Limits of an AI program for solving college math problems

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Abstract

Drori et al. (2022) report that "A neural network solves, explains, and generates university math problems by program synthesis and few-shot learning at human level ... [It] automatically answers 81% of university-level mathematics problems." The system they describe is indeed impressive; however, the above description is very much overstated. The work of solving the problems is done, not by a neural network, but by the symbolic algebra package Sympy. Problems of various formats are excluded from consideration. The so-called "explanations" are just rewordings of lines of code. Answers are marked as correct that are not in the form specified in the problem. Most seriously, it seems that in many cases the system uses the correct answer given in the test corpus to guide its path to solving the problem.

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Specifically, the neural network takes as input word problems from undergraduate math courses. The wording of the problem is first modified by a hand-crafted automated front end. In most cases this involves only adding a few stock phrases, such as "Use sympy"; this is called "zero-shot learning". In some cases, the system automatically finds similar examples in the corpus and adds them to the prompt; this is called "few-shot learning". The modified problem is given as input to the Codex system developed by OpenAI (Chen et al. 2021), and Codex outputs Python code intended to solve the problem.

The system was tested on a data set of word problems taken from six MIT undergraduate math courses (Single Variable Calculus, Multivariable Calculus, Differential Equations, Introduction to Probability and Statistics, Linear Algebra, and Mathematics for Computer Science), one Columbia University course (Computational Linear Algebra), and six topics from the MATH dataset (Hendrycks et al. 2021) (Prealgebra, Algebra, Number Theory, Counting and Probability, Intermediate Algebra, and Precalculus), assembled from high school math competitions. Over this data set, it is claimed the system achieves 71% success rate using zero-shot learning and an additional 10% success rate using few-shot learning.

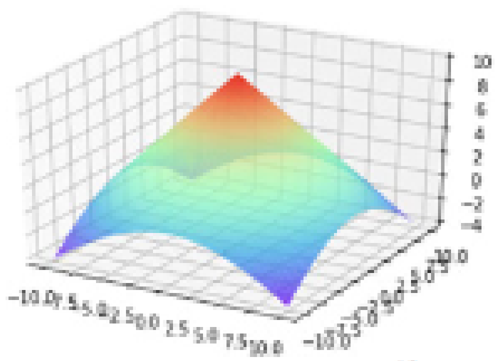
By contrast, the large language model GPT-3 (Brown et al. 2020) achieves only 18% with zero-shot learning and 30.8% using few-shot learning and chain-of-thought prompting. Thus the new system is a major advance in the state of the art.

Unquestionably this is a very impressive accomplishment. However, there are a number of points that should be kept in mind here in evaluating its significance.

First: It is misleading to attribute this success in solving the problems to "a neural network", as in the title. The actual work of finding the mathematical solutions is, in all cases, being done by Sympy, a large and sophisticated Python package for symbolic mathematics that was constructed by hand, building on sixty years of expert development of computational symbolic math systems. What the neural network does is to convert the problems from an English language statement into the proper call to Sympy functions (the additional surrounding Python code is generally trivial.) This itself is of course a very difficult problem, and the success of Codex on these problems is remarkable, but it is important to be clear about which parts of the overall process are being done by what kinds of technology.

Second: As mentioned in the paper (p. 2 and p. 9) the program cannot solve problems where the question involves an image or requires a proof. The claim (p. 1) that the system solves "81% of university-level mathematics problems" therefore has to be qualified. (Moreover, of course, there are many university-level mathematics problems that go beyond the first six introductory courses.)

Third: In a number of the examples given in the paper, the answer provided by the system is not actually in the form requested in the original problem. For instance, Table 1 problem 2 reads "Describe the graph of the function $f(x,y) = 10 - \sqrt{x^2+y^2}$." The answer given by the system is this three-dimensional graph:



Presumably, however, the author of the question did not have in mind producing an image, which many students would find challenging to do by hand, and trivial to do using plotting software. Rather, the question asks for a verbal description: "The graph is a right circular cone, with apex at $\langle 0, 0, 10\rangle$ and axis downward on the z axis" or something of the kind.

Another example: In figure 4, the first question (from course 18.01) reads "Find the derivative of the function using the definition of a derivative: $f(x) = (x * *2 - 1)/(2 * x - 3)$." Presumably the intent of "using the definition of a derivative" is that the student should actually show how the answer can be derived using the limit definition of a derivative (admittedly a somewhat laborious problem, requiring a fair amount of rather fiddly algebra). The system, however, simply ignores the words "using the definition of a derivative" and instructs Sympy to use the built-in symbolic differentiation module.

Fourth: The title states that the system "explains" the problem. In fact, what the system produces is a line-by-line explanation of the Python code that computed the answer, generally rather dreary and unenlightening. For instance in the same example from 18.01 in figure 4 discussed above, the explanation produced is as follows

```
1. We import sympy as sp.

2. We create a symbol x.

3. We create a function f.

4. We print the derivative of f with respect to x.
```

That really doesn't go far toward explaining the problem. The other examples in figure 4 are similar.

Fifth, and perhaps the most serious: The way in which problems are "automatically" chosen for few-shot learning is altogether unclear and may be illegitimate. The paper says (figure 2) "If zero-shot learning does not work, perform few-shot learning," and (p. 4) "If the question is not solved [by zero-shot learning], we do the following [description of the few-shot procedure]". The question is, how does the system know that zero-shot learning has not succeeded? As far as I can see the question is not answered in the paper. Perhaps the system uses some legitimate method; e.g. the Codex system fails to produce executable code. However, if that were the criterion, one would expect that some fraction of the time, zero-shot learning would produce code that executes but is erroneous; and there is no suggestion of that in the paper. What seems much more likely is that the system moves to few shot learning *when zero-shot learning has produced an answer that is incorrect.* That is, the program is using the recorded correct answer to guide its actions. That would be cheating[1] and if that is the case, then all of the results relative to few-shot learning must be thrown out, or at least interpreted with a very large asterisk.

Finally, in one of the examples in the paper, the code is nonsense though it gives the right answer. The question is "Determine whether the alternating series converge or diverge [sic]: $\sum_{n=1}^{\infty}(-1)^{n+1}/n^2$." The Python code produced by Codex is

```
from sympy import Sum, Symbol, oo, limit, init_printing

init_printing()

n = Symbol('n')

s = Sum(((-1)**(n+1))/n**2, (n, 1, oo))

limit(s,n,oo)

"""
The series converges.
"""
```

If you actually run this, the result of the call to "limit" is the symbolic expression $\sum_{n=1}^{\infty}(-1)^{n+1}/n^2$. But having generated this expression, the code proceeds to ignore it and simply print out "The series converges". (The example that it is using for guidance in few shot learning has the same error.)

Even with all this borne in mind, the program remains impressive. My real complaint here is not about the program but about the paper. The paper has eighteen authors, of whom eleven are affiliated with M.I.T., four with Columbia, two with Harvard, and one with Waterloo. Presumably it was read by three reviewers for PNAS. How did all these sloppy errors get past all these readers?

References

Brown, Tom et al. (2020). "Language models are few-shot learners." *Advances in Neural Information Processing Systems,* **33**: 1877-1901.

Chen, Mark et al. (2021). "Evaluating large language models trained on code." arXiv preprint arXiv:2107.03374.

Drori, Iddo et al. (2022) "A neural network solves, explains, and generates university math problems by program synthesis and few-shot learning at human level." *Proceedings of the National Academy of Sciences (PNAS)* 119(32), p.e2123433119.

[1]In the technical sense. I don't, certainly, mean to accuse the authors of deliberate malfeasance; merely of sloppiness and unintentional misrepresentation

Hendrycks, Dan, et al. (2021). "Measuring mathematical problem solving with the math dataset." arXiv preprint arXiv:2103.03874