Lecture 5b:

Parameter Estimation

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Undirected Models

- In general, for undirected models, the parameter estimation problem does not decouple (except for special undirected models, called "decomposable models").
- The problem is the global normalizer (partition function) Z.
- Consider discrete models:

$$\begin{aligned}\ell(\theta; \mathcal{D}) &= \log p(\mathcal{D}|\theta) \\ &= \sum_{cliques} \sum_{\mathbf{x}_C} N_{\mathbf{x}_C} \log \psi_C(\mathbf{x}_C) - M \log Z \\ \frac{\partial \ell}{\partial \psi_C} &= \frac{N_{\mathbf{x}_C}}{\psi_C(\mathbf{x}_C)} - M \frac{p(\mathbf{x}_C)}{\psi_C(\mathbf{x}_C)} \\ \Rightarrow p_{\mathrm{ML}}(\mathbf{x}_C) &= \frac{N_{\mathbf{x}_C}}{M}\end{aligned}$$

- Very important result: For each clique, the model marginals must match the empirical distribution. (more later)