##### My Early Experiences with Scientific Computation

It may be a piece of vanity to write about my personal experiences as opposed to the general state of the art. But I take comfort for this limitation by the words of the great biographer Lytton Strachey:

**"Human beings are too important to be treated as mere symbols of the past. They have a value which is independent of any temporal process… and must be felt for their own sake." -- *Eminent Victorians***

##### Hopefully, the general state of the art in the time frame I have selected will emerge from what follows here.

##### I attended High School in Lawrence, Massachusetts during the years 1935-1939. In our classes in "advanced" mathematics, we did some drilling with logarithms and trigonometric tables. I recall having to use the side bars that gave "proportional parts." Drilling was tedious work. In those years also, I discovered for myself what I later learned was known as the Newton forward difference interpolation formula.

##### Quite by accident, since my elder brother had gone to MIT, I came to own a discarded copy of David [Gibb](http://library.brown.edu/search/aGibb%2C+David/agibb+david/-3,-1,0,B/browse)s' *A* ***course in interpolation and numerical integration for the mathematical laboratory*** London, 1915. The laboratory in question was at the University of Edinborough and run by the distinguished applied mathematician Edmund Whittaker. This laboratory was rare in academic circles. The following quote from Gibbs' book gives a vivid and amusing picture of the state of the art.

##### "Each desk [in the Mathematical Laboratory at the University of Edinburgh] is equipped with a copy of *Barlow's Tables*, (square, cubes, square roots, etc.) a copy of Creole's Tables which gives at sight the product of any two numbers less than 1000. For the neat and methodical arrangement of the work, computing paper is essential. . . . It will be found conducive to speed and accuracy if, instead of taking down a number one digit at a time, the computer takes it down two digits at a time."

##### For a graduation present from High School I asked my elder brother for a good slide rule and he gave me a Keuffel & Esser log-log slide rule -- one on which fractional exponents could be done.

##### I attended Harvard College during the years 1939-1943, majoring in mathematics. Computation of whatever sort was not part of the curriculum. I don't believe the Department of Mathematics had an electric adding machine. One would have to go to the Physics or Astronomy Departments to find one.

There were few relevant English language books on scientific computation. One of them was E. T. Whittaker and G. Robinson's [*The Calculus of Observations; a Treatise on Numerical Mathematics*](http://library.brown.edu/search/awhittaker++e.t./awhittaker+e+t/1%2C2%2C27%2CB/frameset&FF=awhittaker+e+t+edmund+taylor+1873+1956&3%2C%2C25) (1929**). Joseph Lipka's *Graphical and Mechanical Computation,* 1918,** was another. I hardly need to say that there were no courses in numerical analysis (I believe this term was coined in early 1950's by George Forsythe, and the word "algorithm", though of medieval origin, had hardly been employed.)

The social status of computation among theoretical mathematicians was low; this despite the fact that famous mathematicians had worked on numerical problems related to navigation, astronomy, geodesy, tides, etc. Think of "Newton's method" of "Gauss elimination." But if one wanted to learn such material, one had to pick it up oneself.

##### The United States entered WW II in December , 1941. I was then in my junior year in college. Science students were draft - deferred for a while. Professors encouraged us to take a variety of science courses. The younger faculty were dropping out of teaching to join groups that worked on radar, cryptography, meteorology, operations research, feedback and control theory or the Manhattan atomic bomb project.

##### I received my bachelor's degree in mathematics in 1943 and in that summer, entering graduate school, I took mechanics and dynamics with Prof. Garrett Birkhoff. Shortly thereafter, since scientific talent was mobilized for the "war effort," I was alerted to and accepted a position at the National Advisory Committee for Aeronautics (NACA, the precursor of NASA), at its Langley Field, Hampton, Virginia laboratory. Somewhat later I was inducted into the United States Air Force, placed on reserve status, and sent back to the NACA with the equivalent salary of a Second Lieutenant.

As I've said, this was in the middle of WWII with the United States on a total war footing. Though there was social upset with millions of young men and many women in the armed services and such things as food and gas rationing, there was nothing in the United States that compared to the enormous physical, moral, bodily and psychological devastation experienced in Europe.

In moving from Cambridge, Massachusetts to Hampton, Virginia in early 1944, I experienced two kinds of minor shock. One was cultural, the other was scientific. The cultural shock arose from experiencing at first hand the severe prejudice and restrictions then suffered by Afro-Americans in the South of which Virginia was a part.

The scientific or organizational shock arose from this: that I (and all the young scientists who were similarly commandeered) was thrust into a well established scientific-technological environment with well seasoned old-timers, a set of problems and goals, specialized terms and ideas, and a set of preferred practices and strategies for their solution. Now sink or swim! And do more than swim: innovate if you can or if you are allowed to.

No college course can train for all the possibilities and necessities that exist in "real world" practice! In college, as a major in mathematics, I learned that the real number system was a complete Archimedean ordered field. I learned that measurable functions had little to do with physical measurements. Useless information. The noted numerical analyst and information theorist Richard Hamming wrote much later something like: "If the safety of an airplane depended on the distinction between the Riemann and the Lebesgue integrals, I would be afraid to fly."

However, from my college courses experiences, I knew something about Fourier series and complex analytic functions and these theories were quite relevant and useful. [A side remark: I did complex variable theory under David V. Widder --- the "Laplace Transform Man." As reading period assignment, I studied the complex gamma function. My interest in this function continued over the years so that I wrote up as Chapter Six of the famous Abramowitz & Stegun *Handbook of Mathematical Functions*, (1964).]

The NACA was divided into a number of divisions: theoretical, full scale tunnel, compressibility, structures, etc. My job was in NACA's Aircraft Loads Division, which studied dynamic loads on the components of aircraft -- often fighter planes -- during a variety of flight maneuvers. I was partially a computer---working with the raw data provided by flight instrumentation, accelerometers, and pressure gauges--- partially an interpreter of what I had computed. Later on, I thought of myself as an "algorithmiker", i.e., devising computational strategies.

My colleagues and I worked with slide rules, planimeters, charts, nomograms, French curves and other drawing instruments. We had various electromechanical desk-top calculators such as Marchants and Friedens. We had other computational aids: the WPA tables of special functions. We made use of these and other tables computed some years before --- many in England. We had available compendia of formulas. We worked with experimental results from wind tunnel and flight data, "rules of thumb", theoretical books (e.g., the books and lectures of Ludwig Prandtl, the five volumes on aeronautics edited by William F. Durand, or the *Theory of Flight* by Richard von Mises). We worked with published in-house reports or reports from other laboratories that were often mimeographed or photocopied.

In those years, the word "computer" did not designate a mechanical or electronic device; it designated a person who computed. I know this at first hand because my wife (we were married early), and who had had two courses in college mathematics, was employed as a computer in the Structures Division of the NACA. It was widely believed and very likely the case that women were better and more reliable than men in carrying out computations, and in those years there were extensive employment opportunities open to them. My wife adds that the computers were treated as machines by the engineers for whom they worked : do this, do that with hardly any explanation as to what they were doing or why.

In thinking through my work at the NACA which lasted from Spring, 1944 to September, 1946, I can distinguish five major jobs that I was given to work on. The last one led to my first published paper.

1)Experimental pressure distribution on the wing profile during flight maneuvers.

**2)** Finding theoretically the pressure (lift) distribution over a two dimensional airfoil. (Potential flow: no compressibility, no viscosity.)

##### 3) The inverse problem. Find the airfoil shape corresponding to a experimental pressure distribution with compressibility at higher Mach numbers.

4) Analysis of V-G diagrams (i.e., velocity-acceleration) during flight maneuvers,

##### 5) Analysis of the failure of a flying boat tail structure under test maneuvers.

I'll now comment briefly on these jobs from the point of view of the computational procedures used.

1) A series of pressure holes over the wing profile provided the flight data. This pointwise flight data was carefully plotted and then, using French curves, "faired" to provide a continuous record. The area and the moment under the curve were then obtained by running the planimeter over the contour.

From my knowledge of the contents of Gibbs' book I felt certain that approximate integration methods applied directly to the raw data would provide equivalent accuracy, but it was done the way I just described and I had no desire to upset the computational apple cart in the middle of a war. Nonetheless, this experience and later experiences when I was employed at the National Bureau of Standards in Washington during the years of the first generation of electronic digital computers led me to collaborate with Philip Rabinowitz on *Numerical Integration* , a book that has now gone through several editions.

2) In the years 1931-33, Theodorsen and Garrick, both employed at the NACA, had worked out a satisfactory algorithm. They mapped the exterior of the airfoil conformally onto the exterior of the unit circle (where the pressure distribution had long been known) by a rapidly convergent process involving the Joukowsky Transformation. This involved making a harmonic analysis of the airfoil contour. This analysis was accomplished by using blueprinted stencils for 24, 36, 48 point analyses. These stencils derived from the work of the German mathematician Carl Runge : ***Rechenschablonen für harmonische analyse und synthese* *nach Carl Runge*, von P. Terebesi.**

**Runge's insight was to make use of the inherent symmetries in the sine and cosine functions when the number of points employed was highly composite. It took a computer perhaps all day to work through and check a 48 point analysis.**

**Here, then, was an early version of the Cooley-Tukey's FFT (the Fast Fourier Transform) which, now in chipified form, is accomplished in nanoseconds**

3) The inverse problem was: given the experimental pressure distribution from a high speed plane (high Mach numbers had already been achieved in flight) find the theoretical airfoil shape that gave rise to it under the assumptions of 1), i.e., potential flow. There was no theory behind this problem and the numerical methods I employed were essentially those of 1).

I suppose that the purpose of this investigation was to infer something about airfoil shapes that would be efficient at high Mach numbers. Despite vagueness in my mind as to what I had accomplished, I was asked to present my results to an audience of aerodynamicists and Air Force Officers. This was the first time I gave a scientific talk; just placards; no overhead projectors, no Power Point, no subsequent publication.

I've recently learned about the conserable work in the early '70's , both theoretical and algorithmic, of Paul Garabedian , David Korn et alia on the inverse problem of finding shock-free airfoil shapes.

4) The job of V-G analysis involvedfinding significant patterns empirically by more or less eyeballing the diagrams and doing a bit of averaging. Considering the presence of such phenomena as aerodynamic stall, his information was important in setting aerodynamically safe limits to flight maneuvers, particularly in fighter planes such as the P-40 or the P-51.

##### 5) The Glenn Martin "Flying Boat" Mars was designed in the early '40's for the US Navy as a long range "flying dreadnought". During some initial flight-test maneuvers, the Mars, flying flat at low altitudes over Chesapeake Bay, experienced a break in its vertical tail. Why did this happen? My boss, Henry A, Pearson, a man with vast experience in aeronautics, suspected that during the testing process in which the test pilot executed the required fish-tail maneuver of oscillating the rudder, the natural frequency of the plane in flight (considered as a spring system) would be reached with a corresponding large build up of the vertical tail load.

##### Pearson suggested to my colleague John Boshar and me that we set up a mathematical model, use wind tunnel and various parameters from flight records, and see whether we could reproduce the build up computationally. Confining the motion to one degree of freedom (yawing motion) , we set up the dynamic equation as a second order linear differential equation with the rudder deflection as the forcing function on the right hand side.

##### We employed the well-known method of the Duhamel Integral which solves the equation essentially as the convolution of the forcing function against a damped sinusoid , the latter requiring the eigenvalues of the differential operator. The various constants in the equation had first to be calculated as complicated combinations of aerodynamic parameters. Then, the unit impulse response -- the sinusoid was calculated, and finally the convolution integral. Again we worked with a planimeter, Frieden machines and the full panoply of charts, reports, etc. from which we extracted the parameters.

##### Our work was successful in that it showed the possibility of serious tail overloads, and resulted in my first technical paper: *Consideration of Dynamic Loads On the Vertical Tail By The Theory Of Flat Yawing Maneuvers.* NACA, Report No.838, 1946.

##### The possibility and actuality of tail failure in aircraft is today still an ongoing concern. This can be learned from scanning search engine displays under this heading. *Pearson \*v\*ration*

##### In retrospect it would have been an extremely difficult and time consuming job, in those days, to reproduce numerically the three dimensional trajectory of an airplane (pitch, yaw, and roll) corresponding to deflections of its various control surfaces. The development of the electronic digital computer received a tremendous boost from the computational necessities of the airplane and space industries.

##### But lest we be too proud: consider all the buildings, bridges, and planes built before electronic digital computers were available. The first supersonic airplane, the X-1, a joint project of the US Army and Air Force, the NACA, and Bell Aircraft in Buffalo, New York, flew on Oct. 14, 1947 and achieved a speed of Mach 1.06 at a time when the electronic digital computers were just getting started. The "digital wind tunnel" has not yet (2008) replaced physical experimentation, and in the opinion of some authorities, it never will.

##### The computers that have made flights to Mars possible have changed applied mathematics as well as our lives on Earth in ways that could not have been imagined in 1946.