CSC412 – Assignment #1

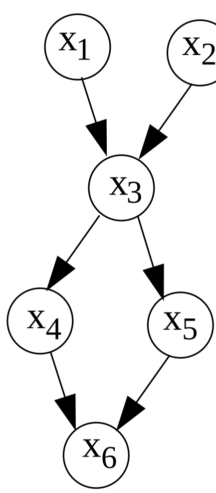
Due: Jan20, 10am at the **START** of class
Worth: 10%
Late assignments not accepted.

1 Graphical Model Distributions

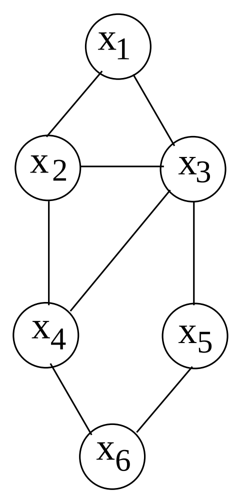
The figure below shows a directed graphical model (D) and an undirected graphical model (U) each representing a joint distribution over six random variables.

- For each of the following statements and each graphical model, say whether the statement *must* be true of the distribution represented by the model, *could be true* but we don't know, or *cannot be true*.

  1. $\mathbf{x}_4$ is conditionally independent of $\mathbf{x}_5$ given $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$
  2. $\mathbf{x}_3$ is conditionally *dependent* on $\mathbf{x}_6$ given $\mathbf{x}_4$ and $\mathbf{x}_5$
  3. $\mathbf{x}_1$ is marginally independent of $\mathbf{x}_2$
  4. $\mathbf{x}_4$ is conditionally independent of $\mathbf{x}_5$ given $\mathbf{x}_3, \mathbf{x}_6$



D



U

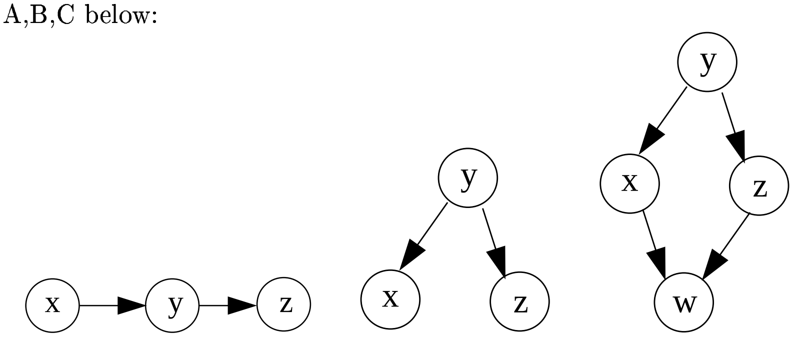
2 Numeric Distributions

This question is to get you comfortable with MATLAB for future assignments.
You shouldn't have to write any programs; you should be able to do everything in the interpreter.
Look at `help sum` to find out how to sum along different dimensions.
You may also find the command `rank` helpful to tell you whether a matrix is the outer product of two vectors or not.

- In MATLAB load the file `a1distribs.mat`.
- Each multidimensional array `pA,pB,pC` represents a joint probability distribution over discrete random variables. For example `pA(3,4,5,1)` is the probability under distribution A that $\mathbf{x}_1$ takes on its third value, $\mathbf{x}_2$ takes on its fourth value, $\mathbf{x}_3$ takes on its fifth value and $\mathbf{x}_4$ takes on its first value.
- Calculate and print out (or copy down) the following distributions. (Only show up to 4 decimal places.)
  1. $pA(\mathbf{x}_3, \mathbf{x}_4 | \mathbf{x}_1$ takes its third value)
  2. $pB(\mathbf{x}_4 | \mathbf{x}_2$ takes its first value)
  3. $pC(\mathbf{x}_1)$
- For each distribution {A,B,C} and each of the following statements, say whether the statement applies.
  Be careful of numerical roundoff errors, when checking if things are equal!
  1. $\mathbf{x}_1$ is conditionally independent of $\mathbf{x}_2$ given $\mathbf{x}_3$
  2. $\mathbf{x}_1$ is conditionally independent of $\mathbf{x}_2$ given $\mathbf{x}_3$ and $\mathbf{x}_4$
  3. $\mathbf{x}_1$ is marginally independent of $\mathbf{x}_2$
  4. $\mathbf{x}_3$ is conditionally independent of $\mathbf{x}_4$ given $\mathbf{x}_1$ and $\mathbf{x}_2$

3 Model Conversions

For each of the directed graphs A,B,C below:



A B C

- Give all possible *topological orderings* of the variables.
- List all conditional independencies implied by the directed model.
- If there is an undirected model which captures exactly the same conditional independencies, draw it.
  If not, remove the fewest possible edges from the directed model until it has an undirected model capturing the equivalent independencies, and show the new directed model and the corresponding undirected model.
  (Hint: consider whether the graph is acyclic if we ignore the direction of the edges
  i.e. are there cycles if you can move either way on any edge.)

4 Complete Graphs

- Draw a directed graphical model on 5 variables which (a) can capture *any* joint distribution and (b) is acyclic.
- Can any edges be added to or removed from your graph and still preserve both the properties (a) and (b) above? If so, show the addition or removal, if not say why not.
- Draw an undirected graphical model on 4 variables which can capture *any* joint distribution.
  List all the maximal cliques.
- Can any edges be added to or removed from your graph and still preserve the above property?
  If so, show the addition or removal. If not, say why not.

5 Incompatible Conditionals

In this question you'll try to convince yourself that the representation

$$p(\mathbf{X}) = \prod_i \mathsf{P}(\mathbf{x}_i|\mathbf{x}_{\text{neighbours}(i)})$$

does not allow arbitrary conditional probabilities $\mathsf{P}(\mathbf{x}_i|\mathbf{x}_{\text{neighbours}(i)})$.

- Consider two binary variables $\mathbf{x}$ and $\mathbf{y}$ with the conditional distributions:

$$p(x=1|y=0) = 3/4 \qquad p(x=1|y=1) = 2/3$$
$$p(y=1|x=0) = 2/3 \qquad p(y=1|x=1) = 4/7$$

- Show that the function $f(x,y) = p(x|y)p(y|x)$ is not a valid joint distribution $p(x,y)$.
- Write down a valid joint distribution $p(x,y)$ that has the given conditionals.
- What condition must the joint $p(x,y)$ satisfy in order to be able to be written as a product of conditionals? Show your reasoning.
- Write down two conditional distributions, like the ones above, on binary variables x and y, so that $p(x|y)p(y|x)$ *is* a valid joint. (Don't set any probabilities to zero or one.)