Syntax of Predicate Calculus

The predicate calculus uses the following types of symbols:

Constants: A constant symbol denotes a particular entity. E.g. John, Muriel, 1.

Functions: A function symbol denotes a mapping from a number of entities to a single entities: E.g. FatherOf is a function with one argument. Plus is a function with two arguments. FatherOf(John) is some person. Plus(2,7) is some number.

Predicates: A predicate denotes a relation on a number of entities. e.g. Married is a predicate with two arguments. Odd is a predicate with one argument. Married(John, Sue) is a sentence that is true if the relation of marriage holds between the people John and Sue. Odd(Plus(2,7)) is a true sentence.

Variables: These represent some undetermined entity. Examples: x, s1, etc.

Boolean operators: ¬, ∨, ∧, ⇒, ↔.

Quantifiers: The symbols ∀ (for all) and ∃ (there exists).

Grouping symbols: The open and close parentheses and the comma.

A term is either

1. A constant symbol; or
2. A variable symbol; or
3. A function symbol applied to terms.

Examples: John, x, FatherOf(John), Plus(x,Plus(1,3)).

An atomic formula is a predicate symbol applied to terms.

Examples: Odd(x). Odd(Plus(2,2)). Married(Sue,FatherOf(John)).

A formula is either

1. An atomic formula; or
2. The application of a Boolean operator to formulas; or
3. A quantifier followed by a variable followed by a formula.

Examples: Odd(x). Odd(x) ∨ ¬Odd(Plus(x,x)). ∃x Odd(Plus(x,y)).
∀x Odd(x) ⇒ ¬Odd(Plus(x,3)).

A sentence is a formula with no free variables. (That is, every occurrence of every variable is associated with some quantifier.)