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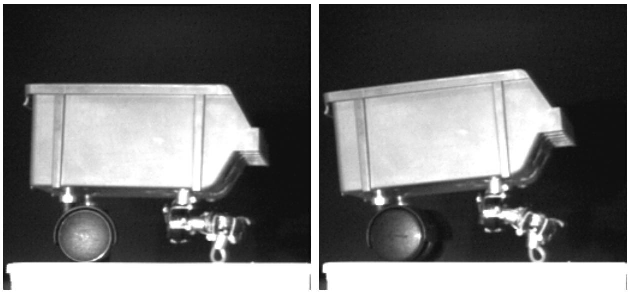

Figure 6: A prototype one legged rickshaw showing the direct drive leg fully compressed (left) and fully extended (right). This robot rickshaw made a few short hops forward during the 90 seconds of runtime before the prototype leg burned up, a result of not taking care of heat dissipation in the coils of this leg. Each leg link is about 4.5 cm in length.

angles of the three links, in order from the hip joint, are $-\pi/2$, $\pi/2$ and $-\pi/2$ respectively. The leg is a three link redundant planar manipulator having the Jacobian matrix

$$J = \ell \begin{pmatrix} -s_1 - s_{12} - s_{123} & -s_{12} - s_{123} & -s_{123} \\ c_1 + c_{12} + c_{123} & c_{12} + c_{123} & c_{123} \end{pmatrix} \tag{2}$$

in a coordinate frame attached to the hip. We can approximate the Jacobian this case, using the fact that the leg joints rotate only slightly around the angles $\theta_1 \approx -\pi/2$, $\theta_2 \approx \pi/2$, and $\theta_3 \approx -\pi/2$, to

$$J \approx \ell \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \tag{3}$$

For the leg depicted in Figure 6 the link length $\ell = 4.5cm$. We have attained joint velocities of more than $2800°/sec$ in our direct drive actuators. Certain commercial permanent magnet motors achieve velocities of $10,000°/sec$. Under the simplifying assumption that all three joints may reach this velocity, i.e. $\theta'_1 = \theta'_2 = \theta'_3 = \theta'$, the foot velocity in the x direction is about $4\theta'$. If θ' reaches $10,000°/sec$, then a small running robot may one day surpass the speed of the fastest running mammals.

5 Conclusion

More than just to position themselves, we want robots to exert forces on the world. Direct drive so far is the most promising means to achieve robot force control, a trend proven by the commercial success of industrial direct drive assembly robots. We are now taking the direct drive concept down to a small but not microscopic scale. The cost of a robot is determined in part by its relative scale: a micromachine is expensive to make, and so is a giant Mars exploration machine, but at the scale of the miniature we find the cheapest robots (see [16]).

Taking advantage of high strength permanent magnets, we have built a family of direct drive actuators at very low cost. We have developed standard universal links that allow us to construct a variety of robot mechanisms. These standardized parts are composed of simple geometric solids, so the link inertia matrix has a compact form.

The main problems we've experienced in developing DD actuators are overheating and underpowering. These problems will be mitigated by further advances in permanent magnets and other motor materials. While these problems may inhibit some immediate commercial applications of miniature DD actuators, they are not as significant for controlled laboratory robotics experiments.

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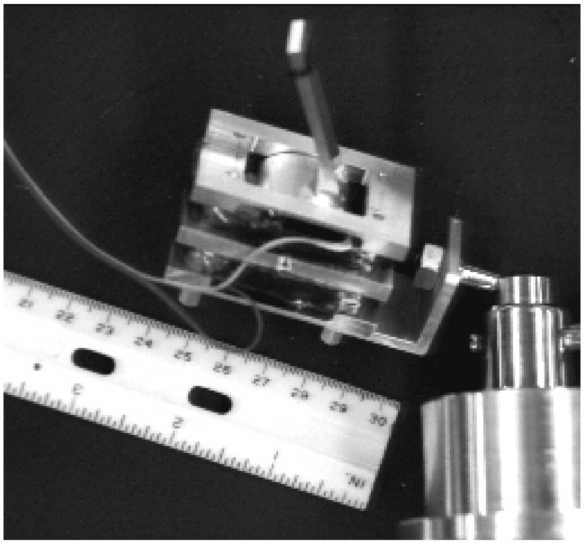

Figure 4: A prototype direct drive finger joint with a 90 degree workspace. This finger joint includes Hall effect sensors to detect rotor position and an integrated power driver chip (L293) and A/D converter on the backplate of the stator.

Mitt allows us to transmit tactile sensations through ordinary phone lines, by encoding a human grasp or squeeze in a modem channel, then using direct drive actuators to output a squeeze to the receiver. Together with Prof. Ken Goldberg of USC, we exhibited the Data Mitt at the 1993 Siggraph Media Culture Show, by demonstrating telerobotic "hand-holding" between New York and Anaheim, CA [28].

4 Direct Drive Leg

Unlike the Spherical Pointing Motor and the Direct Drive Finger, a direct drive leg has less immediate commercial appeal, and its purpose is primarily to answer scientific questions about the feasibility and control of high speed compliant leg mechanisms, Should a direct drive leg be feasible, we have the possibility of building a very fast, small and agile running machine. Inspired by the possibility, we attempted to build a pair of three-link direct drive legs to attach to the rear of a two-wheeled, two-legged rickshaw. Practical problems forced us to abandon one leg, but we found that just one leg could exert enough force to push the vehicle body up and forward about 1 *cm* per step.

The joints in our direct drive leg have $k = 1$, $n = 1$ and $m = 1$. Here the purpose of the stator magnet is not only to provide a neutral rest position, but also to act as a magnetic spring which stores energy during the ground contact phase of walking [29]. The construction process follows the same basic steps involved in building the direct drive finger, except that we simplified the design by using generic identical link pieces we call *universal links*.

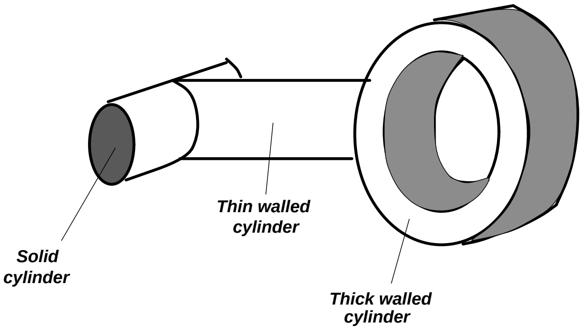

Figure 5: The universal link is a primitive element in a direct drive robot mechanism. The link eliminates all components from the robot except the essential parts: a rotor magnet (solid cylinder), a connecting shaft (thin walled cylinder), and the stator coil for the next link (thick walled cylinder). Because the link is the union of simple solid shapes, the link inertia matrix has a simple form. The coil assembly also contains a rotor shaft, not shown in this figure.

4.1 Universal Links

In the construction of the direct drive leg we attempted to make three identical joints and links. Such a design has not only process technology advantages but also analytical advantages because of simplified kinematics and dynamics. At the same time, we wanted a leg that was "natural", so we targets roughly the size and overall shape as the three primary links of a cat's hind leg. The link element we built is an example of a *universal link*, a primitive link element in a direct drive linkage (see Figure 5). A universal link is one whose Denavit-Hartenberg parameters may be adjusted, either at fabrication time or linkage integration time, so that we can construct a family of parameterized links. Specifically, our prototypes allow the adjustment of the link twist angle α and offset angle θ.

Our universal link fits into the context of research on manipulator design from task specifications [27] [26]. Previous work has concentrated on large scale reconfigurable robot links [33], but our miniaturized universal link opens up new application areas for task based manipulator design.

4.2 A One-legged Rickshaw

To test experimentally the feasibility of a direct drive leg, we built a small robot rickshaw having two caster wheels in front and a single direct drive leg propelling the body from the rear (see Figure 6). The first two links of each leg are identical universal links, having a link length of $\ell = 4.5cm$ and a mass of $75gm$. The last link is identical except that it has no subsequent link and therefore carries no coil. The offset

Figure 3: The direct drive finger is a three link mechanism made of stacked coil and magnet pairs. Maximum force at the fingertip is 1.0 Nt, and fingertip force is determined by controlled torques at the joints. The finger has no gears and therefore has passive dynamics.

between the first and second joint axis and the length of the first link, allowing easy experimentation with various kinematic configurations. When $\alpha = 0$ the finger forms a 3 link planar manipulator, but when $\alpha \neq 0$ the workspace becomes a volume. The lengths of the links are as follows: link 3 is 28 *mm*, link 2 is 45 *mm*, and link 1 is 35 *mm*. The range of motion in each link is about 30°

We made the stator coils of the first two joints using *AWG* 30 bondable magnet wire purchased from *MWS*, a magnet wire manufacturer. To form a coil, we first make a wax spool mold, and wind the magnet wire around the wax spool. After winding several hundred turns, we heat the coil with a heat gun, which causes the wax to melt and the wire to bond into a rigid solid mass. The *AWG* 30 wire has a resistance of about 0.3Ω per meter, and the resistance of the first two joint coils is about 30Ω.

The rotors of the first and second joint are nearly identical and consist of four stacked cylindrical NdFeB magnets of our standard size, 1/2" diameter and 1/4" height. We obtained the magnets from *Armtek/Adams Magnetic Products*.

The third joint coil is a commercial air core inductor manufactured by *Renco Electronics*, having a coil diameter of 1" (≈ 2.54 *cm*), a cylindrical height of 1/2" and a core diameter of 5/8". The third joint rotor contains a single cylindrical NdFeB magnet of our standard size, 1/2" diameter and 1/4" height. Unlike our coils, the *Renco* coil uses *AWG* 20 wire, a larger diameter having lower resistance.

The finger may be manually controlled directly from three polarity reversing variable current power supplies. We supplied the first two joint coils with about 1 *Amp* and the fingertip coil with about 3 *Amps*, and measured force at the fingertip up to 1.0 *Nt*. This arrangement, which allows manual control by varying the current flowing into the coils through a potentiometer, enables us to measure the fingertip forces as a function of known current inputs. For force measurement we used a Mark-10 digital force meter.

3.2 A Precision Finger Joint

Based on our initial success with the 3-jointed finger prototype, we decided to construct a prototype finger joint with a 90° workspace, like a human finger joint. The joint has two electromagnets wired in parallel, and has a terminal resistance of 31Ω.

The finger link length is 5.1 *cm*. The rotor consists of a cylindrical magnet holder and the link arm. Ideally the entire cylinder of the rotor would be a magnet, polarized *across* the cylinder rather than along its axis. Unfortunately we could not locate commercial NdFeB rotor magnets of that type, so we used the same axially polarized magnets used in the other actuators described here. The diameter of the rotor is 2.9 *cm*. The stator coils are just large enough to surround the rotor cylinder. We experimented with machining the magnet pole faces so that they were flush with the cylinder, in order to bring the pole face slightly closer to the stator coils, but we could measure no significant improvement.

The rotor magnet is oriented so that the maximum torque occurs when the finger limb is positioned exactly at 45°. At that position we measured a force of 2.0 *Nt* at the limb tip, using a current of 0.9 *Amp*. The maximum torque constant is 0.12 *Nt-M/Amp*. At the extrema of the workspace, the torque constant is 0.06 *Nt-M/Amp*. The torque constant of the finger joint is higher than the SPM, because the rotor magnet is always closer to the stator coil than the two axis SPM rotor is to its coils.

We have so far measured a top average joint velocity of 2800°/*sec* using an alternating current signal to the coils. In addition we demonstrated the finger joint's throwing power by having it toss small objects such as coins up to 2 meters across the lab floor. Computer control of the finger joint is identical to control of the SPM (see section 2.2).

In addition to the scientific applications motivating robot hands, we have demonstrated one highly successful potential commercial application of the direct drive finger, a device called Data Mitt. The Data electromagnet coils on the faces of that cube. Each pair of opposing coils forms a parallel circuit and generates a magnetic field **B**. Because of their symmetry, each coil pair forms a Helmholtz coil, so that we have

$$\nabla \mathbf{B}(\mathbf{x}) = 0 \text{ and } \nabla^2 \mathbf{B}(\mathbf{x}) = 0$$

for each of the fields **B**. Since the fields sum vectorially, the total field at the center of the SPM is approximately uniform, and the torque equation (1) is fairly accurate.

2.1 Low cost SPM

Very low parts cost spherical pointing motors are possible, and these have applications in education and as props. The most expensive part in our precision SPM (next subsection) is the small ball bearings used in the gimbal, which cost ten times more than the next most expensive part, the rotor magnets. We experimented with several low cost bearing alternatives including threaded rod and bushings. The low cost SPM shown in Figure 2 uses a threaded rod bearing. The rotor is tapped so that a 4–40 threaded rod may be inserted. When the rotor rotates, it actually translates slightly, about 1/4 the rod pitch. This translation is negligible, and the contact between rod and rotor has very low friction.

A thin steel plate on the back of the low cost SPM provides a neutral resting position in the absence of coil current. The rotor magnet is attracted to this plate and tries to align itself orthogonally. One potential problem with the steel plate is hysteresis, induced magnetism in the plate, which may subvert attempts to obtain precise calibration.

The coil and rotor construction technique is essentially the same as that for the direct drive finger, described below in section 3.1. High school interns in our lab learned to make the electromagnets and gimbals for the low cost SPM, giving them first hand experience in direct drive robotics. The existence of low cost direct drive actuators makes possible the dissemination of dynamic robotics to a much larger set of secondary and post-secondary schools than would be possible with conventional industrial robots.

2.2 Precision SPM

For applications demanding high repeatability and accuracy, and for experimental performance data measurement, the low cost SPM is not viable. The precision SPM has a machined aluminum frame made to 10^{-3} *inch* tolerance, holding two coil pairs. We wound the coils by timing a lathe and all four coils are within 0.5Ω of 53Ω. The rotor consists of a machined aluminum gimbal made with high precision ball bearings. The initial construction took about one week of machine shop time, but the parts cost is quite low.

Computer control of the SPM is via the Motorola MC68332 microcontroller, connected to a Sparcstation host. We develop control programs on the host, cross compile and download to the MC68332. The backplate of the precision SPM includes a permanent magnet to provide the reference field, four Hall effect sensors to detect the position of the rotor magnet, a 12 bit serial A/D converter (MAX 188) to read the Hall sensors, and an integrated power driver chip (L293). In a companion paper, we present details of the closed loop feedback control of the precision SPM [23].

Maximum average speed of the rotor measured to date is $500^\circ/sec$. Maximum torque at full deflection angle is 0.02 *Nt-m*, and a torque constant of 0.02 *Nt-m/Amp*. The rotor limb extends 5.0 *cm* from the point of rotation, and is held in place by a magnetic coupling so that we can replace it with other sensor payloads. The rotor inertia is approximately $100gm - cm^2$.

We have identified two practical applications of the spherical pointing motor: as a force feedback joystick, and as a targeting system such as the one under development at Vision Applications, Inc [6].

3 Direct Drive Fingers

Algorithms to use force control fingers are relatively more advanced than robot fingers themselves. Constructive algorithms for stable multifinger grips are known [19] [25]. To execute the polyhedral rotation algorithm of Rus [31], we need a finger that implements position control in two directions and force control in one. The architecture of a hybrid force and position controller is straightforward [11] [30], and may be implemented on the direct drive finger. The importance of force control and compliance in manipulation has influenced several hand and finger designs. Fingertips built using electrorheological fluids provide a means to actively control compliance and damping of fingertips [1]. Ordinary springs are compliant and robot hands with springy fingers exist [13]. Another attempt to achieve compliant finger control utilized a back drivable low inertia rack and pinion assembly [10]. The Stanford-JPL-Salisbury hand[22] integrates strain gauge sensors in the finger tips and implemented force control for grasping.

3.1 Prototype finger

The joints in our prototype direct drive finger have $k = 1$, $n = 1$ and $m = 0$. The finger consists of a base and three links (see Figure 3). The shaft connecting the first and second actuators is a friction fit, providing a manual adjustment of the twist angle α

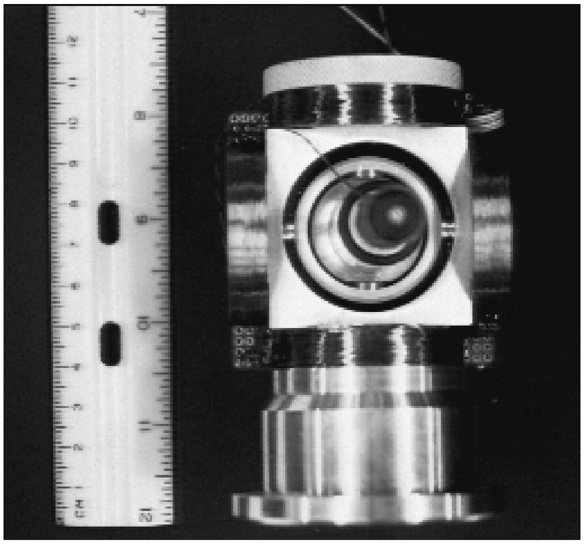

Figure 1: The Precision Spherical Pointing Motor is machined to a high tolerance and has a 75 degree conical workspace. The rotor limb contains a cylindrical permanent magnet, and the stator holds one additional permanent magnet so that the rotor has a neutral resting orientation in the center of its workspace.

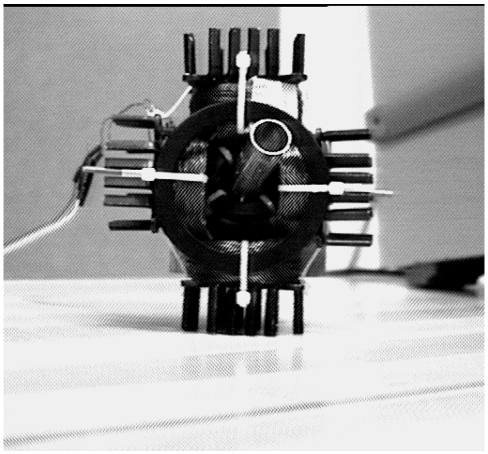

Figure 2: A low cost Spherical Pointing Motor (SPM) The SPM rotor is a permanent magnet attached to a gimbal. The actuator shaft extends outward from the gimbal. The stator consists of a pair of Helmholtz coils mounted orthogonally, and a thin steel plate that holds the magnet shaft in a centered position when no current flows into the coils. In addition to radiating excess heat from the coils, the heat sinks provide a structure to hold the coils.

tors which until then seemed stiff and unnatural because of their gear transmissions. The early attempts to build DD arms were successful, but the designers ended up with bulky manipulators characterized by geometric proportions between successive joint actuators [2] [3] [4]. The direct drive concept was better suited to SCARA manipulators, and several were built [5] [17] [24]. Subsequently the DD SCARA design was accepted by the the marketplace and several commercial examples exist. Another significant development in direct drive robots is the IBM Hummingbird [18], a three axis high speed positioning device applied to the problem of electrical continuity checks in high density circuit elements. Volpe and Khosla [34] experimentally analyzed force control strategies for direct drive robot arms, and concluded that integral force control is the best choice for explicit force control, and their result should apply to DD fingers as well. Volpe [35] also surveyed prior force control work on direct drive manipulators.

Motor manufacturers offer limited angle torque (LAT) motors, which generate torque through a limited range of angles and operate at speeds and torques comparable to our mini DD actuators [12]. The basic components of a LAT and our mini DD actuators are the same: a coil and a rotating magnet, though the arrangement of the LAT is the same as a conventional motor, except that the LAT has only two terminals. Some LAT actuators achieve very high torque (up to 10 $Nt\text{-}m$) through a combination of parallel windings over a multi pole rotor and very high power (up to 440 $Watts$). Apart from costing 100 times more, the main difference between commercial LATs and our actuators is that we have concentrated on making a useful actuator for robot limbs, and eliminated all extraneous motor parts, whereas a LAT is simply a modified conventional motor.

Other actuators can provide force control, specifically pneumatic, hydraulic, and internal combustion systems. But these technologies place formidable engineering requirements on robot builders, and require expensive local resources such as compressors, fluids, and fuels. In their simplest form direct drive actuators can implement force control for less than 1% the cost of competitive actuators, with the tradeoff being decreased power for higher speed.

2 Spherical Pointing Motors

Several implementations of the SPM have demonstrated a variety of improvements, leading to the possibility of more applications than just camera aiming. Since $k = 2$, we need at least $n = 2$ coils. We have built the configurations $(k = 2, n = 2, m = 1)$, $(k = 2, n = 2, m = 0)$, and $(k = 2, n = 3, m = 0)$, the latter having the simplest model.

The SPM rotor is a cylindrical magnet rotating on a gimbal in such a way that the magnet axis pans and tilts around a central point of rotation. Let that center point be $\mathbf{x}$. If $\mathbf{x}$ lies at the center of a a $D \times D \times D$ cube, we can place three pairs of identical diameter D

Miniature Direct Drive Rotary Actuators II: Eye, Finger and Leg *

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Abstract

We have developed miniature direct drive DC motor actuators for robotics. These actuators have low friction, small size, high speed, low construction cost, no gear backlash, operate safely without the use of limit switches and generate moderate torque at a high torque to weight ratio. Our initial experiments indicated the feasibility of constructing a variety of new high speed low cost actuators, for applications in camera pointing, robot hands, and robot legs. In this work we study some prototype devices in each of these categories.

1 Introduction

Electromagnetic devices remain the most viable form of robot actuator, due to their relatively high strength, their well understood characteristics, the ease of interfacing them to electronic control circuits, and the nearly universal availability of electricity as a power source. Moreover, there is ongoing steady advance in electric motor component technology including permanent magnets, permeable materials, power transistors, bearings and magnet wire. These advances made it possible for us to construct a wide variety of direct drive robot mechanisms [7] [8] [36] and below we describe the development of three of those: the Spherical Pointing Motor (SPM) or robot eye motor, a direct drive finger and new finger joints, and a prototype direct drive leg.

1.1 Direct drive

The ideal direct drive actuator would obey the torque equation

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \tag{1}$$

where $\boldsymbol{\mu}$ is the net magnetic field generated by a set of n coil and current pairs plus a set of m fixed permanent magnets, and $\mathbf{B}$ is the field generated by a permanent magnet rotating inside the coils. For coil i, let N_i be the number of turns, A_i be the area of the coil face, I_i be the current, and $\boldsymbol{\mu}_i$ be the unit vector along the coil axis. Let $\mathbf{B}_i$ be the polar moment of magnet i. Because

$$\boldsymbol{\mu} \approx \sum_{i=1}^{n} N_i I_i A_i \boldsymbol{\mu}_i + \sum_{i=1}^{m} \mathbf{B}_i$$

we may control the orientation of the rotor magnet in an ideal torque motor by varying the currents I_i. Notice in the absence of external forces if $m = 0$ the rotor will experience no force without current, but in that case, if $m \geq 1$ then there exists a neutral resting orientation which depends on the geometry of the permanent magnets. Moreover, because we assume the rotor magnet is a simple dipole $\mathbf{B}$, it can rotate with $k = 1$ or 2 degrees of freedom up to axial symmetry, and we can control rotor motion with an appropriate set of $n \geq k$ independent coils, or with $n \geq k$ and $m \geq 1$ if we desire a neutral resting state.

The ideal direct drive motor does not exist, because the magnetic fields are not uniform and nor can they be characterized exactly as dipoles, but we can construct fairly good approximations. Calibration [8] can supply the mapping between desired joint positions and control currents.

1.2 Background

Today, building and controlling miniature direct drive mechanisms is simplified because of the substantial existing body of knowledge about direct drive robot manipulators. By the end of the 1970's it was clear that contemporary direct drive motors could implement force control for motorized robot manipula-

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