- ullet The number is 11 digits (each between 1 and 9).
- The 11th digit is the sum of the first 10, mod 9.
- Example:  $(0+2+5+4+3+7+5+0+5+9) \mod 9 = 40 \mod 9 = 4$





#### LECTURE 14:

REAL WORLD EXAMPLES

November 2, 2005

## COMMON CODES TO PREVENT HUMAN ERRORS

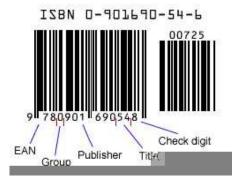
- Many numbers that have to be read and retyped by people include some error correction coding, ususally in the form of a single check digit.
- A common scheme is to make the check digit equal to the sum of the other digits, mod somthing.
- Examples include:
  - Money Orders, Bank Cheques, Traveller's Cheques
  - $-\,\mathsf{UPC}\,\,\mathsf{Symbols},\,\,\mathsf{ISBN}/\mathsf{ISSN}\,\,\,\mathsf{Numbers},\,\,\mathsf{VIN}\,\,\,\mathsf{Numbers}$
  - Credit Cards, Driver's Licenses, Social Security Numbers
  - Airline Tickets, Courier Tracking Numbers, ...

## AIRLINE TICKETS

- The numbers are different lengths, but the whatever number is obtained by dropping the last digit must be congruent to the last digit, mod 7.
- Example:  $3104130444 \mod 7 = 2$



- Each book has a 10 digit ISBN, whose weighted sum must be equal to 0 mod 11. The weighting of the first digit is 10, the second digit 9, and so forth, weighting the last digit by 1.
- Example: 10\*0+9\*9+8\*0+7\*1+6\*6+5\*9+4\*0+3\*5+2\*4+6=198 = 11\*18



• Credit card numbers are 16 digits long (VISA stars with 4, AMEX with 34 or 37, Mastercard with 51-55) and obey the check: 2\*sum-of-odd-positions + sum-of-even-positions +  $number-of-odd-positions>4 = 0 \mod 10$ 

• Example: 2\*(4+8+2+2+7+1+1+7) + 9+8+4+2+2+2+1+5+3= 2\*32+33+3 = 100 = 10\*10



### Bank Identification Numbers

• Each bank has a 9 digit identification number.

• The weighted sum of the first 8 digits must be equal to the last digit, mod 10. The weighting used is [7,3,9,7,3,9,7,3].

• Example: 7\*1+3\*2+9\*1+7\*0+3\*3+9\*1+6\*7+7\*3 = 103



# Universal Product Codes (UPC)

- Many commercial items have a UPC, consisting of a category code, manufacturer's ID (5 digits), product ID (5 digits) and check digit.
- The sum of the even digits plus 3 times the sum of the odd digits must be equal to 0 mod 10.

• Example: 3+4+0+1+7+5+3\*(0+9+0+0+9+2)=80

