-preserving joiners and splitters.

No physical platform. Brooklet programs are completely independent from any actual machines they would run on.

Finite execution. Stream processing applications run conceptually forever, but a Brooklet execution is a finite sequence of steps. One can reason about an infinite execution by induction over each finite prefix [50].

4.1.2 River Concretizations and their Rationale

This section shows how the River intermediate language fills in the holes left by the Brooklet calculus. For each of the abstractions from the previous section, it briefly explains how to concretize it and why. The details and correctness arguments for these points come in later sections.

Atomic steps. Whereas Brooklet executes firings one at a time, albeit in non-deterministic order, River executes them concurrently whenever it can guarantee that the end result is the same. This concurrency is crucial for performance. To guarantee the same end result, River uses a minimum of synchronization that keeps firings conceptually atomic. River shields application developers from the concerns of locking or memory models.

Pure functions. Both the calculus and the execution environment separate state from operators. However, whereas the calculus passes variables in and out of functions by copies, the intermediate language uses pointers instead to avoid the copying cost. Using the fact that state is explicit, River automates the appropriate locking discipline where necessary, thus relieving users from this burden. Furthermore, instead of returning data items to be pushed on output queues, functions in River directly invoke call-backs for the run-time library, thus avoiding copies and simplifying the function implementations.

Opaque functions. Functions for River are implemented in a traditional non-streaming language. They are separated from the River runtime library by a well-defined API. Since atomicity is preserved at the granularity of operator firings, River does not interfere with any local instruction-reordering optimizations the low-level compiler or the hardware may want to perform.

Non-determinism. Being an intermediate language, River ultimately leaves the concrete schedule to the underlying platform. However, it reduces the flexibility for the scheduler by bounding queue sizes, and by using back-pressure to prevent deadlocks when queues fill up.

No physical platform. River programs are target-platform independent. However, at deployment time, the optimizer takes target-platform characteristics into consideration for placement.

Finite execution. River applications run indefinitely and produce timely outputs along the way, fitting the purpose and intent of practical stream processing applications.

In the following sections, we discuss in more detail how River provides a practical realization of the Brooklet calculus. In particular, we discuss how River maximizes the concurrent execution of operators while preserving the sequential semantics of Brooklet (§ 4.1.3); uses back-pressure to avoid buffer overflows in the presence of bounded queues(§ 4.1.4); and provides an implementation language for operator implementation(§ 4.2).

4.1.3 Maximizing Concurrency while Upholding Atomicity

This section gives the details for how River upholds the sequential semantics of Brooklet. In particular, River differs from Brooklet in how it handles state variables and data items pushed on output queues. These differences are motivated by performance goals: they avoid unnecessary copies and increase concurrency.

River requires that each operator instance is single-threaded and that all queue operations are atomic. Additionally, if variables are shared between operator instances, each operator instance uses locks to enforce mutual exclusion. River's locking discipline follows established practice for deadlock prevention, i.e., an operator instance first acquires all necessary locks in a standard order, then performs the actual work, and finally releases the locks in reverse order. Otherwise, River does not impose further ordering constraints. In particular, unless prevented by locks, operator instances may execute in parallel. They may also enqueue data items as they execute and update variables in place without differing in any observable way from River's call-by-value-result semantics. To explain how River's execution model achieves this, we first consider execution without shared state.

Operator instance firings without shared state behave as if atomic. In River, a downstream operator instance o_2 can fire on a data item while the firing of the upstream operator instance o_1 that enqueued it is still in progress. The behavior is the same as if o_2 had waited for o_1 to complete before firing, because queues are one-to-one and queue operations are atomic. Furthermore, since each operator is single-threaded, there cannot be two firings of the same operator instance active simultaneously, so there are no race conditions on operator-instance-local state variables. □

In the presence of shared state, River uses the lock assignment algorithm shown in Figure 4.1. The algorithm finds the minimal set of locks that covers the shared variables appropriately. The idea is that locks form equivalence classes over shared variables: every shared variable is protected

```
1. $AllClasses.add(AllSharedVars)$
2. for all $o \in OpInstances$ do
3.     $UsedByO = sharedVariablesUsedBy(o)$
4.     for all $v \in UsedByO$ do
5.         $EquivV = v.equivalenceClass$
6.         if $EquivV \not\subseteq UsedByO$ then
7.             $AllClasses.remove(EquivV)$
8.             $AllClasses.add(EquivV \cap UsedByO)$
9.             $AllClasses.add(EquivV \setminus UsedByO)$
```

Figure 4.1: Algorithm for assigning shared variables to equivalence classes, that is, locks.

by exactly one lock, and shared variables in the same equivalence class are protected by the same lock.

Two variables only have separate locks if there is an operator instance that uses one but not the other. The algorithm starts with a single equivalence class (lock) containing all variables in line 1. The only way for variables to end up under different locks is by the split in lines 7–9. Without loss of generality, let v be in $EquivV \cap UsedByO$ and w be in $EquivV \setminus UsedByO$. That means there is an operator instance o that uses $UsedByO$, which includes v but excludes w. □

An operator instance only acquires locks for variables it actually uses. Let's say operator instance o uses variable v but not w. We need to show that v and w are under separate locks. If they are under the same lock, then the algorithm will arrive at a point where $UsedByO$ contains v but not w and $EquivV$ contains both v and w. That means that $EquivV$ is not a subset of $UsedByO$, and lines 7–9 split it, with v and w in two separate parts of the split. □

Shared state accesses behave as if atomic. An operator instance locks the equivalence classes of all the shared variables it accesses. □

4.1.4 Bounding Queue Sizes

In Brooklet, communication queues are infinite, but real-world systems have limited buffer space, raising the question of how River should manage bounded queues. One option is to drop data

```
1.  $process(dataItem, submitCallback, variables)$
2.      $lockSet = \{v.equivalenceClass()$ **for** $v \in variables\}$
3.      **for all** $lock \in lockSet.iterateInStandardOrder()$ **do**
4.          $lock.acquire()$
5.      $tmpCallback = \lambda d \Rightarrow tmpBuffer.push(d)$
6.      $opFire(dataItem, tmpCallback, variables)$
7.      **for all** $lock \in lockSet.iterateInReverseOrder()$ **do**
8.          $lock.release()$
9.      **while** $!tmpBuffer.isEmpty()$ **do**
10.         $submitCallback(tmpBuffer.pop())$
```

Figure 4.2: Algorithm for implementing back-pressure.

items when queues are full. But this results in an unreliable communication model, which significantly complicates application development [49], wastes effort on data items that are dropped later on [81], and is inconsistent with Brooklet's semantics. A more attractive option is to automatically apply back-pressure through the operator graph.

A straightforward way to implement back-pressure is to let an operator block during an enqueue operation if its output queue is full. While easy to implement, this approach could deadlock in the presence of shared variables. To see this, consider an operator instance o_1 feeding another operator instance o_2, and assume that both operator instances access a common shared variable. Further assume that o_1 is blocked on an enqueue operation due to back-pressure. Since o_1 holds the shared variable lock during its firing, o_2 cannot proceed while o_1 is blocked on the enqueue operation. On the other hand, o_1 won't be able unblock until o_2 makes progress to open up space in o_1's output queue. They are deadlocked.

The pseudocode in Figure 4.2 presents River's solution to implementing back-pressure. It describes the *process* function, which is called by the underlying streaming runtime when data arrives at a River operator. The algorithm starts in line 2 with an operator's lock set. The lock set is the minimal set of locks needed to protect an operator's shared variables, as described in Section 4.1.3. Before an operator fires, it first must acquire all locks in its lock set, as shown in lines 3-4. Once all locks are held, the process function invokes the operator's *opFire* method, which contains the actual operator logic. The *opFire* does not directly enqueue its resultant data for transport by the runtime. Instead, it writes its results to a dynamically-sized intermediate buffer, which is passed to the *opFire* as a callback. Lines 5-6 show the callback and invocation of the operator logic. Next, lines 7-8 release all locks. Finally, lines 9-10 drain the temporary buffer, enqueuing each data item for transport by calling the streaming runtime's *submit* callback.

The key insight is that lines 9-10 might block if the downstream queue is full, but there is no deadlock because at this point the algorithm has already released its shared-variable locks. Furthermore, *process* will only return after it has drained the temporary buffer, so it only requires enough space for a single firing. If *process* is blocked on a downstream queue, it may in turn block its own upstream queue (i.e. apply back-pressure). The algorithm in Figure 4.2 restricts the scheduling of operator firings. In Brooklet, an operator instance can fire as long as there is at least one data item in one of its input queues. In River, an additional condition is that all intermediate buffers must be empty. This does not impact the semantics of the applications or the programming interface of the operators. It simply impacts the scheduling decisions of the runtime.

4.2 Making Language Development Economic

River is intended as the target of a translation from a source language, which may be an existing streaming language or newly invented. In either case, the language implementer wants to make use of the intermediate language without spending too much effort on the translation. In other words, language development should be economic. We address this requirement by an intermediate language that is easy to target, and in addition, by providing an eco-system of tools and reusable artifacts for developing River compilers. We demonstrate the practicality by implementing three existing languages. The foundation for the translation support is, again, the Brooklet calculus. Hence, we briefly review the calculus first, before exploring what it takes to go from theory to practice in River.

4.2.1 Brooklet Treatment of Source Languages

Chapter 3 contained formalizations, but no implementations, for translating the cores of CQL [8], Sawzall [93], and StreamIt [112] to the Brooklet calculus. This exercise helped prove that the calculus can faithfully model the semantics of two languages that had already been formalized

```
select avg(speed), segNo, dir, hwy
from segSpeed[range 300];
```

(a) One of the Linear-Road queries in CQL.

```
congested: { speed: int; seg_no: int;
             dir: int; hwy: int } relation
= select avg(speed), seg_no, dir, hwy
  from seg_speed[range 300];
```

(b) One of the Linear-Road queries in River-CQL.

```
proto ¨querylog.proto¨
queryOrigins: table sum[url: string]
 of count: int;
queryTargets: table sum[url: string]
 of count: int;
logRecord: QueryLogProto = input;
emit queryOrigins[logRecord.origin] <- 1;
emit queryTargets[logRecord.target] <- 1;
```

(c) Batch log query analyzer in Sawzall.

```
queryOrigins: table sum[url:string]
 of count: int;
queryTargets: table sum[url:string]
 of count: int;
logRecord:{origin:string; target: string}
 = input;
emit queryOrigins[logRecord.origin] <- 1;
emit queryTargets[logRecord.target] <- 1;
```

(d) Batch log query analyzer in River-Sawzall.

```
pipeline {
  pipeline {
    splitjoin {
      split duplicate;
      filter { /* ... */ }
      filter { /* ... */ }
      join roundrobin; }
    filter{ /* ... */ } }
  filter{ /* ... */ } }
```

(e) FM Radio in StreamIt or River-StreamIt

```
work {tf <- low_pass(
 peek(1), peek(2)); push(tf); pop();}
work {tm <- low_pass(
 peek(1)); push(tm); pop();}
work {s,tc <- subtractor(
 s,peek(1)); push(tc); pop();}
work {s,tc <- amplify(s,
 s,peek(1)); push(tc); pop();}
```

(f) Filter bodies for FM Radio in River-StreamIt.

Figure 4.3: Example source code in original languages and their River dialects.

elsewhere, and helped provide the first formal semantics of a third language that had not previously been formalized.

Brooklet translation source. *CQL*, the continuous query language, is a dialect of the widely used SQL database query language for stream processing [8]. CQL comes with rigorously defined semantics, grounded in relational algebra. The additions over SQL are windows for turning streams into tables, as well as operators for observing changes in a table to obtain a stream. *Sawzall* [93] is a language for programming MapReduce [29], a scalable distributed batch processing system. Sawzall is a small and simple language that hides the distribution from the programmer. Finally, *StreamIt* [112] is a synchronous data flow (SDF) [70] language with a denotational semantics [113]. StreamIt relies on fixed data rates to compute a static schedule, thus reducing runtime overheads for synchronization and communication. These three languages have fundamentally different constructs, optimized for their respective application domains. They were developed independently by different communities: CQL originated from databases, Sawzall from distributed computing, and StreamIt from digital signal processing. Chapter 3 abstracted away many details of the source languages that are not relevant for the calculus, such as the type system and concrete operator implementations.

Brooklet translation target. A Brooklet program consists of two parts: the stream graph and the operator functions. For specifying stream graphs, Brooklet provides a topology language, as seen in the IBM market maker example in Figure 3.1. For operator functions, on the other hand, Brooklet does not provide any new notation; instead, it just assumes they are pure opaque functions. Where necessary, Chapter 3 uses standard mathematical notation, including functions defined via currying. Remember that currying is a technique that transforms a multi-argument function into a function that takes some arguments, and returns another residual function for the remaining arguments. This is useful for language translation in that the translator can supply certain arguments statically, while leaving others open to firing time.

Brooklet translation specification. Chapter 3 specifies translations in sequent calculus notation. The translation is syntax-directed, in the sense that each translation rule matches certain syntactic constructs, and translations of larger program fragments are composed from translations of their components. These translations are restricted to the core source language subsets, and are only specified on chapter, not implemented. The mathematical notation and restriction to the essentials make the translations amenable to proofs. However, they leave a lot to be done for a practical implementation, which is what this chapter is about.

4.2.2 River Implementation of Source Languages

As the previous section shows, Brooklet abstracts three aspects of source language support: it simplifies the source languages, it provides a core target language, and it specifies but does not implement translations. The following sections describe how River and its eco-system concretize these three areas, with the goal of economy in language development. In other words, not only does River make it *possible* to implement various streaming languages, it makes it *easier*.

4.2.3 River Translation Source

We implemented three realistic streaming languages on River. They are dialects of CQL [8], Sawzall [93], and StreamIt [112]. The River dialects are more complete than the Brooklet language subsets, but they are not identical to the orignal published versions of the languages. They add some features that were missing in the original versions. For example, we added types to CQL. They omit some infrequently used features from the original version to reduce implementation effort. And they replace some features, notably expressions, with equivalent features to enable code reuse. While we took the liberty to modify the source languages, we retained their essential aspects. In practice, it is not uncommon for source languages to change slightly when they are ported to an intermediate language or virtual execution environment. For example, the JVM supports Jython rather than C Python, and the CLR supports F# rather than OCaml. Those cases, like ours, are motivated by economic language development.

River-CQL. River's dialect of CQL is more complete than the original language. Figures 4.2 (a) and (b) show an example. The original version was lacking types. We added those to make the language more safe, for example, by reporting type errors at compile time. As a side effect, we were able to preserve these types during translation, thus avoiding some overheads at runtime. The type syntax of any language would do for this purpose; we used the syntax for types in River's implementation language, described in Section 4.2.4. Since we already had compiler components for the type sublanguage, we could just reuse those, simplifying source language development.

River-Sawzall. River's dialect of Sawzall replaces protocol buffers by a different notation. Protocol buffers are a data definition language that is part of Google's eco-system. The River eco-system, on the other hand, has its own type notation, which we reuse across source languages. Figures 4.2 (c) and (d) illustrates this change. As this feature was not central to Sawzall to begin with, changing it was justified to ease language development.

River-StreamIt. River's dialect of StreamIt elides the feature called *teleport messaging.* Teleport messages are an escape hatch to send out-of-band messages that side-step the core streaming paradigm. Only very few StreamIt programs use teleport messaging [110]. They require centralized support, and are thus only implemented in the single-node back-end of StreamIt. Since River runs on multi-node clusters, we skipped this feature altogether. Furthermore, another change in River's dialect of StreamIt is that work function implementations are defined separately from their use, rather than in-place, as seen in Figures 4.2 (e) and (f). Since work functions in StreamIt contain traditional non-streaming code, it is a matter of taste where and in what syntax to write them. We chose not to spend time literally emulating a notation that is inessential to the main language.

4.2.4 River Translation Target

A River program consists of two parts: the stream graph and the operator implementations. For the stream graph, River simply reuses the topology language of Brooklet. For operators, on the other hand, River must go beyond Brooklet by supplying an implementation language. The primary requirements for this implementation language are that (1) the creation and decomposition of data items be convenient, to aid in operator implementation, and (2) mutable state be easily identifiable, in keeping with the semantics. An explicit non-goal is support for traditional compiler optimizations, which we leave to an off-the-shelf traditional compiler.

Typed functional languages clearly meet both requirements and a lower-level traditional IL such as LLVM [69] can also meet them, given library support for higher-level language features such as pattern matching. In our current implementation, we rely on OCaml as River's implementation sublanguage. If features a high-quality native code compiler and a simple foreign function interface, which facilitates integration with existing streaming runtimes written in C/C++.

The implementation language of the River IL allows language developers to write language-specific libraries of standard operators, such as select, project, split, join, and aggregate. However, the operator implementations need to be specialized for their concrete application. Consider, for example, an implementation for a selection operator:

```
Bag.filter (fun x -> #expr) inputs
```

where #expr stands for a predicate indicating the filter condition.

How best to support this specialization was an important design decision. One approach would be to rely on language support, i.e., OCaml's support for generic functions and modules (i.e. *functors*) as reflected in the River IL. This approach is well-understood and statically safe. But it also requires abstracting away any application-specific operations in callbacks, which can lead to unwieldy interfaces and slow performance. Instead, we chose to implement common operators as IL templates, which are instantiated inline with appropriate types and expressions. Pattern variables (of form `#expr`) are replaced with concrete syntax at compile time. This eliminates the overhead of abstraction at the cost of code size.

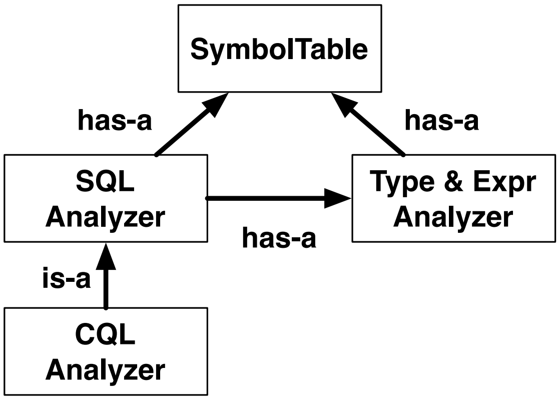
The templates are actually parsed by grammars derived from the original language grammars. As a result, templates benefit from both the convenience of using concrete syntax, and the robustness of static syntax checking. Code generation templates in River play the same role as currying in Brooklet, i.e. they bind the function to its arguments.

4.2.5 River Translation Specification

A language implementer who wants to create a new language translator needs to implement a parser, a type-checker, and a code generator. We facilitate this task by decomposing each language into sublanguages, and then reusing common sublanguage translator modules across languages. Principally, we follow the same approach as the Jeannie language [55], which composes C and Java. However, our application both increases the granularity of the components (e.g., by combining parts of languages) and the number of languages involved. Our use of language composition also differs from prior work because all front-end language translators share a common translation target, the River IL, which significantly reduces the code generation effort. For example, River-CQL, River-Sawzall, and River-StreamIt share the same sublanguage for expressions, leading to code reuse between their translators.

Modular parsers. The parsers use component grammars written as modules for the *Rats!* parser generator [48]. Each component grammar can either *modify* or *import* other grammar modules. For example, the CQL grammar consists of several modules: SQL's select-from-where clauses, streaming constructs modifying SQL to CQL, an imported expression sublanguage for things like projection or selection, and an imported type sublanguage for schemas. The grammar modules for expressions and types are the same as in other River languages. The result of parsing is an abstract syntax tree (AST), which contains tree nodes drawn from each of the sublanguages.

Modular type checkers. Type checkers are also implemented in a compositional style. Type checkers for composite languages are written as groups of visitors. Each visitor is responsible for all AST nodes corresponding to a sublanguage. Each visitor can either dispatch to or inherit from other visitors, and all visitors share a common type representation and symbol table. For example, the CQL analyzer inherits from an SQL analyzer, which in turn dispatches to an analyzer for expressions and types. All three analyzers share a symbol table.



If there are type errors, the type analyzer reports those and exists. Otherwise, it populates the symbol table, and decorates the AST with type annotations.

Modular code generators. Code generation is also simplified by the use of language composition. The input to the code generator is the AST annotated with type information, and the output is a stream graph and a set of operator implementations. Our approach to producing this output is to first create the AST for the stream graph and each operator implementation, and then pretty-print those ASTs. In the first step, we splice together subtrees obtained from code generation templates with subtrees obtained from the original source code. In the second step, we reuse pretty-printers that are shared across source language implementations. Overall, we found that the use of language composition led to a smaller, more consistent implementation with more reuse, making the changes to the source languages well worth it.

4.3 Safe and Portable Optimizations

One of the benefits of an intermediate language is that it can provide a single implementation of an optimization, which benefits multiple source languages. In prior work on stream processing, each source language had to re-implement similar optimizations. The River intermediate language, on the other hand, supports optimization reuse across languages. Here, we are primarily interested in optimizations from the streaming domain, which operate at the level of a stream graph, as opposed to traditional compiler optimizations. By working at the level of a stream graph, River can optimize an entire distributed application. As with the other contributions of River, the Brooklet calculus provides a solid foundation, but new ideas are needed to build an intermediate language upon it.

4.3.1 Brooklet Treatment of Optimizations

Chapter 3 decouples optimizations from their source languages. It specifies each optimization by a *safety guard* and a *rewrite rule.* The safety guard checks whether a subgraph satisfies the preconditions for applying the optimization. It exploits the one-to-one restriction on queues and the fact that state is explicit to establish these conditions. If a subgraph passes the safety guard, the rewrite rule replaces it by a transformed subgraph. Chapter 3 then proceeds to prove that the optimizations leave the observable input/output behavior of the program unchanged.

Chapter 3 discusses three specific optimizations: (1) *Fusion* replaces two operators by a single operator, thus reducing communication costs at the expense of pipeline parallelism. (2) *Fission* replaces a single operator by a splitter, a number of data-parallel replicas of the operator, and a merger. Chapter 3 only permits fission for stateless operators. (3) *Selection hoisting* rewrites a subgraph $A \to \sigma$ into a subgraph $\sigma \to A$, assuming that A is a stateless operator and σ is a selection operator that only relies on data fields unchanged by A. Selection hoisting is profitable if A is expensive and σ reduces the number of data items that A needs to process. Chapter 3 relied on analyzing the function of operator A to establish the safety of selection hoisting.

4.3.2 River Optimization Support

We made the observation that River's source languages are designed to make certain optimizations safe by construction, without requiring sophisticated analysis. For example, Sawzall provides a set of built-in aggregations that are known to be commutative, and partitioned by a user-supplied key, thus enabling fission. Rather than loosing safety information in translation, only to have to discover it again before optimization, we wanted to add it to River's intermediate language (IL). However, at the same time, we did not want to make the IL source-language specific, which would jeopardize the reusability of optimizations and the generality of River.

We solved this dilemma by adding extensible annotations to River's graph language. An annotation next to an operator specifies *policy* information, which encompasses safety and profitability. Safety policies are usually passed down by the translator from source language to IL, such as, which operators to parallelize. Profitability policies usually require some knowledge of the execution platform, such as the number of machines to parallelize on. In this chapter, we use simple heuristics for profitability; prior work has also explored more sophisticated analyses for this, which are beyond the scope of this chapter [46]. Policy is separated from *mechanism*, which implements the actual code transformation that performs the optimization. River's annotation mechanism allows it to do more powerful optimizations than Brooklet. For example, fission in River works not just on stateless operators, but also on stateful operators, as long as the state is keyed and the key fields are listed in annotations. Both CQL and Sawzall are designed explicitly to make the key evident from the source code, so all we needed to do is preserve that information through their translators.

To keep annotations extensible, they share a common, simple syntax, inspired by Java. Each use of an operator is preceded by zero or more annotations. Each annotation is written as an atsign (`@`), an identifier naming the annotation, and a comma-separated list of expressions serving as parameters. In other words, the general annotation syntax is:

$$\begin{array}{lll}
\mathit{operatorUse} & ::= & \mathit{annotation}^{*}\ \ \mathit{opOutputs}\ \ \text{`}\texttt{<-}\text{'}\ \ \mathit{ID}\ \ \mathit{opInputs}\ \ \text{`}\texttt{;}\text{'} \\
\mathit{opOutputs} & ::= & \text{`}\texttt{(}\text{'}\ \ \mathit{ID}\ \ (\text{`}\texttt{,}\text{'}\ \ \mathit{ID})^{*}\ \ \text{`}\texttt{)}\text{'} \\
\mathit{opInputs} & ::= & \text{`}\texttt{(}\text{'}\ \ \mathit{ID}\ \ (\text{`}\texttt{,}\text{'}\ \ \mathit{ID})^{*}\ \ \text{`}\texttt{)}\text{'} \\
\mathit{annotation} & ::= & \text{`}\texttt{@}\text{'}\ \ \mathit{ID}\ \ \text{`}\texttt{(}\text{'}\ \ (\mathit{expr}\ \ (\text{`}\texttt{,}\text{'}\ \ \mathit{expr})^{*})^{?}\ \ \text{`}\texttt{)}\text{'}
\end{array}$$

River currently makes use of the following annotations:

Annotation	Description
`@Fuse(`*ID*`)`	Directive to fuse operators with the same *ID* in the same process.
`@Parallel()`	Directive to perform fission on an operator.
`@Commutative()`	Declares that an operator's function is commutative.
`@Keys(`k_1`,...,`k_n`)`	Declares that an operator's state is partitionable by the key fields $k_1, \ldots, k_n$ in each data item.
`@Group(`*ID*`)`	Directive to place operators with the same *ID* on the same machine.

We anticipate adding more annotations as we implement more source languages and/or more optimizations. The translator from River IL to native code invokes the optimizers one by one, transforming the IL at each step. A specification passed to the translator determines the order in which the optimizations are applied.

4.3.3 Fusion Optimizer

Intuition. Fusion combines multiple stream operators into a single stream operator, to avoid the overhead of data serialization and transport [45, 63].

Policy. The policy annotation is `@Fuse(`*ID*`)`. Operators with the same *ID* are fused. Applying fusion is a tradeoff. It eliminates a queue, reducing communication cost, but it prohibits operators from executing in parallel. Hence, fusion is profitable if the savings in communication cost exceed the lost benefit of parallelism. As shown in the Brooklet calculus, a sufficient safety precondition for fusion is if the fused operators form a straight-line pipeline without side entries or exits.

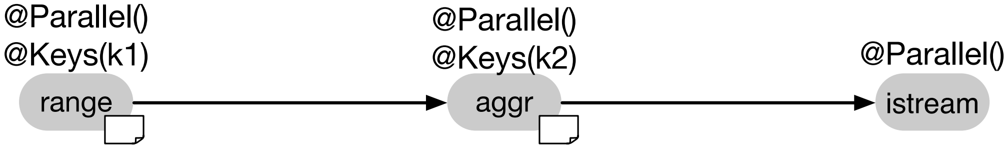
Mechanism. The current implementation replaces internal queues by direct function calls. A possible future enhancement would be to allow fused operators to share the same process but run on different threads. This would reduce the cost for communication, but still maintain the benefits of pipeline parallelism on multi-cores.

4.3.4 Fission Optimizer

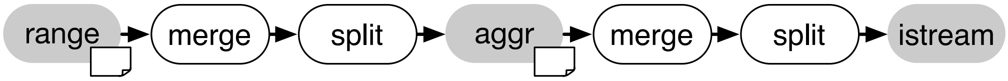
Intuition. Fission replicates an operator or a stream subgraph to introduce parallel computations on subsets of the data [29, 30, 45].

Policy. Fission uses three annotations. The `@Parallel()` annotation is a directive to parallelize an operator. The `@Commutative()` annotation declares that a given operator's function commutes. Commutative operators can be safely parallelized because their functions do not depend on the order of the input data. Finally, the `@Keys(`k_1`,...,`k_n`)` annotation declares that an operator is stateful, but that its state is keyed (i.e. partitioned) by the key in fields k_1,...,k_n. Fission is profitable if the computation in the parallel segment is expensive enough to make up for the the overhead of the inserted split and merge operators. The safety conditions for fission depend on state and order. In terms of state, there must be no memory dependencies between replicas of the operator. This is trivially the case when the operator is stateless. The other way to accomplish this is if the state of the operator can be partitioned by key, such that each operator replica is responsible for a separate portion of the key space. In that case, the splitter routes data items by using a hash on their key fields. When a parallel segment consists of multiple operators, they must be either stateless or have the same key. To understand how order affects correctness, consider the following example. Assume that in the unoptimized program, the operator pushes data item d_1 before d_2 on its output queue. In the optimized program, d_1 and d_2 may be computed by different replicas of the operator, and depending on which replica is faster, d_2 may be pushed first. That would be unsafe if any down-stream operator depends on order. That means that fission is safe with respect to order either if all down-stream operators commute, or if the merger brings data items from different replicas back in the right order. Depending on the source language, the merger can use different ordering strategies: CQL embeds a logical timestamp in every data item that induces an ordering; Sawzall has commutative aggregations and can hence ignore order; and StreamIt only parallelizes stateless operators and can hence use round-robin order.

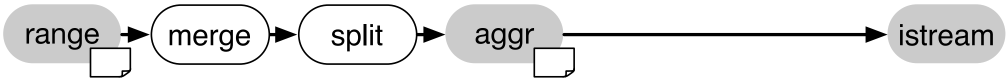
Mechanism. River's fission optimization consists of multiple steps. Consider the following example in which three operators appear in a pipeline. The first two operators, `range` and `aggr`, are stateful, and keyed by k_1 and k_2 respectively. The third, `istream`, is stateless. The figures indicate stateful operators by a rectangle with a folded corner. All three operators have the `@Parallel()` annotation, indicating that fission should replicate them.



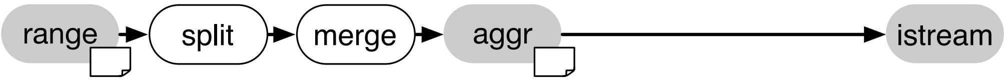
Step 1 adds split and merge operators around parallelizable operators. This trivially parallelizes each individual operator. At the same time, it introduces bottlenecks, as data streamed through adjacent mergers and splitters must pass through a single machine. Note that for source or sink operators, only merge or split, respectively, are needed. This is because the partitioning is assumed to occur outside of the system.



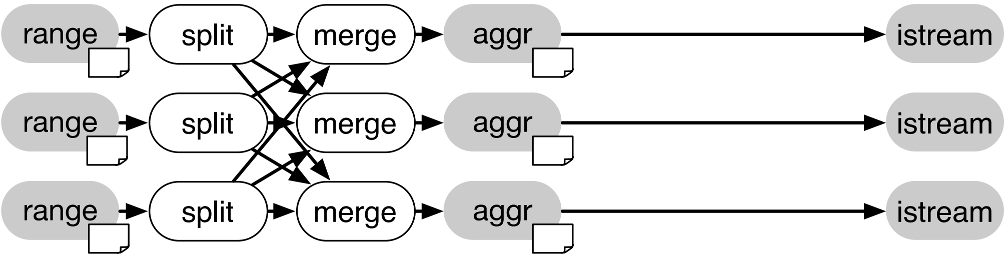
Step 2 removes the bottlenecks. There are two in the example; each calls for a different action. First, the merge and split between `aggr` and `istream` can be safely removed, because `istream` is stateless.



Next, the merge and split between `range` and `aggr` cannot be removed, because both operators partition state by different keys, k_1 and k_2. Instead, we apply a *rotation*. A rotation switches the order of the merge and split to remove the bottleneck. This is the same approach as the *shuffle* step in MapReduce [29].



Finally, Step 3 replicates the operators to the desired degree of parallelism, and inserts `@Group(`*ID*`)` annotations for the placement optimizer, to ensure that replicas actually reside on different machines.



In this example, all operators are parallelized. In other applications, only parts may be parallelized, so performance improvements will be subject to Amdahl's law: for example, if the unoptimized program spends 1/5th of its time in non-parallelizable operators, the optimized program can be no more than 5× faster.

4.3.5 Placement Optimizer

While Brooklet explored selection hoisting, River explores placement instead, because it is of larger practical importance, and illustrates the need for the optimizer to be aware of the platform.

Intuition. Placement assigns operators to machines and cores to better utilize physical resources [116].

Policy. The policy annotation is `@Group(`*ID*`)`. Operators with the same *ID* are assigned to the same machine. Several operators can be assigned the same *ID* to take advantage of machines with multiple cores. Placement is profitable if it reduces network traffic (by placing operators that communicate a lot on the same machine) and/or improves load balance (by placing computationally intensive operators on different machines).

Mechanism. The placement mechanism does not transform the IL, but rather, directs the runtime to assign the same machine to all operators that use the same identifier. Information such as the number of machines is only available at the level of the intermediate language, a trait that River shares with other language-based virtual machines. Placement is complicated if operators share state. In general, River could support sharing variables across machines, but relies on the underlying runtime to support that functionality. Because our current backend does not provide distributed shared state, the placement optimizer has an additional constraint. It ensures that all operators that access the same shared variable are placed on the same machine. Fortunately, none of the streaming languages we encountered so far need cross-machine shared state.

4.3.6 When to Optimize

The Brooklet calculus abstracts away the timing of optimizations. In River, optimizations are applied once the available resources are known, just before running the program. In other words, the translations from source languages to IL happen ahead of time, but the translation from IL to native code for a specific target platform is delayed until the program is launched. That enables River to make more informed profitability decisions. We have not yet taken the next step, which would be *dynamic* optimizations that make decisions at runtime. Dynamic optimizations can use profile information without requiring a separate training run; they can adapt when available resources change, for example, as the result of failure, or due to another application launched on the same system; and they can adapt when load characteristics change, for example, based on the time of day.

A straight-forward approach to dynamic optimizations is to drain all queues in a (sub-)graph of their in-flight data items, take the (sub-)graph offline, restructure it, and then bring it back online [4, 57]. This approach is similar to JIT (just-in-time) compilers for traditional optimizations. However, in the streaming domain, it may be too disruptive, because many streaming applications are required to produce continuous low-latency outputs.

A more nimble approach to dynamic optimizations is to statically create a more general graph,

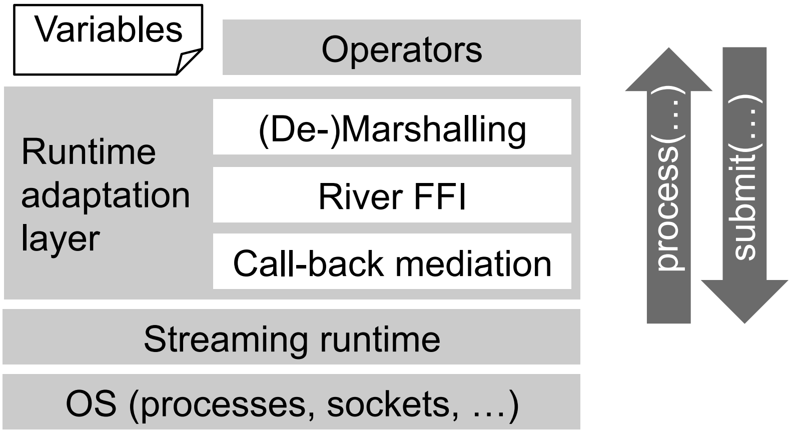


Figure 4.4: Stack of layers for executing River.

and then adapt how data flows through it at runtime. A seminal example for this is the Eddy operator, which performs dynamic operator reordering without physically changing the graph at runtime [12]. While we have not yet implemented optimizations that use this approach, River is well-suited for it. The approach would use annotations to decide where an optimization applies, statically rewrite the graph, and add in control operators that dynamically route data items for optimization.

4.4 Runtime Support

The main goal for River's runtime support is to insulate the IL and the runtime from each other. Figure 4.4 shows the architecture. It consists of a stack of layers, where each layer only knows about the layers immediately below and above it. At the lowest level is the streaming runtime, which provides the software infrastructure for process management, data transport, and distribution. Above that is the runtime adaptation layer, which provides the interface between River operators and the distributed runtime. At the highest level are the operator instances and their associated variables. River's clean separation of concerns ensures that it can be easily ported to additional streaming runtimes. The rest of this section describes each layer in detail.

4.4.1 Streaming Runtime

A distributed streaming runtime for River must satisfy the following requirements. It must launch the right processes on the right machines in the distributed system to host operators and variables. It must provide reliable transport with ordered delivery to implement queues. It must arbitrate resources and monitor system health for reliability and performance. Finally, our placement optimizer relies on placement support in the runtime. There is no strict latency requirement: the semantics tolerate an indefinite transit time.

The streaming runtime sits on top of an operating system layer, which provides basic centralized services to the runtime layer, such as processes, sockets, etc. However, building a high-performance streaming runtime satisfying the above-listed requirements is a significant engineering effort beyond the OS. Therefore, we reuse an existing runtime instead of building our own. We chose System S [42], a distributed streaming runtime developed at IBM that satisfies the requirements, and that is available to universities under an academic license. While System S has its own streaming language, we bypassed that in our implementation, instead interfacing with the runtime's C++ API.

4.4.2 Runtime Adaptation

The runtime adaptation layer provides the interface between River operators and the distributed runtime. As shown in Figure 4.4, it consists of three sub-layers.

Call-Back Mediation Layer. This layer mediates between the runtime-specific APIs and call-backs, and the River-specific functions. The code in this layer peels off runtime-specific headers from a data item, and then passes the data item to the layer above. Similarly, it adds the headers to data on its way down. If there are shared variables, this layer performs locking, and implements output buffers for avoiding back-pressure induced dead-lock as described in Section 4.1.4. The call-back mediation layer is linked into the streaming runtime.

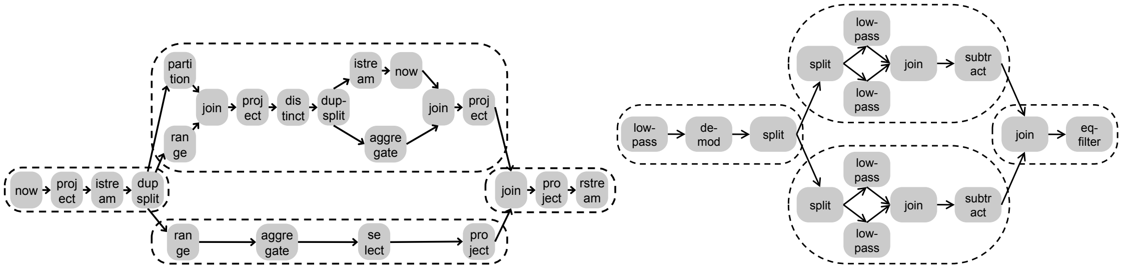
River FFI Layer. A foreign function interface, or FFI for short, enables calls across programming languages. In this case, it enables C++ code from a lower layer to call a River function in an upper layer, and it enables River code in an upper layer to call a C++ function in a lower layer. The River FFI is the same as the OCaml FFI. Each OS process for River contains an instance of the OCaml runtime, which it launches during start-up.

(De-)Marshalling Layer. This layer converts between byte arrays and data structures in River's implementation language. It uses an off-the-shelf serialization module. The code in this layer is auto-generated by the River compiler. It consists of `process(...)` functions, which are called by the next layer down, demarshal the data, and call the next layer up; and of `submit(...)` functions, which are called by the next layer up, marshal the data, and call the next layer down. Since this layer is crucial for performance, we have plans to optimize it further by specialized code generation.

4.4.3 Variables and Operators

As described in Section 4.2.4, operators are generated from templates written by the language developer. We chose to use templates to support operator specialization in River, because they allow for static syntax checking and do not add to performance overhead. Their implementation strikes a balance between the functional purity of operators in Brooklet, and performance demands of a practical IL that needs to update data in place, and make callbacks instead of returning values. Variables and operators are implemented in the implementation sublanguage of River. An operator firing takes the form of a function call from the next lower layer. If the operator accesses variables, then the call passes those as references, so that the operator can perform in-place updates if needed. Instead of returning data as part of the function's return value, the operator invokes a call-back for each data item it produces on an output queue. Note that this simple API effectively hides any lower-layer details from the variables and operators.

4.5 Evaluation



(a) CQL's Linear Road in River.

(b) StreamIt's FM Radio in River.

Figure 4.5: Structural view for the CQL and StreamIt benchmarks, distributed across 4 machines each for task and pipeline parallelism. The dashed ovals group operators that are placed onto the same machine.

We have built a proof-of-concept prototype of River, including front-ends for three source languages, implementations of three optimizations, and a back-end on a distributed streaming system. We have not yet tuned the absolute performance of our prototype; the goal of this chapter was to show its feasibility and generality. Therefore, while this section presents some experimental results demonstrating that the system works and performs reasonably well, we leave further efforts on absolute performance to future work.

All performance evaluations were run on a cluster of 16 machines. Each machine has two 4-core 64-bit Intel Xeon (X5365) processors running at 3.00GHz, where each core has 32K L1i and 32K L1d caches of its own, and a 4MB unified L2 cache that is shared with another core. The processors have a FSB speed of 1,333 MHz and are connected to 16GB of uniform memory. Machines in the cluster are connected via 1Gbit ethernet.

4.5.1 Support for Existing Languages

To verify that River is able to support a diversity of streaming languages, we implemented the language translators described in Section 4.2, as well as illustrative benchmark applications. The benchmarks exercise a significant portion of each language, demonstrating the expressivity of River. They are described below:

CQL Linear Road. Linear Road [8] is a benchmark designed by the authors of CQL. It is a hypothetical application that computes tolls for vehicles traveling on the Linear Road highway. (Figure 4.5 (a) shows the operator graph of the application.) Each vehicle is assigned a unique ID, and its location is specified by three attributes: speed, direction, and position. Each highway in also assigned an ID. Vehicles pay a toll when they drive on a congested highway. A highway is congested if the average speed of all vehicles on the highway over a 5 minute span is less than 40 mph.

StreamIt FM Radio. This benchmark implements a multi-band equalizer [45]. As shown in Figure 4.5 (b), the input passes through a demodulator to produce an audio signal, and then an equalizer. The equalizer is implemented as a splitjoin with two band-pass filters; each band-pass filter is the difference of a pair of low-pass filters.

Sawzall Batch Log Analyzer. Figure 4.2 (d) shows this query, which is based on an exemplar Sawzall query in Pike et al. [93]. It is a batch job that reads its input from the distributed file system. The program analyzes a set of search query logs to count queries per origin based on IP address. The resulting aggregation could then be used to plot query origins on a world map.

CQL Continuous Log Analyzer. This is similar to the Sawzall log query analyzer, but it is a continuous query rather than a batch job. Its input comes from a server farm. Each server reports the origin of the requests it has received, and the analyzer performs an aggregation keyed by the origin over the most recent 5 minute window. Note that the data is originally partitioned by the target (server) address, so the application must shuffle the data.

The benchmark applications exercise significant portions of each language. This demonstrates that River is flexible enough to express three source languages, CQL, StreamIt, and Sawzall, that occupy diverse points in the design space for streaming languages.

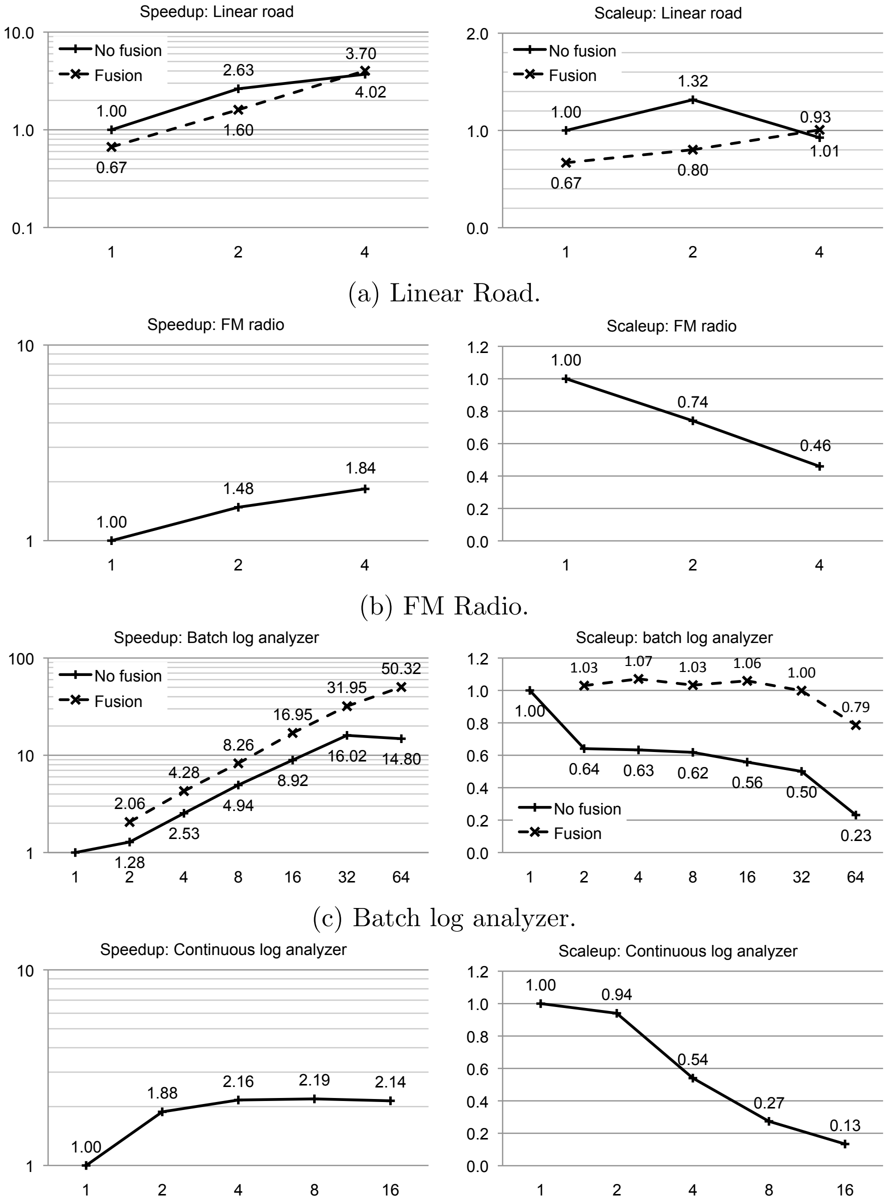
4.5.2 Suitability for Optimizations

To verify that River is extensible enough to support a diverse set of streaming optimizations, we implemented each of the optimizations described in Section 4.3. We then applied the different optimizations to the benchmark applications from Section 4.5.1.

Placement. Our placement optimizer distributes an application across machines. Operators from each application were assigned to groups, and each group was executed on a different machine. As a first step, we used the simple heuristic of assigning operators to groups according to the branches of the top-level split-merge operators, although there has been extensive prior work [116] on determining the optimal assignments. In the non-fused version, each operator had its own process, and it was up to the OS to schedule processes to cores. Figure 4.6 (a) and (b) show the results of running both Linear Road and the FM Radio applications on 1, 2, and 4 machines. These experiments demonstrate that distribution improves applications performance. Figure 4.5 shows the partitioning scheme for the 4-machine case using dashed ovals. These results are particularly exciting because the original implementation of CQL was not distributed. Despite the fact that the Linear Road application shows only limited amounts of task and pipeline parallelism, the first distributed CQL implementation achieves a 3.70× speedup by distributing execution on 4 machines. The FM Radio application exhibits a 1.84× speedup on 4 machines.

Fission. Our fission optimizer replicates operators, and then distributes those replicas evenly across available machines. We tested two applications, the Sawzall batch log analyzer and the CQL continuous log analyzer, with increasing amounts of parallelism. In these experiments, the degree of parallelism corresponded to the number of available machines, from 1 to 16. We additionally ran the Sawzall query on 16 machines with 16, 32, 48, and 64 degrees of parallelism, distributing the computation across cores. The results are shown in Figures 4.6 (c) and (d).

Fission adds split and merge operators to the stream graph. Therefore, in the non-fissed case, there are fewer processing steps. Despite of this, the Sawzall program's throughput increased when the degree of parallelism went from 1 to 2. As the degree of parallelism and the number of



(a) Linear Road.

(b) FM Radio.

(c) Batch log analyzer.

(d) Continuous log analyzer.

Figure 4.6: Speedup is the throughput relative to single machine. Scaleup is the speedup divided by number of machines in (a) and (b), and the speedup divided by the degree of parallelism in (c) and (d).

machines increased from 2 to 16, the increase in throughput was linear, with an 8.92× speedup on 16 machines. When further distributed across cores, the Sawzall program also experience a large performance increase. However, the 32 replicas case showed better performance than 64 replicas. This makes sense, since the unfused Sawzall query has 5 operators, each of which was replicated 64 times ($64 * 5 = 320$), while the total number of available cores across all machines was $16 * 8 = 128$.

The CQL continuous log analyzer saw a performance improvement with fission, but only achieved at best a 2.19× speedup, with no benefit past 6 machines. Unlike Sawzall, all data items in CQL are timestamped. To maintain CQL's deterministic semantics, mergers must wait for messages from all input ports for a given timestamp. The lesson we learned was that when implementing a distributed, data-parallel CQL, we need to give more consideration to how to enforce deterministic execution efficiently. Unlike our River-CQL implementation, the original CQL did not do fission and thus did not encounter this issue.

Fusion. The Linear Road results in Figure 4.6 (a) illustrate the tradeoffs during fusion, which often comes at the expense of pipeline parallelism. The fusion optimization only improves performance in the 4-machine case, where it achieves a 4.02× speedup, which is overall better than the 4-machine results without fusion. Figure 4.6 (c) shows that fusion is particularly beneficial to the Sawzall log query analyzer. In this case, fusion eliminates much of the per-data item processing overhead, and therefore allows the fission optimization to yield much better results. Fusion combines each mapper with its splitter and each reducer with its merger. With both fusion and fission, the Sawzall log query analyzer speeds up 50.32× on 16 machines with 64 degrees of parallelism.

Calibration to Native Implementation. The placement, fission, and fusion optimizations demonstrate that River can support a diverse set of streaming optimizations, thus validating our design. While we did not focus our implementation efforts on absolute performance metrics, we were interested to see how River's performance compared to a native language implementation. Native Sawzall programs are translated to MapReduce jobs. We therefore implemented the Sawzall web log analyzer query as a Hadoop job. We ran both Hadoop and River on a cluster of 16 machines, with an input set of 10^9 data items, around 24GB. Hadoop was able to process this data in 96 seconds. We observed that the Hadoop job started 92 mappers during its execution, so we ran River with 92 mappers, and computed the results in 137 seconds. However, we should point out that the execution model for both systems differs. While Hadoop is a batch processing system and stores its intermediate results to disk, River is a streaming system, in which the mapper and the reducer are running in parallel, and intermediate results are kept in memory. Given that there are 16 machines with 8 cores each, we ran River again with 64 mappers and 64 reducers ($16 * 8 = 64 + 64 = 128$). Under this scenario, River completed the computation in 115 seconds.

Despite the fact that Hadoop is a well-established system that has been heavily used and optimized, the River prototype ran about 83% as fast. There are two reasons for this performance difference. First, our current fission optimizer always replicates the same number of mappers as reducers, which may not be an optimal configuration. Additional optimization work is needed to adjust the map-to-reduce ratio. Second, our implementation has some unnecessary serialization overheads. A preliminary investigation suggests that eliminating those would give us performance on par with Hadoop.

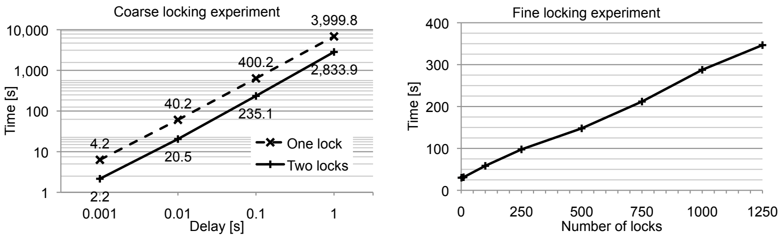
4.5.3 Concurrency

This section explores the effectiveness of the locking algorithm from Figure 4.1.

Coarse-Grained Locking. For the coarse-grained locking experiment, we used the following setup, described in River's topology language:

```
(q2, $v1)      <- f1(q1, $v1     );
(q3, $v1, $v2) <- f2(q2, $v1, $v2);
(q4,      $v2) <- f3(q3,      $v2);
```

In this example, `f1` and `f3` are expensive (implemented by sleeping for a set time), whereas `f2` is cheap (implemented as the identity function). Operator instances `f1` and `f2` share the variable `$v1`, and `f2` and `f3` share the variable `$v2`. With naive locking, we would put all three variables under a single lock. That means that all three operator instances are mutually exclusive, and



(a) Coarse locking

(b) Fine locking

Figure 4.7: Locking experiments

you would expect the total execution time to be approximately:

$$cost(\texttt{f1}) + cost(\texttt{f2}) + cost(\texttt{f3})$$

On the other hand, with our smarter locking scheme, `f1` and `f3` have no locks in common and can therefore execute in parallel. Hence, you would expect an approximate total execution time of:

$$\max\{cost(\texttt{f1}), cost(\texttt{f3})\} + cost(\texttt{f2})$$

We tested this by varying the cost (i.e. delay) of the operators `f1` and `f3` over a range of 0.001 to 1 seconds. The results, in Figure 4.7 (a), behave as we expected, with our locking scheme performing approximately twice as fast as with the naive version.

Fine-Grained Locking. As a second micro-benchmark for our locking algorithm, we wanted to quantify the overhead of locking on a fine-grained level. We used the following setup, described in River's topology language:

```
(q2, $v1, ..., $vn) <- f1(q1, $v1, ..., $vn);
(q3, $v1, ..., $vn) <- f2(q2, $v1, ..., $vn);
```

Operators `f1` and `f2` share variables `$v1` … `$vn`, where we incremented `n` from 1 to 1,250. With naive locking, each variable would be protected by its own lock, as opposed to our locking scheme, which protects all `n` variables under a single lock. Figure 4.7 (b) shows that the overhead of the naive scheme grows linearly with the number of locks.

4.6 Chapter Summary

This chapter presents River, an intermediate language for distributed stream processing. River is founded on the Brooklet calculus. Stream processing is widely used, easy to distribute, and has language diversity. By providing an intermediate language for streaming, we are making a lot of common infrastructure portable and reusable, and thus facilitating further innovation. And by building our intermediate language on a calculus, we are giving a clear definition of how programs should behave, thus enabling reasoning about correctness.

One contributions of this chapter is to show how to maintain the properties of the calculus in the intermediate language. The Brooklet calculus has a small-step operational semantics, which models execution as a sequence of atomic operator firings using pure functions. In River, operator firings still behave as if atomic, but are concurrent and do not need to be pure.

A second contribution of this chapter is to make source language development economic. We show how to reuse common language components across diverse languages, thus limiting the effort of writing parsers, type checkers, and code generators to the unique features of each language. The River intermediate language includes an intermediate language for describing stream graphs and operator implementations. We use code generation for operator implementations based on a simple templating mechanism.

A third contribution of this chapter is to provide safe optimizations that work across different source languages. Each source language is designed to make the safety of certain optimizations evident. We provide an annotation syntax, so that on the one hand, translators from the source languages can retain the safety information, while on the other hand, optimizers can work at the IL level without being source-language specific. Each optimizer uses the annotations as policy and implements the program manipulation as mechanism.

To conclude, River is an intermediate language for running multiple streaming languages on a distributed system, and comes with an eco-system for making it easy to implement and optimize multiple source-languages. River is based on a calculus, giving it a clear semantics and strong correctness guarantees.

5
Related Work

5.1 Streaming Languages

Stream processing has a long history in computer science. This thesis primarily uses CQL [8], Sawzall [93], and StreamIt [112] as representative examples of **streaming languages**, but there are many more. Prominent early examples include Signal [15] and Lustre [51], which, like StreamIt, explore a synchronous programming model. Spade [42] and it successor SPL [54] are streaming languages for composing parallel and distributed flow graphs for System S, IBM's scalable data processing middleware. Gigascope [28] is a lightweight streaming database designed for network monitoring and analysis. It is programmed with GSQL, a query language with SQL-like syntax. Hancock [27] is a query language designed for data-mining in the telecommunications domain. Pig Latin [88] is one of the languages designed to compose MapReduce or Hadoop jobs. DryadLINQ [123] runs imperative code on local machines and uses integrated SQL to generate distributed queries. Several languages, including Brook [19], CUDA [86], and OpenCL [64] are targeted at programming GPUs.

5.2 Surveys on Stream Processing

Chapter 2 presents a catalog of stream processing optimizations. Existing **surveys on stream processing** do not focus on optimizations [106, 13, 61], and existing catalogs of optimizations do not focus on stream processing. Chapter 2 provides both. The presentation is inspired by catalogs for design patterns [40] and for refactorings [38].

5.3 Semantics of Stream Processing

Chapter 3 presents the Brooklet calculus. Brooklet takes inspiration from Featherweight Java [59], in that they both define core minimal languages that allow us to reason about correctness. There has been extensive prior work in the **semantics of stream processing**. Brooklet is designed to facilitate reasoning about language and optimization implementation in distributed systems, and therefore chooses different abstractions from other formalisms. Notably, the π-calculus does not model state [79]. The ambient calculus omits state and makes computations mobile [22]. Kahn process networks [62], such as Unix pipes, assume deterministic execution. Synchronous data flow [70] models, such as StreamIt, assume fixed buffer sizes and static communication patterns. Hoare's communicating sequential process [56] assumes no buffering, and synchronous communication. Finally, Gurevich et al. present an abstract formalism without state and communications [50].

5.4 Continuous Queries

The database literature often refers to streaming applications as **continuous queries** [25, 109]. Surprisingly, there is little work from the database community on optimizations of queries with side effects. Two exceptions are a study of XQuery with side effects [43] and a study of object-oriented databases [35].

5.5 Intermediate Language for Streaming

Chapter 4 presents River, an **intermediate language for streaming**, which runs on a distributed system and supports multiple source languages. SVM, the stream virtual machine, is a C framework for streaming on both CPU and GPU back-ends, but the paper does not describe any source-language support [66]. MSL, the multicore streaming layer, executes the StreamIt language on the Cell architecture [124]. Erbium is a set of intrinsic functions for a C compiler for streaming on an x86 SMP, but the paper describes no source-language support [80]. And Lime is a new streaming language with three back-ends: Java, C, and FPGA bit-files [11]. None of SVM, MSL, Erbium, or Lime are distributed on a cluster, none of them are founded on a calculus, and they all have at most a single source language each. While we are not aware of prior work on intermediate languages for distributed streaming, there are various execution environments for distributed batch dataflow. MapReduce [29] has emerged as a de-facto execution environment for various batch-processing languages, including Sawzall [93], Pig Latin [88], and FlumeJava [24]. Dryad [60] is a batch execution environment that comes with its own language Nebula, but is also targeted by DryadLINQ [123]. CIEL is a batch execution environment with its own language SkyWriting [84]. Like MapReduce, Dryad, and CIEL, River runs on a shared-nothing cluster, but unlike those works, River is designed for continuous streaming, and derived from a calculus.

5.6 Economic Source-Language Development

River comes with an eco-system for **economic source-language development**. The LINQ framework (language integrated query) also facilitates front-end implementation, but using a different approach: LINQ embeds SQL-style query syntax in a host language such as C#, and targets different back-ends such as databases or Dryad [76]. Our approach follows more traditional compiler frameworks such as Polyglot [87] or MetaBorg [17]. We use the Rats! parser generator to modularize grammars [48]. Our approach to modularizing type-checkers and code-generators uses the same approach as Jeannie [55], but River composes more language components at a finer granularity.

5.7 Streaming Optimizations

Several communities have come up with similar **streaming optimizations**, but unlike River, they do not decouple the optimizations from the source-language translator and reuse them across different source languages. In *parallel databases* [30], the IL is relational algebra. Similarly to the fission optimization in this paper, parallel databases use hash-splits for parallel execution. But to do so, they rely on algebraic properties of a small set of built-in operators, unlike River, which supports an unbounded set of user-defined operators. There has been surprisingly little work on generalizing database optimizations to the more general case [35, 43], and that work is still limited to the database domain. The StreamIt compiler implements its own variant of fission [45]. It relies on fixed data rates and stateless operators for safety, and indeed, the StreamIt language is designed around making those easy to establish. Our fission is more general, since it can parallelize even in the presence of state. MapReduce has data-parallelism hard-wired into its design. Safety relies on keys and commutativity, but those are up to the user or a higher-level language to establish. River supports language-independent optimization by making such language-specific safety properties explicit in the IL.

6
Limitations and Future Work

There are three components to this thesis: a catalog of stream processing optimizations; the Brooklet calculus for stream processing; and the River intermediate language. Taken collectively, these components provide a reusable infrastructure for stream processing languages. However, this thesis does not address all the needs for stream processing with regards to infrastructure support. This chapter identifies some of the limitations and key avenues for future work for each of the three components, primarily the support for dynamic optimizations.

Optimizations Catalog. As a survey of the most prominent streaming optimizations, this chapter is fairly conprehensive. However, it also offers some broader observations, and proposes several avenues for future research. Since they have already been discussed in detail, we only list the proposals here with references to the relevant sections. The avenues for future work include: further study on how to augment languages to support optimization (§ 2.12.1); determining the best order for applying optimizations (§ 2.12.2); new compiler analysis for enabling optimizations (§ 2.12.3); extending existing optimizations beyond their current restrictions (§ 2.12.5); exploring how to support dynamic optimizations (§ 2.12.4); and defining standard benchmarks for streaming (§ 2.12.6).

Brooklet. There are many formal models for streaming systems [9, 62, 68, 70]. Additional work is needed to formalize the relationship between Brooklet and these other process calculi. For example, by mapping (if possible) a Brooklet program to the π-calculus [79] or the actor model [53], and vice-versa.

Many streaming applications have time constraints. In healthcare, there might be real-time constraints for reporting critical events. In finance, trades might need to happen within a certain time window. For streaming media applications, there might be limits on latency to ensure stutter free playback. Brooklet does not have a way to formalize these time constraints in the calculus, but it would clearly be useful for implementing real systems.

Chapter 2 surveys eleven prominent streaming optimizations. So far, we have only formalized three of them using Brooklet. More work is needed to formalize the remaining ones. One challenge, as discussed in Section 2.13, is that information about non-determinism, state, and communication is not sufficient to model all of the optimizations in the catalog. We addressed this problem in the River IL with the use of extensible annotations. How to address this in the calculus remains an open question.

Section 2.13 also identifies support for dynamic optimization as one of the requirements for a streaming IL. This is one of the major limitations for both Brooklet and River. We discuss this in more detail below.

River. River is currently limited in that it only supports static optimizations i.e., those that are performed *before* execution. These optimizations are easily modelled as translations from IL to IL. Dynamic optimizations, i.e., those that are performed *during* execution, are somewhat more challenging. One solution would be a global optimization engine, which observes all running operators across all system nodes and then modifies the operator graph and its mapping to machines in order to achieve its performance goals. While certainly feasible, this approach introduces another large component into the architecture. Furthermore, that component would need to coordinate across all nodes in the system.

A better way to address this limitation would be to take a cue from previous work [3, 12, 99] and, as much as possible, implement dynamic optimizations as operators. Such optimizing operators are statically injected into an application and then dynamically change the application by re-routing data or modifying operators and their placement. In effect, the stream processing application becomes self-monitoring and self-adjusting.

Another limitation of River is its performance. Although the system works and the optimizations demonstrate improvements in relative performance, the absolute performance is still short of expectations. Performance could be improved by using a different implementation sublanguage; by using better serialization; and by implementing a larger set of optimizations.

Finally, we have used River as the target for dialects of three existing streaming languages, letting their requirements guide the design of the IL. In the future, we would like to leverage our experience with translating front-ends and implementing optimizations into the design of a new streaming language from scratch using River. This would close the feedback loop, allowing the IL to guide the design of the language.

7 Conclusion

This thesis explores how to provide a reusable infrastructure for stream processing. Stream processing applications have become ubiquitous and essential to a variety of application domains, including entertainment, finance, and healthcare. They are the product of a paradigm shift towards data-centric applications that are expected to run on multiple processors or multiple machines. This paradigm shift has a profound impact on the design of both programming languages and optimizations. In order to support continued innovation in streaming languages and optimizations, language implementors need the proper infrastructure. Unfortunately, existing ILs are ill-suited for stream processing, so an intermediate language for stream processing does not exist.

This thesis has three components. A catalog of stream processing optimizations identifies what information a streaming IL needs to provide in order to support streaming optimizations. The Brooklet calculus provides a formal semantics that is general enough to express a wide range of streaming languages, and enables reasoning about language implementation and optimization correctness on distributed systems. Finally, the River IL builds on Brooklet by addressing the real-world details that the calculus elides. The result is a practical intermediate language with a rigorously defined semantics.

We evaluated our system by implementing front-ends for three diverse streaming languages (CQL, StreamIt, and Sawzall), and three important optimizations (operator fusion, fission, and placement). Additionally, we wrote a back-end for River on the System S distributed streaming runtime. We measured the performance improvements for benchmark applications written in the streaming languages when the three optimizations were applied. River effectively decouples the optimizations from the language front-ends, and thus makes them reusable across front-ends, reducing the overall implementation effort for language implementors.

Finally, this dissertation has suggested directions for future work in regards to infrastructure support for stream processing. Most importantly, we advocate further research in how to support dynamic optimizations, which can better take advantage of information only available at runtime, and the design of new streaming languages that are better suited to make use of the support that the infrastructure provides.

Collectively, these contributions demonstrate that an intermediate language designed to meet the requirements of stream processing can serve as a common substrate for critical optimizations; assure implementation correctness; and reduce overall implementation effort. Overall, this work enables further advances in language and optimization design, and encourages innovation in the vital domain of stream processing.

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Appendices

A
CQL Translation Correctness

This section proves Theorem 3.1 in Section 3.3.4.

A.1 Background on CQL Formal Semantics

Before we can prove that the semantics of CQL on Brooklet are equivalent to previously specified semantics of CQL, we recapitulate those semantics from [9].

A.1.1 CQL Function Environment.

The CQL function environment maps names for stream-relational operators to functions. These functions are used both to define the CQL denotational semantics [9], and to define our semantics by translation to Brooklet. In [9] the CQL function environment is written $\mathcal{M}$, but we will write it as F_c here for consistency with the other languages in this chapter.

As shown in Fig. 3.2, the signature of an S2R operator, a.k.a. a window, is $\mathcal{S} \times \mathcal{T} \to \Sigma$. Three common windows are:

$$F_c(\texttt{Now})(s, \tau) = \{e : \langle e, \tau \rangle \in s\}$$

(The `Now` window returns tuples from the current time stamp τ.)

$$F_c(\texttt{Range(T)})(s, \tau) = \{e : \langle e, \tau' \rangle \in s \text{ and } \max\{\tau - T, 0\} \leq \tau' \leq \tau\}$$

(The `Range(T)` window returns tuples from time stamps up to `T` in the past up to the current time stamp τ.)

$$F_c(\texttt{Rows(T)})(s, \tau) = \{e : \langle e, \tau' \rangle \in s \text{ and } \tau' \leq \tau \text{ and } N \geq |\{\langle e, \tau'' \rangle : \tau' \leq \tau'' \leq \tau\}|\}$$

(The `Rows(N)` window returns the last `N` tuples before the current time stamp τ.)

As shown in Fig. 3.2, the signature of an R2S operator is $\Sigma \times \Sigma \to \Sigma$. Three common R2S operators are:

$$F_c(\texttt{IStream})(\sigma_{\text{new}}, \sigma_{\text{old}}) = \{e : e \in \sigma_{\text{new}} \text{ and } e \notin \sigma_{\text{old}}\}$$

(The `IStream` operator monitors insertions into a relation.)

$$F_c(\texttt{DStream})(\sigma_{\text{new}}, \sigma_{\text{old}}) = \{e : e \notin \sigma_{\text{new}} \text{ and } e \in \sigma_{\text{old}}\}$$

(The `DStream` operator monitors deletions from a relation.)

$$F_c(\texttt{RStream})(\sigma_{\text{new}}, \_) = \sigma_{\text{new}}$$

(The `RStream` operator streams the current instantaneous relation.)

As shown in Fig. 3.2, the signature of an R2R operator is $\Sigma^n \to \Sigma$. Two common R2R operators are:

$$F_c(\texttt{Join(C)})(\sigma_{\text{lhs}}, \sigma_{\text{rhs}}) = \{e_{\text{lhs}} \oplus e_{\text{rhs}} : e_{\text{lhs}} \in \sigma_{\text{lhs}} \text{ and } e_{\text{rhs}} \in \sigma_{\text{rhs}} \text{ and } \texttt{C}(e_{\text{lhs}}, e_{\text{rhs}})\}$$

(The binary `Join(C)` operator joins tuples from two relations when they satisfy the join condition `C`. In the example in Section 3.3.1.1, the join condition `C` was `quotes.ask <= history.low`. In [9], the binary R2R operators are illustrated with a semi-join, but we chose a theta-join here, because we used it for the algorithmic trading example.)

$$F_c(\texttt{Select(C)}(\sigma) = \{e : e \in \sigma \text{ and } \texttt{C}(e)\}$$

(The unary `Select(C)` operator is a filter that returns only tuples that satisfy the selection condition `C`. In [9], this operator is called `Filter`, but we chose to call it `Select` here to match the terminology in the optimizations section.)

A.1.2 CQL Execution Semantics Function.

The CQL execution semantics function is written as $\mathcal{M}$ in [9], but we will denote it as $\rightarrow_c^*$ here to avoid confusion with the function environment $\mathcal{M}$ and for consistency with the other languages in this chapter. It takes as inputs a CQL function environment F_c, program P_c, and input I_c, and returns a CQL output O_c. The CQL semantics are big-step denotational, meaning that they define a mapping from an entire input to an entire output. Therefore, the CQL input and output domains are defined in a way that is easy to represent globally.

CQL domains for input and output:

$$\begin{array}{lll} \varrho & \in \mathit{RName} \rightarrow \mathcal{R} & \textit{Relations store} \\ \varsigma & \in \mathit{SName} \rightarrow \mathcal{S} & \textit{Streams store} \\ I_c & \in (\mathit{RName} \rightarrow \mathcal{R}) \times (\mathit{SName} \rightarrow \mathcal{S}) & \textit{CQL input} \\ O_c & \in \mathcal{R} \mid (\mathcal{T} \rightarrow \mathcal{S}) & \textit{CQL output} \end{array}$$

A CQL input I_c consists of two maps ϱ and ς, mapping relation names to time-varying relations and stream names to CQL streams, respectively. A CQL output is either a time-varying relation or a mapping from time stamps to streams, depending on whether the CQL program is a relation query or a stream query.

There are three kinds of relation queries, for which the semantics function returns a time-varying relation (domain $\mathcal{R}$):

$$\rightarrow_c^* (F_c, \mathit{RName}, I_c) = \text{let } \varrho, \varsigma = I_c \text{ in } \varrho(\mathit{RName})$$

(The semantics of a relation name just retrieve the time-varying relation from the relation store part of the input.)

$$\rightarrow_c^* (F_c, \mathit{S2R}(P_{cs}), I_c)(\tau) = F_c(\mathit{S2R})(\rightarrow_c^* (F_c, P_{cs}, I_c)(\tau), \tau)$$

(The semantics of a window recursively invokes $\rightarrow_c^*$ to obtain the subquery result, and invokes a window function from the function environment F_c. Recall from Fig. 3.2 that the non-terminal P_{cs} denotes a CQL query returning a stream.)

$$\begin{aligned}\rightarrow_c^* (F_c, \mathit{R2R}(\overline{P_{cr}}), I_c)(\tau) = \quad & \text{let } \forall i \in 1 \ldots |\overline{P_{cr}}| : r_i = \rightarrow_c^* (F_c, P_{cr_i}, I_c) \\ & \text{in } F_c(\mathit{R2R})(r_1(\tau), \ldots, r_{|\overline{P_{cr}}|}(\tau))\end{aligned}$$

(The semantics of an *R2R* operator uses the R2R function from the function environment F_c on all the instantaneous relations for the same time stamp.)

There are two kinds of stream queries, for which the semantics function returns a mapping from time stamps to stream contents up to and including that time stamp (domain $\mathcal{T} \rightarrow \mathcal{S}$).

$$\rightarrow_c^* (F_c, \mathit{SName}, I_c)(\tau) = \text{let } \varrho, \varsigma = I_c \text{ in } \{\langle e, \tau' \rangle : \langle e, \tau' \rangle \in \varsigma(\mathit{SName}) \text{ and } \tau' \leq \tau\}$$

(The semantics of a stream name takes a time stamp τ as a parameter, and returns the stream of all tuples with lesser or equal time stamps from the input mapping from stream names to streams).

$$\begin{aligned}\rightarrow_c^* (F_c, \mathit{R2S}(P_{cr}), I_c)(\tau) = \quad & \text{let } r = \rightarrow_c^* (F_c, P_{cr}, I_c) \\ & \text{in } \cup \{F_c(\mathit{R2S})(r(\tau'), r(\tau' - 1)) : 1 < \tau' \leq \tau\}\end{aligned}$$

(The semantics of an *R2S* operator recursively invoke $\rightarrow_c^*$ to obtain the subquery result, and then feed the instantaneous relation from the previous and the current time stamp into the operator function from the function environment F_c).

A.1.3 CQL Input and Output Translation.

The CQL domains are designed as global mathematical structures representing everything that ever happens in a program execution. This representation suits itself well to modeling with denotational semantics, but it is a mismatch for an actual implementation, where data arrives piecemeal and must be processed incrementally. It comes therefore as no surprise that the representation is also a mismatch for the small-step operational semantics of Brooklet, since they are designed to facilitate reasoning about implementation.

Due to this representational mismatch, we need a couple of conversion functions that we shall use in our translation correctness proof. The function $[\![\, I_c \,]\!]_c^i$ converts a CQL input to a Brooklet

CQL input translation: $\llbracket I_c \rrbracket^i_c = \langle V, Q\rangle$

$$\frac{\begin{array}{c}\varrho, \varsigma = I_c \qquad T = \mathit{time}(\varrho) \cup \mathit{time}(\varsigma)\\ V = \mathit{initVars}() \qquad Q = \llbracket \varrho, T \rrbracket^i_c \cup \llbracket \varsigma, T \rrbracket^i_c\end{array}}{\llbracket I_c \rrbracket^i_c = \langle V, Q\rangle} \qquad (\mathrm{T}^i_c)$$

$$\mathit{time}([id \mapsto r]\varrho) = \mathit{time}(\varrho) \cup \mathit{time}(r) \qquad (\mathrm{T}^i_c\text{-}\textsc{Time}\text{-}\varrho)$$

$$\mathit{time}(r) = \{\tau : r(\tau) \neq r(\tau - 1)\} \qquad (\mathrm{T}^i_c\text{-}\textsc{Time}\text{-}r)$$

$$\mathit{time}([id \mapsto s]\varsigma) = \mathit{time}(\varsigma) \cup \mathit{time}(s) \qquad (\mathrm{T}^i_c\text{-}\textsc{Time}\text{-}\varsigma)$$

$$\mathit{time}(s) = \{\tau : \exists e : \langle e, \tau\rangle \in s\} \qquad (\mathrm{T}^i_c\text{-}\textsc{Time}\text{-}s)$$

$$\mathit{time}(\varnothing) = \varnothing \qquad (\mathrm{T}^i_c\text{-}\textsc{Time}\text{-}\varnothing)$$

$$\frac{Q = \llbracket \varrho, T \rrbracket^i_c \qquad b = \llbracket r, T \rrbracket^i_c}{\llbracket [id \mapsto r]\varrho, T \rrbracket^i_c = [id \mapsto b]Q} \qquad (\mathrm{T}^i_c\text{-}\textsc{Data}\text{-}\varrho)$$

$$\frac{\tau > \max T \qquad b = \llbracket r, T \rrbracket^i_c}{\llbracket r, \{\tau\} \cup T \rrbracket^i_c = b, \langle r(\tau), \tau\rangle} \qquad (\mathrm{T}^i_c\text{-}\textsc{Data}\text{-}r)$$

$$\frac{Q = \llbracket \varsigma, T \rrbracket^i_c \qquad b = \llbracket s, T \rrbracket^i_c}{\llbracket [id \mapsto s]\varsigma, T \rrbracket^i_c = [id \mapsto b]Q} \qquad (\mathrm{T}^i_c\text{-}\textsc{Data}\text{-}\varsigma)$$

$$\frac{\begin{array}{c}\tau > \max T \qquad b = \llbracket s, T \rrbracket^i_c\\ d = \langle \{e : \langle e, \tau\rangle \in s\}, \tau\rangle\end{array}}{\llbracket s, \{\tau\} \cup T \rrbracket^i_c = b, d} \qquad (\mathrm{T}^i_c\text{-}\textsc{Data}\text{-}s)$$

$$\llbracket \varnothing, \_ \rrbracket^i_c = \varnothing \qquad (\mathrm{T}^i_c\text{-}\textsc{Data}\text{-}\varnothing_1)$$

$$\llbracket \_, \varnothing \rrbracket^i_c = \bullet \qquad (\mathrm{T}^i_c\text{-}\textsc{Data}\text{-}\varnothing_2)$$

CQL output translation: $\llbracket V, Q \rrbracket^o_c = O_c$

$$\begin{array}{c}\llbracket V, Q \rrbracket^o_c = \llbracket Q(q_{out}) \rrbracket^o_{cr}\\ \llbracket V, Q \rrbracket^o_c = \llbracket Q(q_{out}) \rrbracket^o_{cs}\end{array} \qquad (\mathrm{T}^o_c)$$

$$\frac{\begin{array}{c}r = \llbracket b \rrbracket^o_{cr}\\ r' = [\tau \mapsto \sigma]r\end{array}}{\llbracket (b, \langle \sigma, \tau\rangle) \rrbracket^o_{cr} = r'} \qquad (\mathrm{T}^o_c\text{-}\textsc{Relation})$$

$$\llbracket \bullet \rrbracket^o_{cr} = \varnothing \qquad (\mathrm{T}^o_c\text{-}\textsc{Relation}\text{-}\bullet)$$

$$\frac{\begin{array}{c}O_c = \llbracket b \rrbracket^o_{cs}\\ O'_c = [\tau \mapsto \mathit{stream}((b, \langle \sigma, \tau\rangle))]O_c\end{array}}{\llbracket (b, \langle \sigma, \tau\rangle) \rrbracket^o_{cs} = s'} \qquad (\mathrm{T}^o_c\text{-}\textsc{Stream})$$

$$\llbracket \bullet \rrbracket^o_{cs} = \varnothing \qquad (\mathrm{T}^o_c\text{-}\textsc{Stream}\text{-}\bullet)$$

$$\frac{\begin{array}{c}s = \mathit{stream}(b)\\ s' = s \cup \{\langle e, \tau\rangle : e \in \sigma\}\end{array}}{\mathit{stream}(b, \langle \sigma, \tau\rangle) = s'} \qquad (\mathrm{T}^o_c\text{-}\textsc{Stream}\text{-}\textsc{Aux})$$

$$\mathit{stream}(\bullet) = \varnothing \qquad (\mathrm{T}^o_c\text{-}\textsc{Stream}\text{-}\textsc{Aux}\text{-}\bullet)$$

Figure A.1: CQL input and output translation.

input I_b, and the function $\llbracket O_b \rrbracket^o_c$ converts a Brooklet output back to CQL output O_c. Denoting them with semantic brackets $\llbracket \cdot \rrbracket$ is a slight abuse of notation, because the functions do not incorporate any deep semantics, but rather, are a mechanical (though tedious) conversion from one data format to another. However, since we denoted program translation as $\llbracket \cdot \rrbracket^p_c$, denoting input and output translation as $\llbracket \cdot \rrbracket^i_c$ and $\llbracket \cdot \rrbracket^o_c$ leads to more consistent notation. Note that besides the subscript $_c$ for CQL, we introduce the superscripts ^{p,i,o} for program, input, and output, respectively.

The left column of Figure A.1 defines the input translation. It first obtains the set T of all

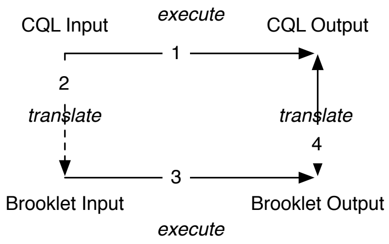

Figure A.2: CQL translation correctness, structural induction base.

"interesting" time stamps, at which either a relation changes, or a stream sends a tuple. Then, it creates Brooklet queue contents, which are sequences of data items where each data item is a tuple of an instantaneous relation and a time stamp in T. All variables are initialized to the empty set, and all input queues are initialized to queue contents.

The right column of Figure A.1 defines the output translation. Depending on whether the CQL program is a relation query or a stream query, the output translation turns the contents of the Brooklet output queue either into a time-varying relation, or into a function from time stamps to the stream of all tuples seen up to that time stamp.

A.2 CQL Main Theorem and Proof

Given the background definitions, we can re-state Theorem 3.1 more clearly:

Theorem 3.1 (CQL translation correctness). For all CQL function environments F_c, programs P_c, and inputs I_c:

$$\to_c^* (F_c, P_c, I_c) = [\![\to_b^* ([\![F_c, P_c]\!]_c^p, [\![I_c]\!]_c^i)]\!]_c^o$$

In other words, executing under CQL semantics ($\to_c^*$) yields the same result as translating the program and the input from CQL to Brooklet ($[\![\cdot]\!]_c^{p,i}$), then executing under Brooklet semantics ($\to_b^*$), and finally translating the output from Brooklet back to CQL ($[\![\cdot]\!]_c^o$). Fig. A.2 illustrates this graphically.

Theorem 3.1. We use an outer structural induction over the query tree, with the two base cases SName (Lemma A.2, stream identity) and RName (Lemma A.3, relation identity), and the three recursive cases S2R, R2S, and R2R (Lemmas A.4, A.5, and A.6). Each of the five cases does an inner induction over time stamps. Each inner base case is the empty set $\varnothing$ of time stamps. Each inner inductive step assumes that the translation is correct for all time stamps in a set T, and proves that the translation is correct when adding another time stamp $\tau > \max T$. □

A.3 Detailed Inductive Proof of CQL Correctness

Lemma A.1 (CQL Program Shape Correspondence)**.** *Every CQL program P_c forms a logical query plan. For all $P_b = [\![\, P_c \,]\!]^p_c$,*

1. *P_b and P_c are trees with equivalent shapes.*

2. *Each node in P_b is a Brooklet operator that corresponds to exactly one CQL operator in P_c.*

3. *Each edge in P_c corresponds to exactly one queue in P_b.*

4. *Each node in P_b has a variable. No variables are shared.*

5. *There is a single root node in both P_b and P_c.*

Lemma A.1. By construction. □

Lemma A.2 (SName Translation Correctness)**.** *For all CQL function environments F_c, inputs I_c, and primitive programs P_c consisting of just a single stream name SName, the first part of the lemma states:*

$$\rightarrow^*_c (F_c, \mathit{SName}, I_c) = [\![\rightarrow^*_b ([\![\, F_c, \mathit{SName} \,]\!]^i_c, [\![\, I_c \,]\!]^i_c) \,]\!]^o_c$$

This equality is a special case of Theorem 3.1. In terms of Fig. A.2, it amounts to showing that $\rightarrow$ = ↓→↑ *in the case where the program P_c consists of a single stream name SName.*

The second part of the lemma states:

$$[\![\rightarrow_c^* (F_c, \mathit{SName}, I_c)]\!]_c^i = \rightarrow_b^* ([\![F_c, \mathit{SName}]\!]_c^i, [\![I_c]\!]_c^i)$$

This equality is used as the assumption for peer lemmas in the outer induction. In terms of Fig. A.2, it amounts to showing that ↴= ↳ *in the case where the program P_c consists of a single stream name SName.*

Lemma A.2. We do an inner induction over the set T of time stamps.

I. Basis: $T = \varnothing$.

→: $\rightarrow_c^* (Q_s, I_c)$

$= \varnothing$

by $\rightarrow_c^* (\mathit{SName})$

↓ : $[\![F_c, \mathit{SName}]\!]_c^i, [\![I_c]\!]_c^i$

$= P_b, Q(\mathit{SName}) = \bullet$

by rule T$_c^i$-Data-s.

↳: $\rightarrow_b^* ([\![F_c, Q_s]\!]_c^i, [\![I_c]\!]_c^i)$

$= Q(\mathit{SName}) = \bullet$

because no Brooklet semantic rule fires.

↴: $[\![\rightarrow_c^* (F_c, \mathit{SName}, I_c)]\!]_c^i$

$= Q(\mathit{SName}) = \bullet$

by rule T$_c^i$-Data-s.

↳↑: $[\![\rightarrow_b^* ([\![F_c, \mathit{SName}]\!]_c^i, [\![I_c]\!]_c^i)]\!]_c^o$

$= \varnothing$

by rule T$_{cs}^o$-$\bullet$

To show: (→ = ↳↑)

$\rightarrow_c^* (F_c, \mathit{SName}, I_c) = \varnothing$

$[\![\rightarrow_b^* ([\![F_c, \mathit{SName}]\!]_c^i, [\![I_c]\!]_c^i)]\!]_c^o = \varnothing$

So, $\rightarrow_c^* (\mathit{SName}, I_c) = \llbracket \rightarrow_b^* (\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) \rrbracket_c^o$.

To show: (↴ = ↳)

$\llbracket \rightarrow_c^* (F_c, \mathit{SName}, I_c) \rrbracket_c^i = \bullet$

$\rightarrow_b^* (\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) = \bullet$

So, $\llbracket \rightarrow_c^* (F_c, \mathit{SName}, I_c) \rrbracket_c^i = \rightarrow_b^* (\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i)$

II. Inductive step: $T' = \{\tau\} \cup T$ where $\tau > \max T$.

i. Assume for T:

→: $\rightarrow_c^* (Q_s, I_c)$

$= s$

↓ : $\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i$

$= P_b, Q(\mathit{SName}) = b_s,$

↳: $\rightarrow_b^* (\llbracket F_c, Q_s \rrbracket_c^i, \llbracket I_c \rrbracket_c^i)$

$= Q(\mathit{SName}) = b_s$

↴: $\llbracket \rightarrow_c^* (F_c, \mathit{SName}, I_c) \rrbracket_c^i$

$= Q(\mathit{SName}) = b_s$

↳↑: $\llbracket \rightarrow_b^* (\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) \rrbracket_c^o$

$= s$

ii. Prove for T':

→: $\rightarrow_c^* (Q_s, I_c)$

$= s \cup s'$

where s is defined in the assumption as the stream of tuples at time T, and s' is the bag of additional tuples for time $\{\tau\}$, according to $\rightarrow_c^* (\mathit{SName})$.

↓ : $\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i$

$= P_b, Q(\mathit{SName}) = b_s, b_{s'}$

by rule T_c^i-Data-s.

↳: $\rightarrow_b^* (\llbracket F_c, Q_s \rrbracket_c^i, \llbracket I_c \rrbracket_c^i)$

$= Q(\mathit{SName}) = b_s, b_{s'}$

because no Brooklet semantic rule fires.

↴: $\llbracket \rightarrow_c^* (F_c, \mathit{SName}, I_c) \rrbracket_c^i$

$= Q(\mathit{SName}) = b_s, b_{s'}$

by rule T$_c^i$-Data-s.

↳↑: $\llbracket \rightarrow_b^* (\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) \rrbracket_c^o$

$= s \cup s'$

by T$_c^o$-Stream

To show: (→ = ↳↑)

$\rightarrow_c^* (F_c, \mathit{SName}, I_c) = s \cup s'$

$\llbracket \rightarrow_b^* (\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) \rrbracket_c^o = s \cup s'$

So, $\rightarrow_c^* (\mathit{SName}, I_c) = \llbracket \rightarrow_b^* (\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) \rrbracket_c^o$.

To show: (↴ = ↳)

$\llbracket \rightarrow_c^* (F_c, \mathit{SName}, I_c) \rrbracket_c^i = b_s, b_{s'}$

$\rightarrow_b^* (\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) = b_s, b_{s'}$

So, $\llbracket \rightarrow_c^* (F_c, \mathit{SName}, I_c) \rrbracket_c^i = \rightarrow_b^* (\llbracket F_c, \mathit{SName} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i)$ □

Lemma A.3 (RName Translation Correctness)**.** *This is the other base case of the outer induction, and is formulated analogously to the SName case in Lemma A.2.*

Lemma A.3. We do an inner induction over the set T of time stamps, which is analogous to the proof for Lemma A.2. □

Lemma A.4 (S2R Translation Correctness)**.** *For all CQL function environments F_c, CQL inputs I_c, CQL stream queries P_{cs}, and CQL S2R operators, assume:*

$$\llbracket \rightarrow_c^* (F_c, P_{cs}, I_c) \rrbracket_c^i = \rightarrow_b^* (\llbracket F_c, P_{cs} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i)$$

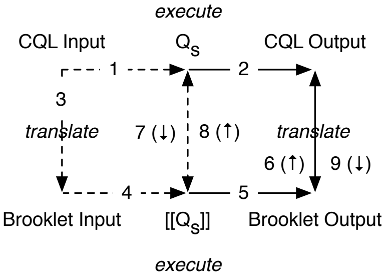

Figure A.3: CQL translation correctness, structural induction step.

This equality is the outer induction assumption, and is proven as part of the peer lemmas for any queries that return streams. In terms of Fig. A.3, it amounts to assuming that $\rightarrow\downarrow = \downarrow\rightarrow$ *for the left part of the diagram.*

The first part of the lemma states:

$$\rightarrow_c^* (F_c, S2R(P_{cs}), I_c) = [\![\rightarrow_b^* ([\![F_c, S2R(P_{cs})]\!]_c^i, [\![I_c]\!]_c^i)]\!]_c^o$$

This equality is a special case of Theorem 3.1. In terms of Fig. A.3, it amounts to showing that $\rightarrow\rightarrow = \downarrow\rightarrow\rightarrow\uparrow$ *in the case where the program* P_c *has the shape* $S2R(P_{cs})$*.*

The second part of the lemma states:

$$[\![\rightarrow_c^* (F_c, S2R(P_{cs}), I_c)]\!]_c^i = \rightarrow_b^* ([\![F_c, S2R(P_{cs})]\!]_c^i, [\![I_c]\!]_c^i)$$

This equality is used as the assumption for peer lemmas in the outer induction. In terms of Fig. A.3, it amounts to showing that $\rightarrow\rightarrow\downarrow = \downarrow\rightarrow\rightarrow$ *in the case where the program* P_c *has the shape* $S2R(P_{cs})$*.*

Lemma A.4. We do an inner induction over the set T of time stamps.

I. Basis: $T = \varnothing$.

$\rightarrow$: $\rightarrow_c^* (P_{cs}, I_c)$

$= \varnothing$

by $\rightarrow_c^* (P_{cs})$

→→: $\rightarrow_c^* \, (S2R(P_{cs}), I_c)$

$= \varnothing$

by $\rightarrow_c^* \, (S2R(P_{cs}))$

→→↓: $\llbracket \rightarrow_c^* \, (S2R(P_{cs}), I_c) \rrbracket_c^i$

$= \bullet$ by rule T_c^i-Data-$\varnothing_2$

↓ : $\llbracket F_c, S2R(P_{cs}) \rrbracket_c^i, \llbracket I_c \rrbracket_c^i$

$= P_b, V(v) = \varnothing, Q(q_o) = \bullet, Q(q_o') = \bullet$

by rule T_c^i-Data-r.

↓→: $\rightarrow_b^* \, (\llbracket F_c, P_{cs} \rrbracket_c^i, \llbracket I_c \rrbracket_c^i)$

$= V(v) = \varnothing, Q(q_o) = \bullet, Q(q_o') = \bullet$

because no Brooklet semantic rule fires.

↓→→: $\rightarrow_b^* \, (\llbracket F_c, S2R(P_{cs}) \rrbracket_c^i, \llbracket I_c \rrbracket_c^i)$

$= V(v) = \varnothing, Q(q_o) = \bullet, Q(q_o') = \bullet$

because no Brooklet semantic rule fires.

↓→→↑: $\llbracket \rightarrow_b^* \, (\llbracket F_c, S2R(P_{cs}) \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) \rrbracket_c^o$

$= \varnothing$

by rule T_{cr}^o-$\bullet$

To show: (→→ = ↓→→↑)

$\rightarrow_c^* \, (S2R(P_{cs}), I_c) = \varnothing$

$\llbracket \rightarrow_b^* \, (\llbracket F_c, S2R(P_{cs}) \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) \rrbracket_c^o = \varnothing$

So, $\rightarrow_c^* \, (S2R(P_{cs}), I_c) = \llbracket \rightarrow_b^* \, (\llbracket F_c, S2R(P_{cs}) \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) \rrbracket_c^o$.

To show: (→→↓ = ↓→→)

$\llbracket \rightarrow_c^* \, (S2R(P_{cs}), I_c) \rrbracket_c^i = \bullet$

$\rightarrow_b^* \, (\llbracket F_c, S2R(P_{cs}) \rrbracket_c^i, \llbracket I_c \rrbracket_c^i) = \bullet$

So,$\llbracket \rightarrow_c^* \, (S2R(P_{cs}), I_c) \rrbracket_c^i = \rightarrow_b^* \, (\llbracket F_c, S2R(P_{cs}) \rrbracket_c^i, \llbracket I_c \rrbracket_c^i)$.

II. Inductive step: $T' = \{\tau\} \cup T$ where $\tau > max\ T$.

i. Assume for T:

→: $\rightarrow_c^* (P_{cs}, I_c)$

$= s$

→→: $\rightarrow_c^* (S2R(P_{cs}), I_c)$

$= r$

→→↓: $[\![\rightarrow_c^* (S2R(P_{cs}), I_c)]\!]_c^i$

$= b_r$

$= \langle r(min\ T), min\ T\rangle \ldots \langle r(max\ T), max\ T\rangle$

↓ : $[\![F_c, S2R(P_{cs})]\!]_c^i, [\![I_c]\!]_c^i$

$= P_b, V, Q$

↳: $\rightarrow_b^* ([\![F_c, P_{cs}]\!]_c^i, [\![I_c]\!]_c^i)$

$V(v) = \varnothing$

$Q(q_o) = b_s$

$Q(q_o') = \bullet$

↳→: $\rightarrow_b^* ([\![F_c, S2R(P_{cs})]\!]_c^i, [\![I_c]\!]_c^i)$

$V(v) = s$

$Q(q_o) = \bullet$

$Q(q_o') = b_r$

↳→↑: $[\![\rightarrow_b^* ([\![F_c, S2R(P_{cs})]\!]_c^i, [\![I_c]\!]_c^i)]\!]_c^o$

$= r$

ii. Prove for T':

→: $\rightarrow_c^* (P_{cs}, I_c)$

$= s \cup s'$

where s is defined in the assumption as the stream of tuples at time T, and s' is the bag of additional tuples for time $\{\tau\}$, according to $\rightarrow_c^* (P_{cs})$.

$\rightarrow\rightarrow$: $\rightarrow_c^* (S2R(P_{cs}), I_c)$

$= F_c(S2R)(s \cup s', \tau)$

$= [\tau \mapsto F_c(S2R)(s \cup s', \tau)]r$

by $\rightarrow_c^* (S2R(P_{cs}))$ and assumption.

$\rightarrow\rightarrow\downarrow$: $[\![\rightarrow_c^* (S2R(P_{cs}), I_c)]\!]_c^i$

$= b_r, [\![F_c(S2R)(s \cup s', \tau)]\!]_c^i$

by assumption and rule T_c^i-Data-r.

$\downarrow$: $[\![F_c, S2R(P_{cs})]\!]_c^i, [\![I_c]\!]_c^i$

$= P_b$

$V(v) = \varnothing$ by rule T_c^i

$Q(q_o) = \bullet$ by rule T_c^i-Data-$\varnothing_2$

$Q(q_o') = \bullet$ by rule T_c^i-Data-$\varnothing_2$

$\downarrow\rightarrow$: $\rightarrow_b^* ([\![F_c, P_{cs}]\!]_c^i, [\![I_c]\!]_c^i)$

$V(v) = \varnothing$ because, by construction, nothing fires that modifies v.

$Q(q_o) = b_s, b_{s'}$ by assumption and E-FireQueue.

$Q(q_o') = \bullet$

$\downarrow\rightarrow\rightarrow$: $\rightarrow_b^* ([\![F_c, S2R(P_{cs})]\!]_c^i, [\![I_c]\!]_c^i)$

$V(v) = s \cup s'$ by rule W_c-S2R.

$Q(q_o) = \bullet$ by rule E-FireQueue.

$Q(q_o') = b_r, [\![F_c(S2R)(s \cup s', \tau)]\!]_c^i$ by assumption, E-FireQueue, W_c-S2R, and T_c^i-Data-r.

$\downarrow\rightarrow\rightarrow\uparrow$: $[\![\rightarrow_b^* ([\![F_c, S2R(P_{cs})]\!]_c^i, [\![I_c]\!]_c^i)]\!]_c^o$

$= [\![b_r, [\![F_c(S2R)(s \cup s', \tau)]\!]_c^i]\!]_c^o$

$= [\tau \mapsto F_c(S2R)(s \cup s', \tau)]r$ because

$F_c(S2R)(s, \tau)$ is a curried version of the meaning function $\rightarrow_c^* (S2R)$ which returns the tuples for time parameter τ rather than all the time stamps in the time duration $\tau_1 \ldots \tau$.

$[\![b_r]\!]_c^o = r$ by assumption, and

$$\llbracket\, \llbracket\, F_c(S2R)(s \cup s', \tau)\, \rrbracket^i_c\, \rrbracket^o_c = F_c(S2R)(s \cup s', \tau) \text{ by } \mathrm{T}^o_c\text{-}\textsc{Relation} \text{ and } \mathrm{T}^i_c\text{-}\textsc{Data-}r$$

To show: (→→ = ↓→→↑)

$\rightarrow^*_c (S2R(P_{cs}), I_c) = [\tau \mapsto F_c(S2R)(s \cup s', \tau)]r$

$\llbracket \rightarrow^*_b (\llbracket F_c, S2R(P_{cs}) \rrbracket^i_c, \llbracket I_c \rrbracket^i_c) \rrbracket^o_c = [\tau \mapsto F_c(S2R)(s \cup s', \tau)]r$

So, $\rightarrow^*_c (S2R(P_{cs}), I_c) = \llbracket \rightarrow^*_b (\llbracket F_c, S2R(P_{cs}) \rrbracket^i_c, \llbracket I_c \rrbracket^i_c) \rrbracket^o_c$.

To show: (→→↓ = ↓→→)

$\llbracket \rightarrow^*_c (S2R(P_{cs}), I_c) \rrbracket^i_c = b_r, \llbracket F_c(S2R)(s \cup s', \tau) \rrbracket^i_c$

$\rightarrow^*_b (\llbracket F_c, S2R(P_{cs}) \rrbracket^i_c, \llbracket I_c \rrbracket^i_c) = b_r, \llbracket F_c(S2R)(s \cup s', \tau) \rrbracket^i_c$

So,$\llbracket \rightarrow^*_c (S2R(P_{cs}), I_c) \rrbracket^i_c = \rightarrow^*_b (\llbracket F_c, S2R(P_{cs}) \rrbracket^i_c, \llbracket I_c \rrbracket^i_c)$.

□

Lemma A.5 (R2S Translation Correctness)**.** *This is the second recursive case of the outer induction, and is formulated analogously to the S2R case in Lemma A.4.*

Lemma A.5. We do an inner induction over the set T of time stamps, which is analogous to the proof for Lemma A.4. □

Lemma A.6 (R2R Translation Correctness)**.** *This is the third recursive case of the outer induction, and is formulated analogously to the S2R case in Lemma A.4.*

Lemma A.6. We do an inner induction over the set T of time stamps, which is analogous to the proof for Lemma A.6. □

B

StreamIt Mapping Details

Below is our syntax and translation for the SDF subset of StreamIt, which is similar to the "literal algebra in canonical form" in the earlier formalization of StreamIt semantics [113]. This section is organized analogously to Fig. 3.2 and Fig. 3.4, except that we moved out the program example into Section 3.3.2.1.

StreamIt syntax:

$$
\begin{array}{rcll}
P_s & ::= & \mathit{ft} \mid \mathit{pl} \mid \mathit{sj} \mid \mathit{fl} & \textit{StreamIt program} \\
\mathit{ft} & ::= & \texttt{filter \{ } \overline{s} \texttt{ work \{ } a\ \overline{ps}\ \overline{pp} \texttt{ \} \}} & \textit{Filter} \\
a & ::= & \overline{s}, \overline{t} \leftarrow f\ \texttt{(}\ \overline{s}, \overline{pk}\ \texttt{);} & \textit{Assign} \\
pk & ::= & \texttt{peek(} x \texttt{);} & \textit{Peek} \\
ps & ::= & \texttt{push(} t \texttt{);} & \textit{Push} \\
pp & ::= & \texttt{pop();} & \textit{Pop} \\
pl & ::= & \texttt{pipeline \{ } \overline{P_s} \texttt{ \}} & \textit{Pipeline} \\
sj & ::= & \texttt{splitjoin \{ } sp\ \overline{P_s}\ jn \texttt{ \}} & \textit{Split-join} \\
\mathit{fl} & ::= & \texttt{feedbackloop \{ } jn \texttt{ body } P_s \texttt{ loop } P_s\ sp \texttt{ \}} & \textit{Feedback loop} \\
sp & ::= & \texttt{split } (\ \texttt{duplicate} \mid \texttt{roundrobin}\ )\texttt{ ;} & \textit{Split} \\
jn & ::= & \texttt{join roundrobin;} & \textit{Join} \\
f|s|t & ::= & \mathit{id} & \textit{Function/state/temporary name} \\
x & ::= & \mathit{int} & \textit{Number}
\end{array}
$$

StreamIt program translation: $[\![\, F_s, P_s \,]\!]^p_s = \langle F_b, P_b \rangle$

$$
\frac{q_{out} = \mathit{freshId}() \qquad q_{in} = \mathit{freshId}() \qquad F_b, \overline{op} = [\![\, F_s, P_s, q_{out}, q_{in} \,]\!]^p_s}{[\![\, F_s, P_s \,]\!]^p_s = F_b, \texttt{output } q_{out}\texttt{; input } q_{in}\texttt{; } \overline{op}} \tag{T^p_s}
$$

$$
\frac{\begin{array}{c} \overline{s}, \overline{t} \leftarrow f\,\texttt{(}\overline{s}, \overline{pk}\texttt{)} = a \qquad f_b = \mathit{freshId}() \\ \forall i \in 1 \ldots |\overline{s}| + 1 : v_i = \mathit{freshId}() \\ F_b = [f_b \mapsto \mathit{wrapFilter}(F_s, a, \overline{ps}, \overline{pp})] \qquad op = \texttt{(}q_{out}, \overline{v}\texttt{)} \leftarrow f_b\texttt{(}q_{in}, \overline{v}\texttt{);} \end{array}}{[\![\, F_s, \texttt{filter\{}\overline{s}\texttt{ work\{}a\,\overline{ps}\,\overline{pp}\texttt{\}\}}, q_{out}, q_{in} \,]\!]^p_s = F_b, op} \tag{T^p_s-Ft}
$$

$$\frac{\begin{array}{c} n = |\overline{P_s}| \qquad \forall i \in 1 \ldots n-1 : q_i = \mathit{freshId}() \\ \forall i \in 1 \ldots n : F_{b_i}, \overline{op}_i = \llbracket\, F_s, P_{s_i}, q_i, q_{i-1} \,\rrbracket^p_s \end{array}}{\llbracket\, F_s, \texttt{pipeline\{}\overline{P_s}\texttt{\}}, q_n, q_0 \,\rrbracket^p_s = \cup\overline{F_b}, \overline{op}_1 \ldots \overline{op}_n} \qquad (\text{T}^p_s\text{-P}\textsc{l})$$

$$\frac{\begin{array}{c} n = |\overline{P_s}| \qquad \forall i \in 1 \ldots n : q_i = \mathit{freshId}() \qquad \forall i \in 1 \ldots n : q'_i = \mathit{freshId}() \\ F_{b_s}, op_s = \llbracket\, F_s, sp, \overline{q}, q_a \,\rrbracket^p_s \qquad \forall i \in 1 \ldots n : F_{b_i}, \overline{op_i} = \llbracket\, F_s, P_{s_i}, q'_i, q_i \,\rrbracket^p_s \\ F_{b_j}, op_j = \llbracket\, F_s, jn, q_z, \overline{q}' \,\rrbracket^p_s \qquad F_b = F_{b_s} \cup (\cup\overline{F_b}) \cup F_{b_j} \qquad \overline{op} = op_s\ \overline{op}\ op_j \end{array}}{\llbracket\, F_s, \texttt{splitjoin \{}sp\ \overline{P_s}\ jn\texttt{\}}, q_z, q_a \,\rrbracket^p_s = F_b, \overline{op}} \qquad (\text{T}^p_s\text{-S}\textsc{j})$$

$$\frac{\begin{array}{c} \forall i \in 1 \ldots 4 : q_i = \mathit{freshId}() \\ F_{b_j}, op_j = \llbracket\, F_s, jn, q_1, (q_0, q_4) \,\rrbracket^p_s \qquad F_{b_b}, \overline{op_b} = \llbracket\, F_s, P_s, q_2, q_1 \,\rrbracket^p_s \\ F_{b_l}, \overline{op_l} = \llbracket\, F_s, P'_s, q_4, q_3 \,\rrbracket^p_s \qquad F_{b_s}, op_s = \llbracket\, F_s, sp, (q_3, q_5), q_2 \,\rrbracket^p_s \\ F_b = F_{b_j} \cup F_{b_b} \cup F_{b_s} \cup F_{b_l} \qquad \overline{op} = op_j\ \overline{op_b}\ op_s\ \overline{op_l} \end{array}}{\llbracket\, F_s, \texttt{feedbackloop\{}jn\ \texttt{body}\ P_s\ \texttt{loop}\ P'_s\ sp\texttt{\}}, q_5, q_0 \,\rrbracket^p_s = F_b, \overline{op}} \qquad (\text{T}^p_s\text{-F}\textsc{l})$$

$$\frac{\begin{array}{c} f = \mathit{freshId}() \qquad op = \texttt{(}\overline{q}\texttt{)} \leftarrow f\texttt{(}q_a\texttt{);} \\ F_b = [f \mapsto \mathit{wrapDupSplit}(|\overline{q}|)] \end{array}}{\llbracket\, F_s, \texttt{split duplicate;}, \overline{q}, q_a \,\rrbracket^p_s = F_b, op} \qquad (\text{T}^p_s\text{-D}\textsc{up}\text{-S}\textsc{plit})$$

$$\frac{\begin{array}{c} f = \mathit{freshId}() \qquad v = \mathit{freshId}() \\ F_b = [f \mapsto \mathit{wrapRRSplit}(|\overline{q}|)] \qquad op = \texttt{(}\overline{q}, v\texttt{)} \leftarrow f\texttt{(}q_a, v\texttt{);} \end{array}}{\llbracket\, F_s, \texttt{split roundrobin;}, \overline{q}, q_a \,\rrbracket^p_s = F_b, op} \qquad (\text{T}^p_s\text{-RR-S}\textsc{plit})$$

$$\frac{\begin{array}{c} f = \mathit{freshId}() \qquad \forall i \in 0 \ldots |\overline{q}'| : v_i = \mathit{freshId}() \\ F_b = [f \mapsto \mathit{wrapRRJoin}(|\overline{q}'|)] \qquad op = \texttt{(}q_z, \overline{v}\texttt{)} \leftarrow f\texttt{(}\overline{q}', \overline{v}\texttt{);} \end{array}}{\llbracket\, F_s, \texttt{join roundrobin;}, q_z, \overline{q}' \,\rrbracket^p_s = F_b, op} \qquad (\text{T}^p_s\text{-RR-J}\textsc{oin})$$

StreamIt domains:

$z \in \mathcal{Z}$	*Data item*
$\ell \in \mathcal{Z}^*$	*List of data items*
$x \in \mathbb{N}$	*Natural number (peek number)*

StreamIt operator signatures:

$$\text{filter} : \mathcal{Z}^* \times \mathcal{Z}^* \rightarrow \mathcal{Z}^*$$

StreamIt operator wrapper signatures:

$$\begin{array}{ll}
\text{wrapFilter} & : \mathcal{Z} \times \{1\} \times \mathcal{Z}^* \to \mathcal{Z}^* \times \mathcal{Z}^* \\
\text{wrapDupSplit} & : \mathcal{Z} \times \{1\} \to \mathcal{Z}* \\
\text{wrapRRSplit} & : \mathcal{Z} \times \{1\} \times \mathbb{N} \to \mathcal{Z}^* \times \mathbb{N} \\
\text{wrapRRJoin} & : \mathcal{Z}^* \times \{1\} \times \mathbb{N} \to \mathcal{Z} \times \mathbb{N}
\end{array}$$

StreamIt operator wrappers:

$$\frac{\begin{array}{c}\overline{s}, \overline{t} \leftarrow f(\overline{s}, \overline{\texttt{peek}(x)}) = a \qquad \overline{z}, \ell = \overline{d}_v \qquad \ell' = \ell, d_{in} \\ |\ell'| \geq |\overline{pp}| \qquad \forall i \in 1 \ldots |\overline{x}| : |\ell'| \geq x_i \qquad \forall i \in 1 \ldots |\overline{x}| : d_i = \ell'_{x_i} \\ \overline{z}', \overline{d}_q = f(\overline{z}, \overline{d}) \qquad r = |\ell'| - |\overline{pp}| \qquad \ell'' = \ell'_{r+1} \ldots \ell'_{|\ell'|}\end{array}}{\mathit{wrapFilter}(a, \overline{ps}, \overline{pp})(d_{in}, \_, \overline{d}_v) = \overline{d}_q, \overline{z}', \ell''} \qquad (\text{W}_s\text{-}\textsc{Filter-Ready})$$

$$\frac{\begin{array}{c}\overline{s}, \overline{t} \leftarrow f(\overline{s}, \overline{\texttt{peek}(x)}) = a \qquad \overline{z}, \ell = \overline{d}_v \qquad \ell' = \ell, d_{in} \\ |\ell'| < |\overline{pp}| \text{ or } \exists i \in 1 \ldots |\overline{x}| : |\ell'| < x_i\end{array}}{\mathit{wrapFilter}(a, \overline{ps}, \overline{pp})(d_{in}, \_, \overline{d}_v) = \bullet, \overline{z}, \ell'} \qquad (\text{W}_s\text{-}\textsc{Filter-Wait})$$

$$\frac{\forall i \in 1 \ldots N : b_i = d_{in}}{\mathit{wrapDupSplit}(N)(d_{in}, \_) = \overline{b}} \qquad (\text{W}_s\text{-}\textsc{Dup-Split})$$

$$\frac{c' = c + 1 \bmod N \qquad b_v = d_{in} \qquad \forall i \in 1 \ldots N, i \neq c : b_i = \bullet}{\mathit{wrapRRSplit}(N)(d_{in}, \_, c) = \overline{b}, c'} \qquad (\text{W}_s\text{-}\textsc{RR-Split})$$

$$\frac{\begin{array}{c}d'_i = d_{in}, d_i \qquad \forall j \neq i \in 1 \ldots N : d'_j = d_j \\ d''_c, d_{out} = d'_c \qquad \forall j \neq c \in 1 \ldots N : d''_j = d'_j \\ b_{out}, c', \overline{d}''' = \mathit{wrapRRJoin}(N)(\bullet, i, c + 1 \bmod N, \overline{d}'')\end{array}}{\mathit{wrapRRJoin}(N)(d_{in}, i, c, \overline{d}) = (b_{out}, d_{out}), c', \overline{d}'''} \qquad (\text{W}_s\text{-}\textsc{RR-Join-Ready})$$

$$\frac{\forall j \neq i \in 1 \ldots N : d'_j = d_j \qquad d'_i = d_{in}, d_i \qquad d_c = \bullet}{\mathit{wrapRRJoin}(N)(d_{in}, i, c, \overline{d}) = \bullet, c, \overline{d}'} \qquad (\text{W}_s\text{-}\textsc{RR-Join-Wait})$$

C
StreamIt Translation Correctness

This section proves Theorem 3.2 in Section 3.3.4.

C.1 Background on StreamIt Formal Semantics

Before we can prove that the semantics of StreamIt on Brooklet are equivalent to previously specified semantics of StreamIt, we recapitulate those semantics from [113]. The StreamIt semantics are specified over three algebras: the *literal algebra*, which is a subset of StreamIt; the *intermediate algebra*, which looks more like Lisp; and the *transform algebra*, which is a closed-form function. We will use the function from the transform algebra to define the StreamIt execution semantics function $\rightarrow_s^*$.

C.1.1 StreamIt Function Environment.

The previous StreamIt semantics are defined on a simple SDF core of StreamIt, called *literal algebra* in [113]. It is mostly identical to the StreamIt syntax that we showed in Section B, except that it does not model state in filters. That means that the canonical form of a filter *ft* looks like this:

```
filter {
  work {
    ($t_1$, ..., $t_{x_{\mathrm{PUSH}}}$) $\longleftarrow$ $f$(peek(0), ..., peek($x_{\mathrm{PEEK}} - 1$));
    push($t_1$); ...; push($t_{x_{\mathrm{PUSH}}}$);
    pop(); ...; pop(); /* $x_{\mathrm{POP}}$ times */
  }
}
```

Filters differ in the integers x_{PUSH}, x_{PEEK}, and x_{POP}, and in the name of the function f. The StreamIt semantics treat the function f as a black-box. We denote by F_s the StreamIt function store, which maps from function names to opaque functions. If a filter peeks and pushes x_{PEEK}

and x_{PUSH} data items, respectively, and the filter uses the function name f, then the function $F_s(f)$ in the function environment has the following signature:

$$F_s(f) : \mathcal{Z}^{x_{\text{PEEK}}} \to \mathcal{Z}^{x_{\text{PUSH}}}$$

This is a pure function invoked each time the filter receives a data item. Our formalization resembles the formalization in [113] in that it uses black-box functions to abstract away local deterministic computation. Our formalization of the literal algebra differs from that in [113] in that we have a single function that returns multiple values to push, instead of having a separate function for each value to push. We adjust the remainder of this section to account for this difference. We also picked different letters in some cases, such as x for integers and $\mathcal{Z}$ for the domain of StreamIt data items.

C.1.2 StreamIt Intermediate Algebra.

The StreamIt intermediate algebra (SIA) has a Lisp-like syntax:

StreamIt intermediate algebra syntax:

$$\begin{array}{lll}
P_{SIA} & ::= ft_{SIA} \mid pl_{SIA} \mid sj_{SIA} \mid fl_{SIA} & \textit{SIA program} \\
ft_{SIA} & ::= \texttt{(filter}\ x\ x\ x\ \overline{f}\texttt{)} & \textit{SIA filter} \\
pl_{SIA} & ::= \texttt{(pipeline}\ \overline{P_{SIA}}\texttt{)} & \textit{SIA pipeline} \\
sj_{SIA} & ::= \texttt{(splitjoin}\ sp_{SIA}\ \overline{P_{SIA}}\ jn_{SIA}\texttt{)} & \textit{SIA split-join} \\
fl_{SIA} & ::= \texttt{(feedbackloop}\ jn_{SIA}\ P_{SIA}\ P_{SIA}\ sp_{SIA}\texttt{)} & \textit{SIA feedback loop} \\
sp_{SIA} & ::= \texttt{duplicate} \mid \texttt{roundrobin} & \textit{SIA split type} \\
jn_{SIA} & ::= \texttt{roundrobin} & \textit{SIA join type}
\end{array}$$

The formalization in [113] only sketches an incomplete translation from the literal algebra to the intermediate algebra. Here, we present the complete translation, filling in the missing details and expressing everything in a notation consistent with the rest of this chapter. We will denote the translation as $[\![\,\cdot\,]\!]^{p}_{SIA}$. The input is a StreamIt function environment F_s and program P_s, and the output is an SIA function environment F_{SIA} and program P_{SIA}.

$$\frac{\overline{z}' = f(z_{g_{\text{local}}(0)}, \ldots, z_{g_{\text{local}}(x_{\text{PEEK}}-1)})}{\mathit{wrap}_{SIA}(f, i, x_{\text{PEEK}})(g_{\text{local}})(\overline{z}) = z'_i} \qquad (\text{W}_{SIA}\text{-}\textsc{Ft})$$

$$\frac{\begin{array}{c}\forall i \in 1 \ldots |ps| : f_i = \mathit{freshId}() \\ F_{SIA} = [f_1 \mapsto \mathit{wrap}_{SIA}(F_s(f), 1, |pk|), \ldots, f_{|ps|} \mapsto \mathit{wrap}_{SIA}(F_s(f), |ps|, |pk|)] \\ P_{SIA} = \texttt{(filter}\ |ps|\ |pp|\ |pk|\ \overline{f}\texttt{)}\end{array}}{\llbracket\, F_s, \texttt{filter\{work\{}\overline{t}\texttt{<-}f\texttt{(}\overline{pk}\texttt{);}\overline{ps}\ \overline{pp}\texttt{\}\}}\,\rrbracket^p_{SIA} = \langle F_{SIA}, P_{SIA} \rangle} \tag{T$^p_{SIA}$-Ft}$$

The function environment F_{SIA} of the intermediate algebra is populated with one function for each `push` statement in the literal syntax. These functions differ from the functions in the original environment in that they obtain their parameters directly from a local index transform g_{local} and a tape $\overline{z}$. An *index transform* is a function from integers to integers, and a tape is a sequence of data items. Therefore, the signature of a function $F_{SIA}(f)$ is:

$$F_{SIA}(f) : (\mathbb{N} \to \mathbb{N}) \to (\mathcal{Z}^* \to \mathcal{Z})$$

The local index transform $g_{\text{local}} \in (\mathbb{N} \to \mathbb{N})$ maps the peek index $0 \le i < x_{\text{PEEK}}$ to a tape index, and the tape $\overline{z} \in \mathcal{Z}^*$ is the sequence of all data items that ever travel on some stream. Hence, the indirect subscript $z_{g_{\text{local}}(i)}$ reads a data item from the tape.

We just saw the $\llbracket \cdot \rrbracket^p_{SIA}$ translation rule for filters. The translation rules for recursive syntax (pipeline, split-join, and feedback loop) are fairly straightforward, since they only make superficial syntactic changes:

$$\frac{\begin{array}{c}\forall i \in 1 \ldots |\overline{P_s}| : \langle F_{SIA_i}, P_{SIA_i} \rangle = \llbracket\, F_s, P_{s_i} \,\rrbracket^p_{SIA} \\ F_{SIA} = \cup\overline{F_{SIA}} \qquad P_{SIA} = \texttt{(pipeline}\ \overline{P_{SIA}}\texttt{)}\end{array}}{\llbracket\, F_s, \texttt{pipeline\{}\overline{P_s}\texttt{\}}\,\rrbracket^p_{SIA} = \langle F_{SIA}, P_{SIA} \rangle} \tag{T$_{SIA}$-Pl}$$

$$\frac{\begin{array}{c}\forall i \in 1 \ldots |\overline{P_s}| : \langle F_{SIA_i}, P_{SIA_i} \rangle = \llbracket\, F_s, P_{s_i} \,\rrbracket^p_{SIA} \\ F_{SIA} = \cup\overline{F_{SIA}} \qquad P_{SIA} = \texttt{(splitjoin}\ \llbracket\, sp \,\rrbracket^p_{SIA}\ \overline{P_{SIA}}\ \llbracket\, jn \,\rrbracket^p_{SIA}\texttt{)}\end{array}}{\llbracket\, F_s, \texttt{splitjoin\{}sp\ \overline{P_s}\ jn\texttt{\}}\,\rrbracket^p_{SIA} = \langle F_{SIA}, P_{SIA} \rangle} \tag{T$_{SIA}$-Sj}$$

$$\frac{\begin{array}{c}\langle F'_{SIA}, P'_{SIA} \rangle = \llbracket\, F_s, P'_s \,\rrbracket^p_{SIA} \qquad \langle F''_{SIA}, P'_{SIA} \rangle = \llbracket\, F_s, P''_s \,\rrbracket^p_{SIA} \\ F_{SIA} = F'_{SIA} \cup F''_{SIA} \qquad P_{SIA} = \texttt{(feedbackloop}\ \llbracket\, jn \,\rrbracket^p_{SIA}\ P'_{SIA}\ P''_{SIA}\ \llbracket\, sp \,\rrbracket^p_{SIA}\texttt{)}\end{array}}{\llbracket\, F_s, \texttt{feedbackloop\{}jn\ P'_s\ P''_s\ sp\texttt{\}}\,\rrbracket^p_{SIA} = \langle F_{SIA}, P_{SIA} \rangle} \tag{T$_{SIA}$-Fl}$$

$$\begin{array}{c}\llbracket\, \texttt{split duplicate} \,\rrbracket^p_{SIA} = \texttt{duplicate} \\ \llbracket\, \texttt{split roundrobin} \,\rrbracket^p_{SIA} = \texttt{roundrobin}\end{array} \tag{T$_{SIA}$-Sp}$$

$$[\![\,\texttt{join roundrobin}\,]\!]^{p}_{SIA} = \texttt{roundrobin} \qquad (\textsc{T}_{SIA}\text{-}\textsc{Jn})$$

C.1.3 StreamIt Execution Semantics Function.

The previous formal semantics for StreamIt [113] is a denotational semantics with a meaning function $\mathcal{M}$. We denote the meaning function as $\rightarrow_s^*$ instead for consistency with the rest of the chapter. It is defined by a translation $[\![\,\cdot\,]\!]^{p}_{STA}$, where STA stands for *StreamIt transform algebra.* Here is the definition:

$$\begin{aligned} \rightarrow_s^* (F_s, P_s, I_s) = \quad & \text{let } \langle F_{SIA}, P_{SIA} \rangle = [\![\, F_s, P_s \,]\!]^{p}_{SIA} \text{ in} \\ & \text{let } f = [\![\, F_{SIA}, P_{SIA} \,]\!]^{p}_{STA} \text{ in} \\ & f(g_{\text{id}})(I_s) \end{aligned}$$

In other words, first use $[\![\,\cdot\,]\!]^{p}_{SIA}$ to transform from the literal algebra to the intermediate algebra, then use $[\![\,\cdot\,]\!]^{p}_{STA}$ to transform from the intermediate algebra to the transform algebra. The transform algebra result is a higher-order function f, which depends on a global stream transform for taking care of any index shifts incurred by splitters. Since we are at the outermost level, we pass in the identity index transform g_{id}. After applying the index transform, the resulting function f takes a sequence I_s of data items as a parameters, which is the tape that serves as the single input to the entire StreamIt program. In other words, the signature of $f = [\![\,\cdot\,]\!]^{p}_{STA}$ is:

$$f : (\mathbb{N} \rightarrow \mathbb{N}) \rightarrow (\mathcal{Z}^* \rightarrow \mathcal{Z}^*)$$

Next, we look at the $[\![\,\cdot\,]\!]^{p}_{STA}$ rule for filters.

$$\mathit{sdep}((\texttt{filter}\; x_{\text{PUSH}}\; x_{\text{POP}}\; x_{\text{PEEK}}\; \texttt{\_}))(i'_{\text{global}}) = \lceil \tfrac{i'_{global}}{x_{\text{PUSH}}} \rceil \cdot x_{\text{POP}} + x_{\text{PEEK}} - x_{\text{POP}} \qquad (\textsc{W}_{STA}\text{-}\textsc{Ft})$$

$$\frac{i_{\text{global}} = g_{\text{global}}(\mathit{sdep}(\mathit{ft}_{SIA})(i'_{\text{global}}) - i_{\text{local}})}{\mathit{localIndexTransform}(\mathit{ft}_{SIA}, g_{\text{global}}, i'_{\text{global}})(i_{\text{local}}) = i_{\text{global}}} \qquad (\textsc{W}_{STA}\text{-}\textsc{Ft-Local})$$

$$\frac{\begin{array}{c} g_{\text{local}} = \mathit{localIndexTransform}(\mathit{ft}_{SIA}, g_{\text{global}}, i'_{\text{global}}) \\ (\texttt{filter}\; x_{\text{PUSH}}\; \texttt{\_}\;\texttt{\_}\; \overline{f}) = \mathit{ft}_{SIA} \end{array}}{\left([\![\, F_{SIA}, \mathit{ft}_{SIA} \,]\!]^{p}_{STA}(g_{\text{global}})(\overline{z})\right)_{i'_{\text{global}}} = F_{SIA}(f_{i'_{\text{global}} \bmod x_{\text{PUSH}}})(g_{\text{local}})(\overline{z})} \qquad (\textsc{T}^{p}_{STA}\text{-}\textsc{Ft})$$

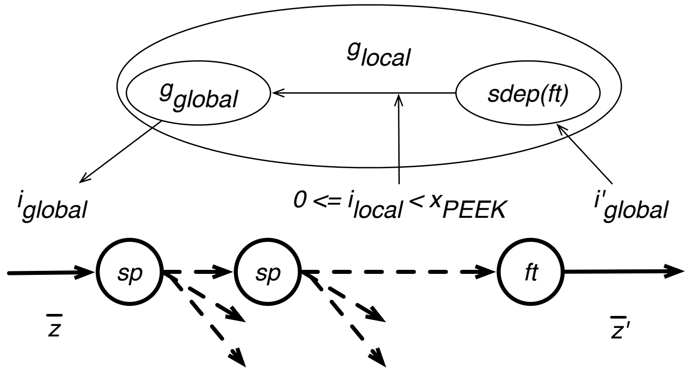

Figure C.1: StreamIt index transforms for a filter.

The local index $0 \leq i_{\text{local}} < x_{\text{PEEK}}$ is the peek number. The global index i'_{global} is the index on the output from the filter, and the global index i_{global} is the index on an upstream tape $\overline{z}$ that is separated from the filter by zero or more splitters. The local index transform g_{local} transforms the local index i_{local} to i_{global}. It uses a helper function $\mathit{sdep}(\mathit{ft}_{SIA})$ that turns a global index at a filter output into the global index at the filter input that it depends on. The $\mathit{sdep}(\mathit{ft}_{SIA})$ function is computed based on the data rates x_{PUSH}, x_{POP}, and x_{PEEK} of the filter. The local index transform also subtracts the local index i_{local} to peek into the past, and finally uses the global index transform g_{global} to take care of any transformation imposed by the context of splitters. Figure C.1 illustrates these transformations. Dashed lines represent intermediate streams that the index transforms skip past.

Next, we look at the $[\![\cdot]\!]^p_{STA}$ rule for pipelines.

$$[\![F_{SIA}, \texttt{(pipeline } P_{SIA}\texttt{)}]\!]^p_{STA}(g_{\text{global}}) = [\![F_{SIA}, P_{SIA}]\!]^p_{STA}(g_{\text{global}}) \qquad (\text{T}_{STA}\text{-}\textsc{Pl-Base})$$

$$\frac{\begin{array}{c}\overline{z}' = [\![F_{SIA}, \texttt{(pipeline } \overline{P_{SIA}}\texttt{)}]\!]^p_{STA}(g_{\text{global}})(\overline{z}) \\ \overline{z}'' = [\![F_{SIA}, P_{SIA}]\!]^p_{STA}(g_{\text{id}})(\overline{z}')\end{array}}{[\![F_{SIA}, \texttt{(pipeline } \overline{P_{SIA}}, P_{SIA}\texttt{)}]\!]^p_{STA}(g_{\text{global}})(\overline{z}) = \overline{z}''} \qquad (\text{T}_{STA}\text{-}\textsc{Pl})$$

The base case of a pipeline with just one stage translates that stage in the context of the global index transform. The recursive case transforms the input sequence $\overline{z}$ of data items by first running it through all pipeline stages but one to get $\overline{z}'$, and then running it through the last pipeline stage to get $\overline{z}''$, in the context of an identity index transform g_{id} because there is no

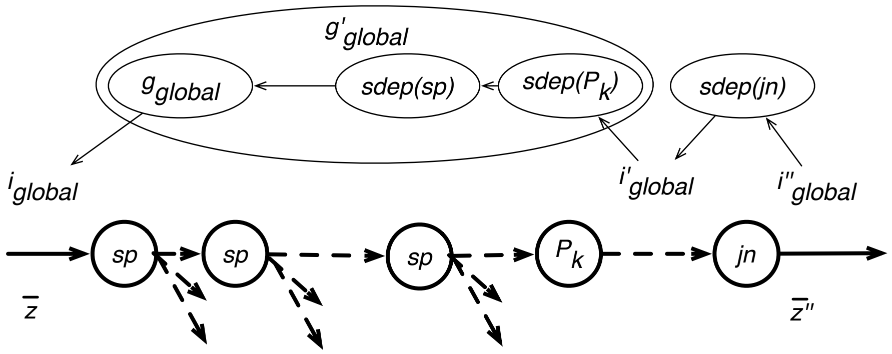

Figure C.2: StreamIt index transforms for a split-join.

additional splitter before the last pipeline stage.

Before we can develop the $[\![\cdot]\!]^p_{STA}$ rule for split-joins, we need another *sdep* function that describes how to turn the global index at the output of a pipeline into the global index at its input. The function is defined using a simple recursion:

$$sdep(\texttt{(pipeline } P_{SIA}\texttt{)})(i'_{\text{global}}) = sdep(P_{SIA})(i'_{\text{global}}) \qquad (\text{W}_{STA}\text{-PL-BASE})$$

$$sdep(\texttt{(pipeline } \overline{P_{SIA}}, P_{SIA}\texttt{)})(i'_{\text{global}}) = sdep(P_{SIA})(sdep(\texttt{(pipeline } \overline{P_{SIA}}\texttt{)})(i'_{\text{global}})) \qquad (\text{W}_{STA}\text{-PL})$$

To develop the *sdep* function for a split-join, we develop separate *sdep* functions for splitters and joiners.

$$sdepSplit(\texttt{duplicate}, \_, \_)(i_{\text{global}}) = i_{\text{global}} \qquad (\text{W}_{STA}\text{-SP-DUP})$$

$$sdepSplit(\texttt{roundrobin}, n, k)(i_{\text{global}}) = n \cdot i_{\text{global}} + k \qquad (\text{W}_{STA}\text{-SP-RR})$$

$$\frac{i_{\text{global}} \bmod n < k}{sdepJoin(\texttt{roundrobin}, n, k)(i_{\text{global}}) = \lfloor \frac{i_{\text{global}}}{n} \rfloor + 0} \qquad (\text{W}_{STA}\text{-JN-RR-0})$$

$$\frac{i_{\text{global}} \bmod n \geq k}{sdepJoin(\texttt{roundrobin}, n, k)(i_{\text{global}}) = \lfloor \frac{i_{\text{global}}}{n} \rfloor + 1} \qquad (\text{W}_{STA}\text{-JN-RR-1})$$

$$\frac{\begin{array}{ccc} n = |\overline{P_{SIA}}| & k = |\overline{P_{SIA}}| - 1 & i''_{\text{global}} = sdepJoin(jn_{SIA}, n, k)(i'''_{\text{global}}) \\ i'_{\text{global}} = sdep(P_{SIA_k})(i''_{\text{global}}) & & i_{\text{global}} = sdepSplit(sp_{SIA}, n, k)(i'_{\text{global}}) \end{array}}{sdep(\texttt{(splitjoin } sp_{SIA}\ \overline{P_{SIA}}\ jn_{SIA}\texttt{)})(i'''_{\text{global}}) = i_{\text{global}}} \qquad (\text{W}_{STA}\text{-SJ})$$

Now, we can formulate the translation $[\![\cdot]\!]^p_{STA}$ rule for split-joins.

$$\frac{\begin{array}{c}(\texttt{splitjoin}\ sp_{SIA}\ \overline{P_{SIA}}\ jn_{SIA}) = sj_{SIA} \qquad n = |\overline{P_{SIA}}| \qquad k = i''_{\text{global}} \bmod n \\ f_{STA} = [\![\, F_{SIA}, P_{SIA_k} \,]\!]^p_{STA} \qquad i'_{\text{global}} = sdep(jn_{SIA}, n, k)(i''_{\text{global}}) \\ g'_{\text{global}} = g_{\text{global}} \circ sdep(sp_{SIA}, n, k) \circ sdep(P_{SIA_k})\end{array}}{\big([\![\, F_{SIA}, sj_{SIA} \,]\!]^p_{SIA}(g_{\text{global}})(\overline{z})\big)_{i''_{\text{global}}} = \big(f_{STA}(g'_{\text{global}})(\overline{z})\big)_{i'_{\text{global}}}} \qquad (\mathrm{T}_{STA}\text{-}\mathrm{SJ})$$

Fig. C.2 illustrates the index transformations. The data item at index i''_{global} on the output from the split-join is the same as the data item at index i'_{global} on the output from the kth subprogram. The kth subprogram operates in the context of an index transform that incorporates the context of the split-join, the index shift of the splitter, and the index shift of the subprogram itself.

We do not model feedback loops here, because they were missing from the transform algebra in [112].

C.1.4 StreamIt Input and Output Translation.

In general, the StreamIt domains for input and output are as follows:

StreamIt domains for input and output:

$I_s \in (id \to z) \times \mathcal{Z}^*$	*StreamIt input*
$O_s \in \mathcal{Z}^*$	*StreamIt output*

The input I_s consists of a variable store V in the domain $id \to z$ and a sequence $\overline{z}$ of data items $\overline{z} \in \mathcal{Z}^*$. The variable store contains initial contents of explicit state variables, and contains the data to kick-start feedback loops. However, since the StreamIt semantics in [112] do not model explicit variables or feedback loops, they only refer to the sequence $\overline{z}$ of input data items. The output O_s, in their case as well as ours, is a simple sequence $\overline{z}'$ of data items.

StreamIt input translation: $[\![\, I_s \,]\!]^i_s = \langle V, Q\rangle$

$$\frac{V, \overline{z} = I_s \qquad Q = [q_{in} \mapsto \overline{z}]}{[\![\, I_s \,]\!]^i_s = \langle V, Q\rangle} \qquad (\mathrm{T}^i_s)$$

StreamIt output translation: $[\![\, V, Q \,]\!]^o_s = O_s$

$$[\![\, V, Q \,]\!]^o_s = Q(q_{out}) \qquad (\mathrm{T}^o_s)$$

Figure C.3: StreamIt input and output translation.

The input translation $[\![\, I_s \,]\!]^i_s$ translates a StreamIt input I_s into a Brooklet input by copying the variable store and by initializing the input queue q_{in} with the sequence of data items. The output translation $[\![\, O_b \,]\!]^o_s$ translates a Brooklet output O_b into a StreamIt output by retrieving the sequence of data items from the output queue q_{out}. Note the superscripts ^{i,o} that distinguish

$$\begin{tikzcd}
\text{Input} & \text{Execute} & \text{Output} \\
F_s, P_s, I_s \arrow[rr, "\rightarrow_s^*"] \arrow[d, "\text{Translate}\ \llbracket \cdot \rrbracket_s^{p,i}"'] & & O_s \\
F_b, P_b, I_b \arrow[rr, "\rightarrow_b^*"'] & & O_b \arrow[u, "\llbracket \cdot \rrbracket_s^{o}\ \text{Translate}"'] \\
\text{Input} & \text{Execute} & \text{Output}
\end{tikzcd}$$

Figure C.4: StreamIt translation correctness.

input and output translation from program translation $\llbracket \cdot \rrbracket^p$. The translations are formalized in Fig. C.3.

C.2 StreamIt Main Theorem and Proof

The previous denotational semantics for StreamIt [112] model neither stateful filters nor feedback loops. In our semantics by translation to Brooklet, on the other hand, we model both features. This was easy to do with our semantics, because it is small-step operational. But because the features are missing from the previous semantics, a proof can only show equivalence for programs that do not use them. With that in mind, we can re-state Theorem 3.2 more clearly:

Theorem 3.2 (StreamIt translation correctness. For all StreamIt function environments F_s, programs P_s, and inputs I_s, where the program P_s uses neither stateful filters nor feedback loops:

$$\rightarrow_s^* (F_s, P_s, I_s) = \llbracket \rightarrow_b^* (\llbracket F_s, P_s \rrbracket_s^p, \llbracket I_s \rrbracket_s^i) \rrbracket_s^o$$

In other words, executing under StreamIt semantics ($\rightarrow_s^*$) yields the same result as translating the program and the input from StreamIt to Brooklet ($\llbracket \cdot \rrbracket_c^{p,i}$), then executing under Brooklet semantics ($\rightarrow_b^*$), and finally translating the output from Brooklet back to StreamIt ($\llbracket \cdot \rrbracket_c^o$). Fig. C.4 illustrates this graphically.

Theorem 3.2. We use an outer structural induction over the program topology, with the base case Filter (Lemma C.1) and the two recursive cases Pipeline and Split-Join (Lemmas C.2 and C.3; there is no lemma for feedback loops, because they are missing in [112]). □

C.3 Detailed Inductive Proof of StreamIt Correctness

Lemma C.1 (Filter Translation Correctness)**.** *Theorem 3.2 holds for the special case where the StreamIt program P_s is a simple stateless filter.*

Lemma C.1. Without loss of generality, we assume that the filter *ft* is in canonical form, repeated here for convenience:

```
filter { work {
  ($t_1$, ..., $t_{x_{\mathrm{PUSH}}}$) $\longleftarrow$ $f$(peek(0), ..., peek($x_{\mathrm{PEEK}} - 1$));
  push($t_1$); ...; push($t_{x_{\mathrm{PUSH}}}$);
  pop(); ...; pop(); /* $x_{\mathrm{POP}}$ times */
} }
```

Let $\overline{z} = I_s$ be the StreamIt input and $\overline{z}' = O_s$ be the StreamIt output, and let i be an index in the program output. We will derive the result for z_i along the different edges in the commuting diagram in Fig. C.4.

$\rightarrow$: Along this edge, we have $z_i = \big(\rightarrow_s^* (F_s, ft, \overline{z}) \big)_i$, which we can rewrite directly by expanding out the definition of $\rightarrow_s^*$. This rewriting eventually leads us to the following closed-form expression for z_i:

$$\begin{aligned}
z_i &= \big(\rightarrow_s^* (F_s, \mathit{ft}, \overline{z})\big)_i \\
&= \big(\llbracket \llbracket F_s, \mathit{ft} \rrbracket^p_{SIA} \rrbracket^p_{STA}(g_{id})(\overline{z})\big)_i \\
&= \big(\llbracket F_{SIA}, \mathit{ft}_{SIA} \rrbracket^p_{STA}(g_{id})(\overline{z})\big)_i \\
&= F_{SIA}(f_{i \bmod x_{\mathrm{PUSH}}})(\mathit{localIndexTransform}(\mathit{ft}_{SIA}, g_{id}, i))(\overline{z}) \\
&= F_{SIA}(f_{i \bmod x_{\mathrm{PUSH}}})(\lambda i_{local} : g_{id}(\mathit{sdep}(\mathit{ft}_{SIA})(i) - i_{local})(\overline{z}) \\
&= F_{SIA}(f_{i \bmod x_{\mathrm{PUSH}}})(\lambda i_{local} : \mathit{sdep}(\mathit{ft}_{SIA})(i) - i_{local}))(\overline{z}) \\
&= \mathit{wrap}_{SIA}(F_s(\mathtt{f}), i \bmod x_{PUSH}, x_{\mathrm{PEEK}})(\lambda i_{local} : \mathit{sdep}(\mathit{ft}_{SIA})(i) - i_{local}))(\overline{z}) \\
&= F_s(\mathtt{f})(z_{\mathit{sdep}(\mathit{ft}_{SIA})(i)-0}, \dots, z_{\mathit{sdep}(\mathit{ft}_{SIA})(i)-x_{\mathrm{PEEK}}+1})_{(i \bmod x_{\mathrm{PUSH}})} \\
&= F_s(\mathtt{f})\big(z_{(\lceil \frac{i}{x_{PUSH}} \rceil \cdot x_{\mathrm{POP}} + x_{\mathrm{PEEK}} - x_{\mathrm{POP}} - 0)}, \dots, z_{(\lceil \frac{i}{x_{PUSH}} \rceil \cdot x_{\mathrm{POP}} + x_{\mathrm{PEEK}} - x_{\mathrm{POP}} - x_{\mathrm{PEEK}} + 1)}\big)_{(i \bmod x_{\mathrm{PUSH}})} \\
&= F_s(\mathtt{f})\big(z_{(\lceil \frac{i}{x_{PUSH}} \rceil \cdot x_{\mathrm{POP}} + x_{\mathrm{PEEK}} - x_{\mathrm{POP}})}, \dots, z_{(\lceil \frac{i}{x_{PUSH}} \rceil \cdot x_{\mathrm{POP}} - x_{\mathrm{POP}} + 1)}\big)_{(i \bmod x_{\mathrm{PUSH}})}
\end{aligned}$$

↓ : This edge performs the program translation and input translation from StreamIt to Brooklet. We end up with the following Brooklet function environment F_b, program P_b, and input $I_b = \langle Q, V\rangle$:

$$\begin{aligned}
F_b &= [\mathtt{f} \mapsto \mathit{wrapFilter}(\mathit{ft})] \\
P_b &= \mathtt{output}\ \mathtt{q_{out}}\mathtt{;}\ \mathtt{input}\ \mathtt{q_{in}}\mathtt{;}\ \mathtt{(q_{out},v)} \longleftarrow \mathtt{f(q_{in},v);} \\
Q &= [\mathtt{q_{in}} \mapsto \overline{z}, \mathtt{q_{out}} \mapsto \bullet] \\
V &= [v \mapsto \bullet]
\end{aligned}$$

↳ : Along this edge, we are determining the ith data item that gets pushed onto the output queue $\mathtt{q_{out}}$. This data item occurs when rule W$_s$-FILTER-READY has fired $\lceil \frac{i}{x_{\mathrm{PUSH}}} \rceil$ times. At that point, the contents of the variable $V(\mathtt{v})$ together with the last data item that triggered the firing consists of x_{PEEK} data items. These data items come from consecutive indices of $Q(\mathtt{q_{in}})$ by construction of the wrappers. The index of the last data item that triggered the firing is $\langle \frac{i}{x_{\mathrm{PUSH}}} \rangle + 1 - x_{\mathrm{POP}})$, because at each firing of the W$_s$-FILTER-READY rule, we pop x_{POP} inputs and push x_{PUSH} outputs. Hence, the data items that the wrapper function $\mathit{wrapFilter}(\mathit{ft})$ passes to the wrappee function $F_s(\mathtt{f})$ are:

$$z_{(\langle\frac{i}{x_{\text{PUSH}}}\rangle+x_{\text{PEEK}}-x_{\text{POP}})}, \ldots, z_{(\langle\frac{i}{x_{\text{PUSH}}}\rangle+1-x_{\text{POP}})}$$

In other words, we invoke the same function on the same parameters as in the → case. The result is the sequence of data items to push on the output, and we get the same result z_i as in the → case by subscripting with $(i \bmod x_{\text{PUSH}})$.

↳↑: Along this edge, we have:

$$([\![\, O_b \,]\!]_b^o)_i = (Q(\mathtt{q}_{\mathtt{out}})_i)$$

which is the same data item computed by the ↳ step.

Both of the calculations (→ and ↳↑) result in the same data item for the ith position of the output. Since our argument holds for any i, the outputs are fully equal. □

Lemma C.2 (Pipeline Translation Correctness)**.** *Assuming the translations for all sub-programs in the pipeline are correct, Theorem 3.2 holds for the special case where the StreamIt program P_s is a pipeline.*

Lemma C.2. We use the assumption from the outer induction to equate the → and ↳↑ cases for each individual stage. We do an inner induction over the number of pipeline stages. In each stage, the output queue contents of the previous stage serve as inputs to the next stage. □

Lemma C.3 (Split-Join Translation Correctness)**.** *Assuming the translations for all sub-programs in the split-join are correct, Theorem 3.2 holds for the special case where the StreamIt program P_s is a split-join.*

Lemma C.3. We chose an arbitrary but fixed subprogram index. We express the contents of the input tape of that subprogram by using an index transform on the StreamIt side, but direct split operator execution on the Brooklet side. Then, we use the assumption from the outer induction to equate the → and ↳↑ cases for that subprogram. Finally, we express the contents of the output tape of the entire split-join by using, again, an index transform on the StreamIt side and direct join operator execution on the Brooklet side. □

D

Data Parallelism Optimization Correctness

This section sketches the proof for Theorem 3.3 in Section 3.4.1.

Let b_{in} and b_{out} be the sequences of all data items that ever appear on queues q_{in} and q_{out}, respectively. Because every Brooklet queue is defined only once and because *op* is stateless, b_{in} fully determines b_{out}. Since f commutes, b_{out} has the same contents in both P_b and P'_b. Since round-robin split-joins preserve order, b_{out} has the same order in both P_b and P'_b.

E

Fusion Optimization Correctness

This section sketches the proof for Theorem 3.4 in Section 3.4.2.

Let b_{in} and b_{out} be the sequences of all data items that ever appear on queues q_{in} and q_{out}, respectively. Because every queue is defined only once and because only op_1 and op_2 write v_1 and v_2, b_{in} fully determines b_{out}. We can show that b_{out} is the same for both P_b and P'_b by induction over b_{in}.

F

Selection Hoisting Optimization Correctness

This section sketches the proof for Theorem 3.5 in Section 3.4.3.

The input data on $\overline{q}$ fully determines the output data on q_{out}, because the operators are stateless. We can show by induction over the input data that the output data is the same for both P_b and P'_b. The proof relies on the fact that f_1 only reads data forwarded unmodified by f_2 and vice versa.