CSCI-UA.0201

Computer Systems Organization

Concurrency – Correctness of Concurrent Objects

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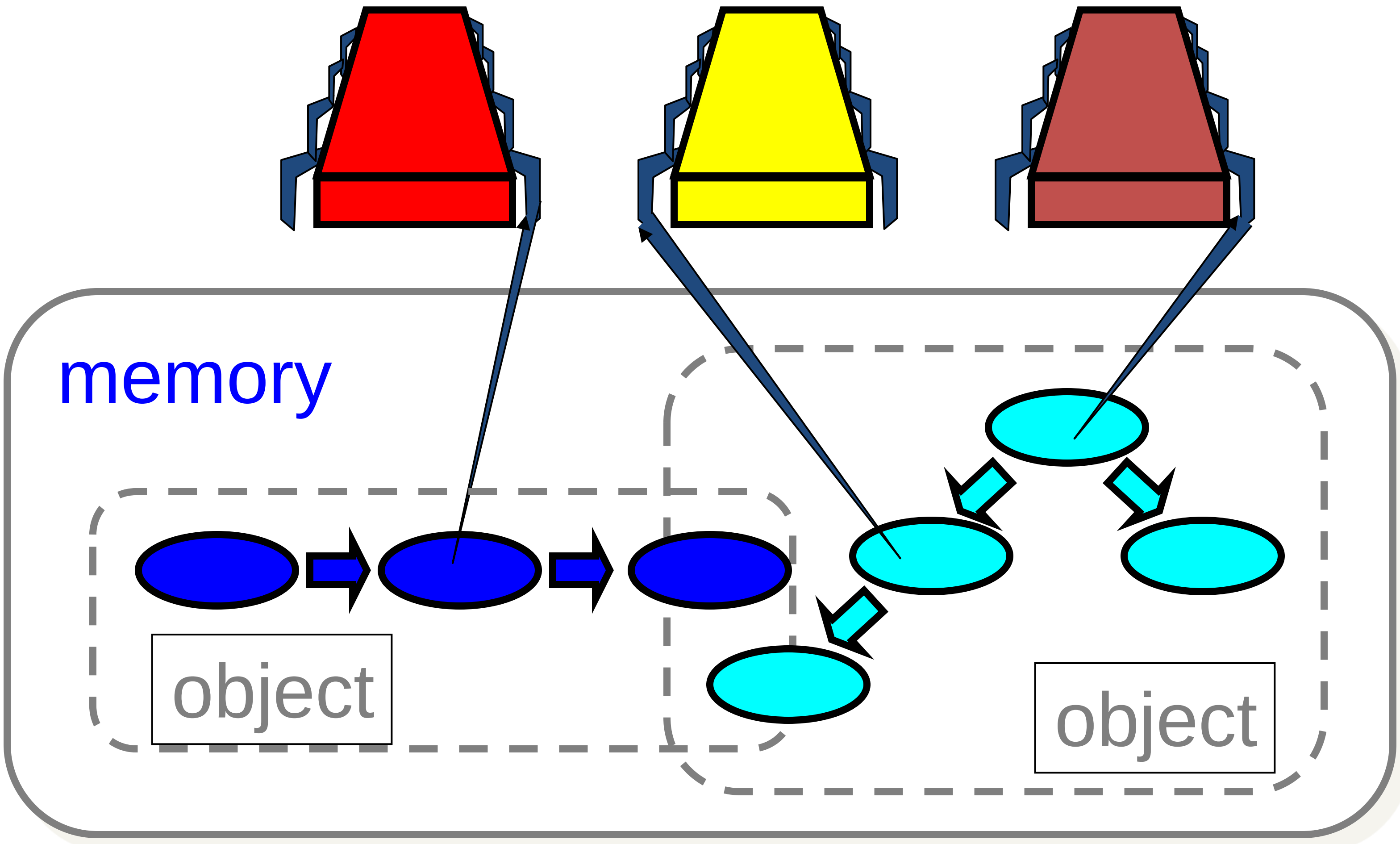
Concurrent Computation



Objectivism

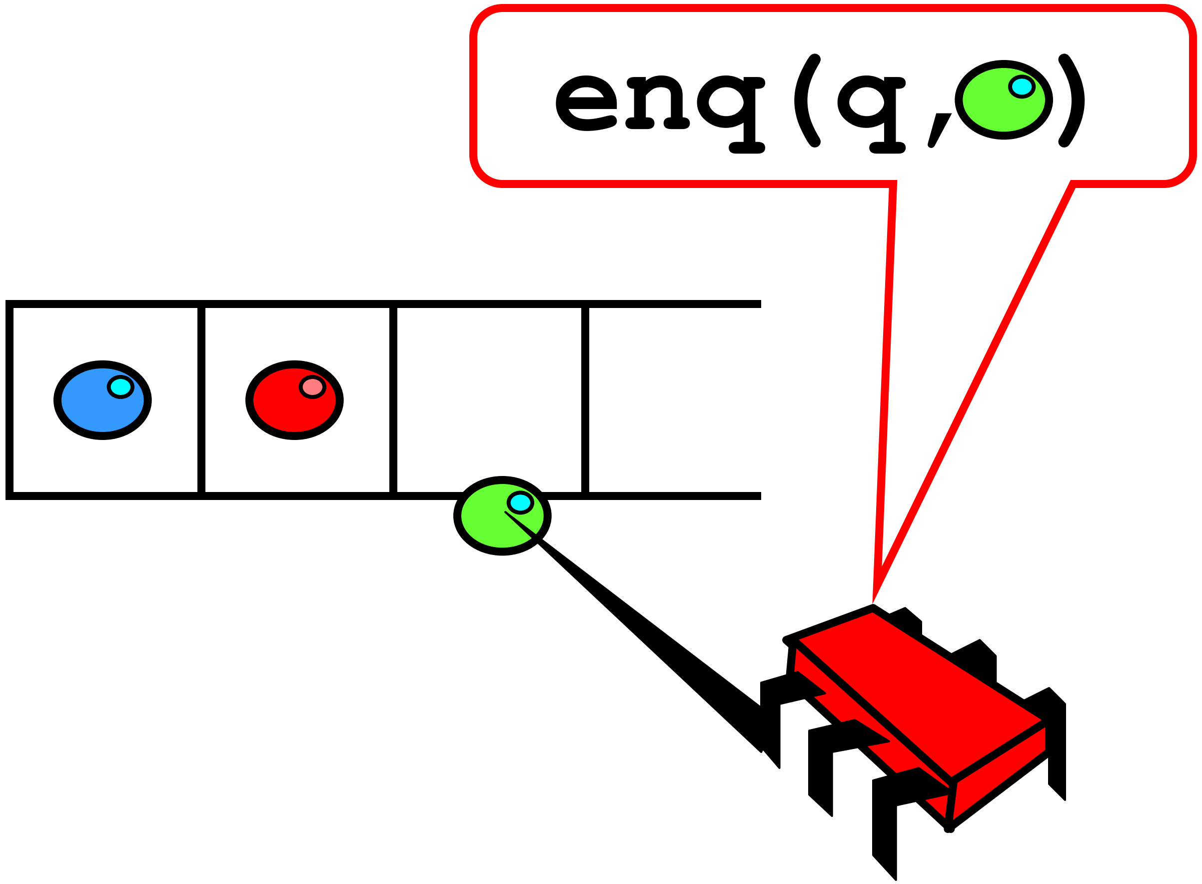
- What is a concurrent object?
  - How do we **describe** one?
  - How do we **implement** one?
  - How do we **tell if it is correct**?

Objectivism

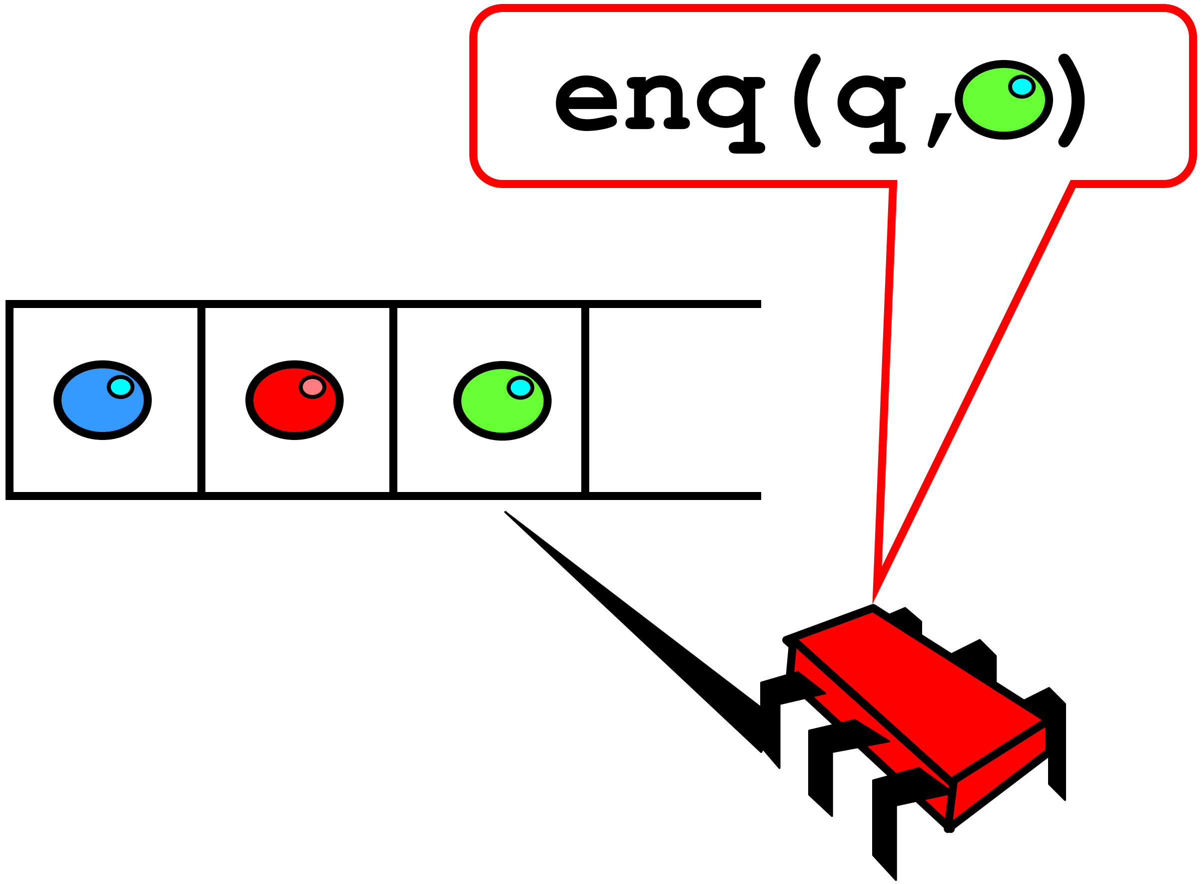
- What is a concurrent object?
  - How do we **describe** one?

  - How do we **tell if it is correct**?

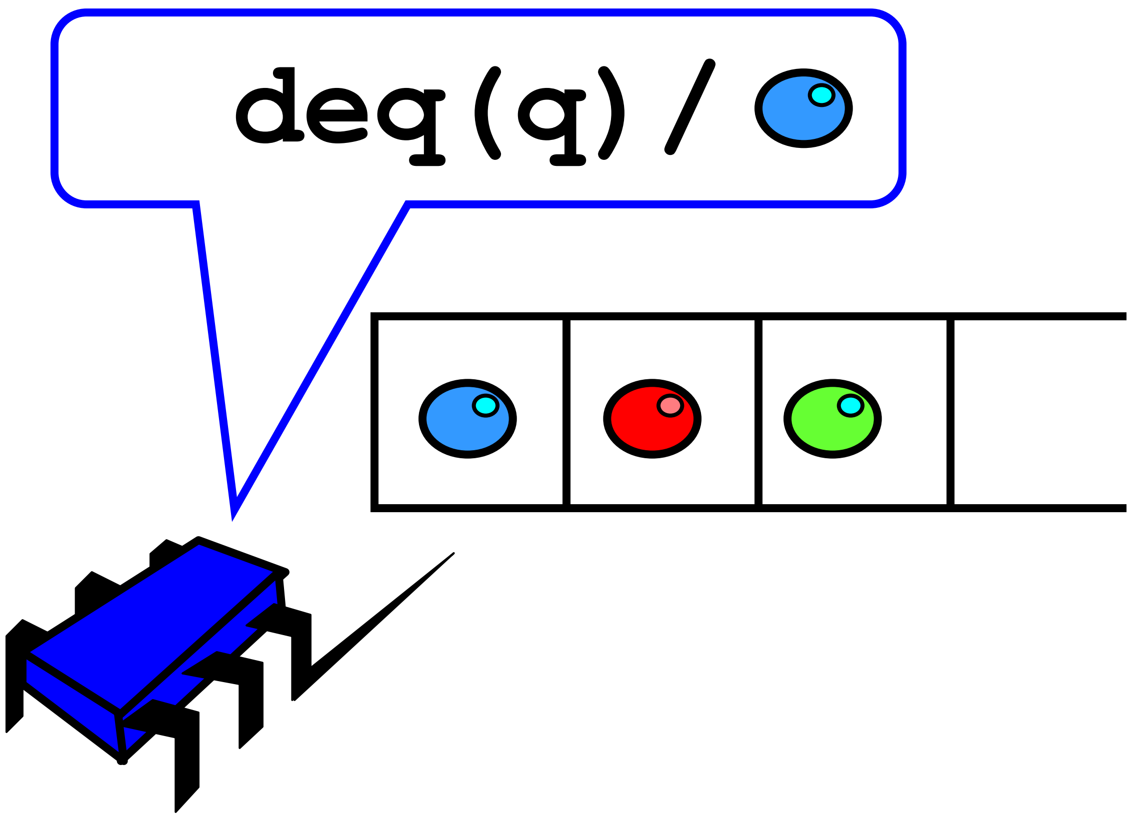
FIFO Queue: Enqueue Method



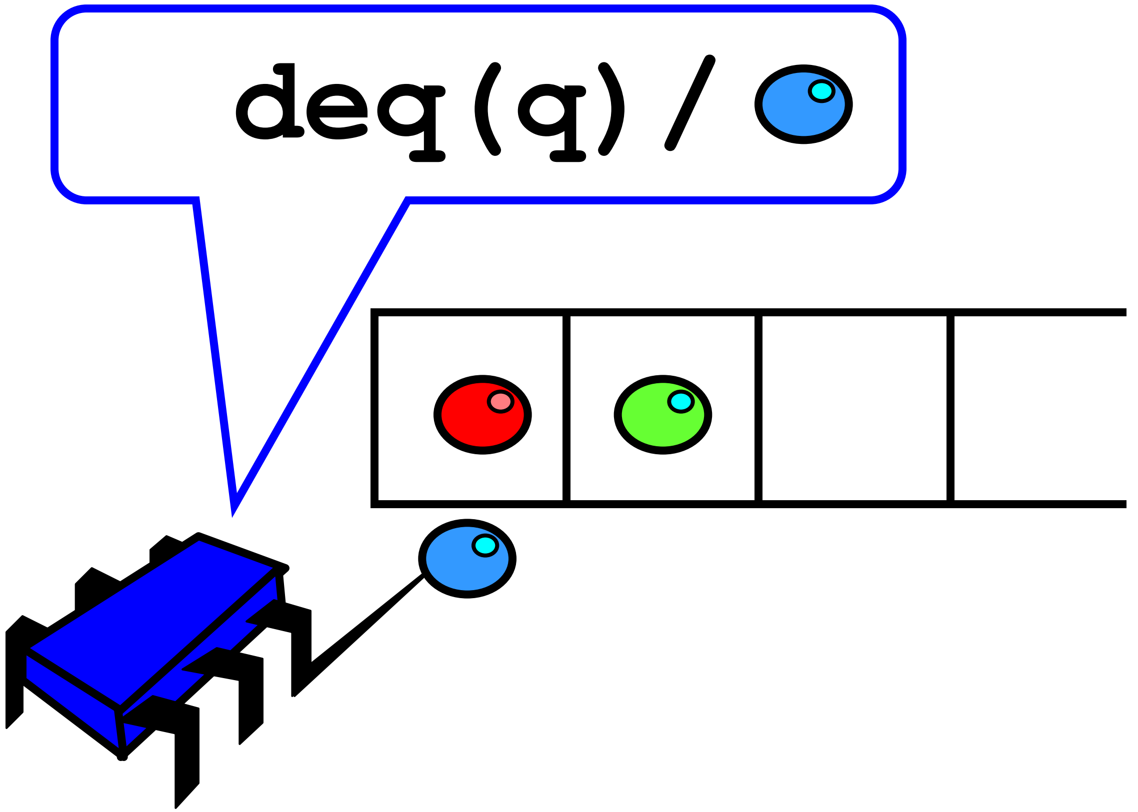
FIFO Queue: Enqueue Method



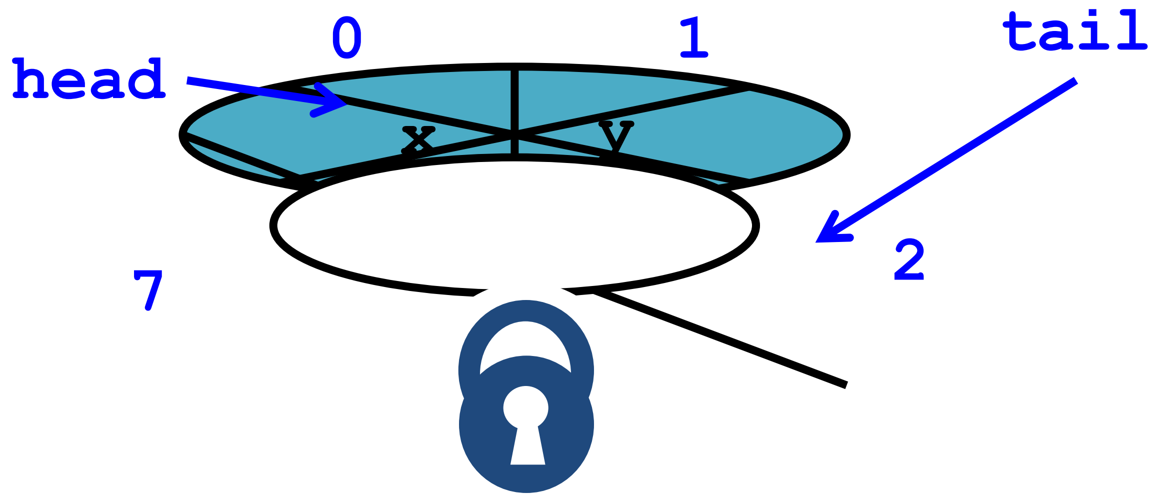
FIFO Queue: Dequeue Method



FIFO Queue: Dequeue Method

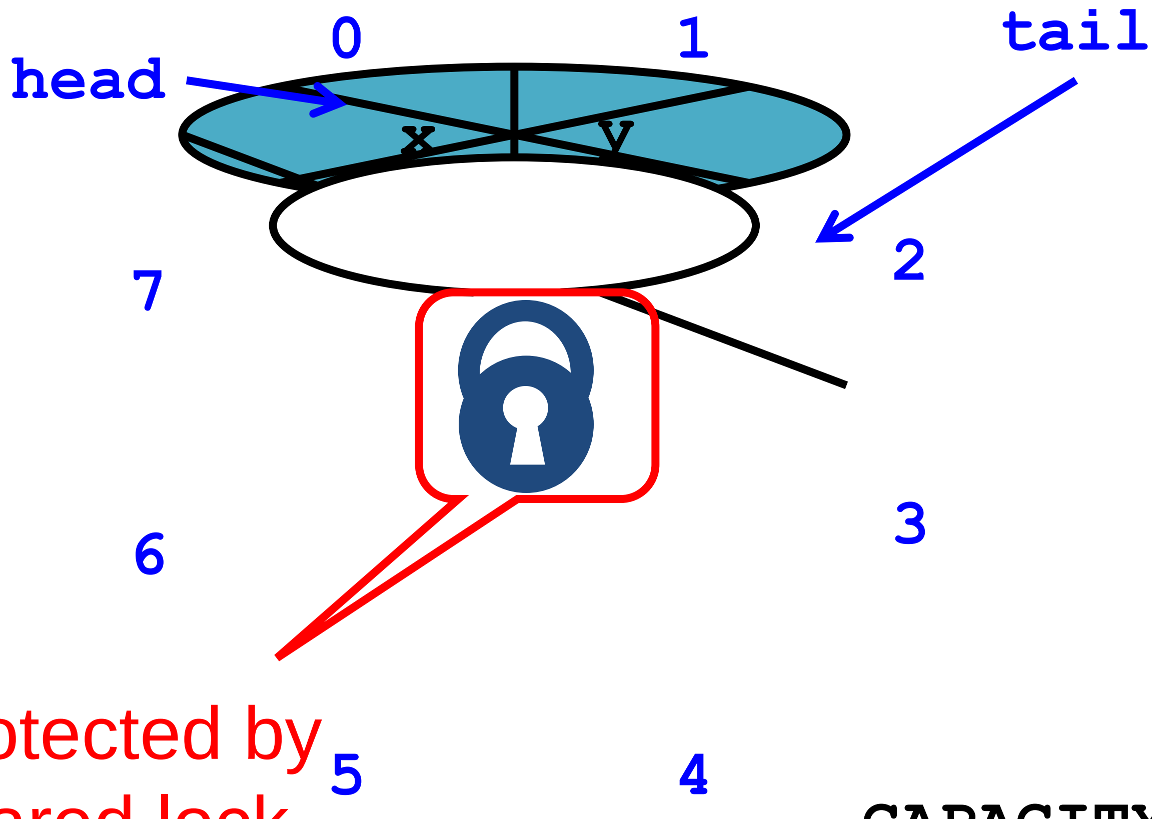


Lock-Based Queue



CAPACITY = 8

Lock-Based Queue

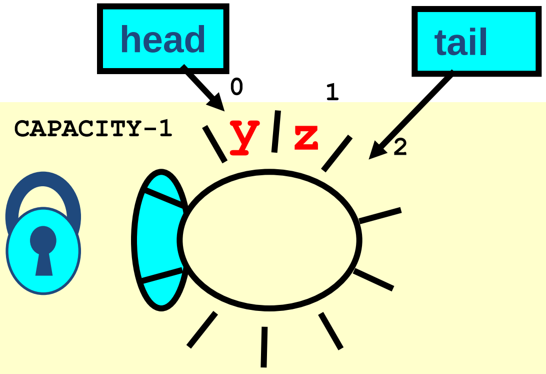


Fields protected by single shared lock

CAPACITY = 8

A Lock-Based Queue

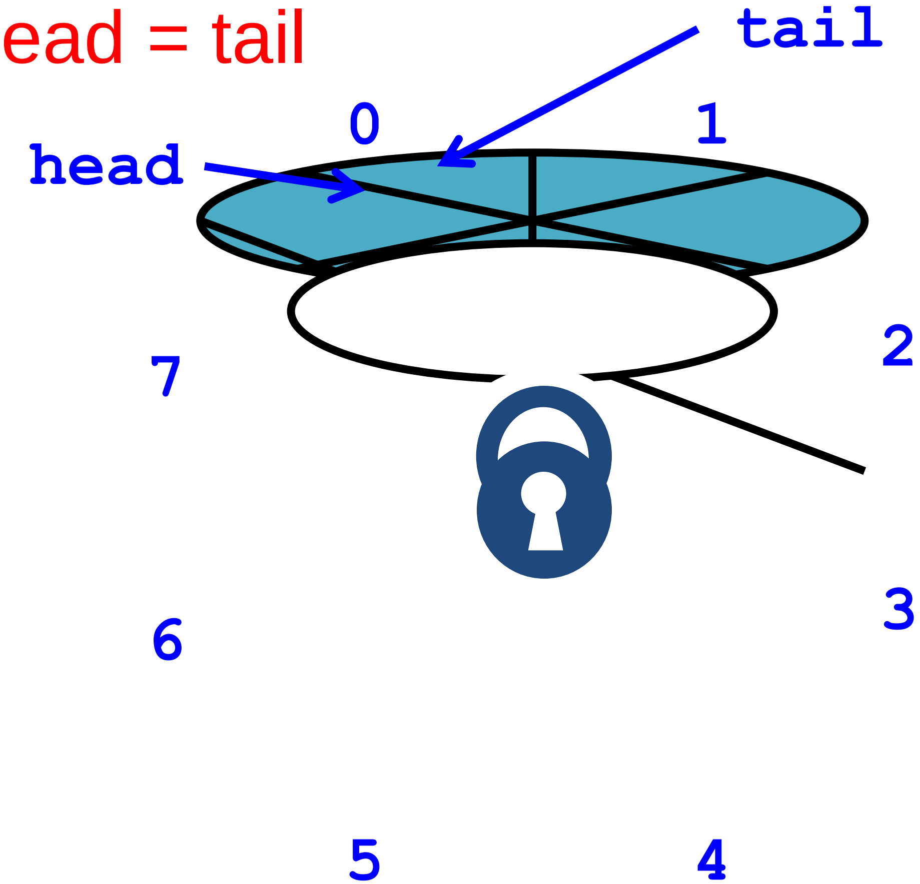
```
typedef struct {
  int head, tail;
  void* items[CAPACITY];
  phread_mutex_t lock;
} queue_t;
```



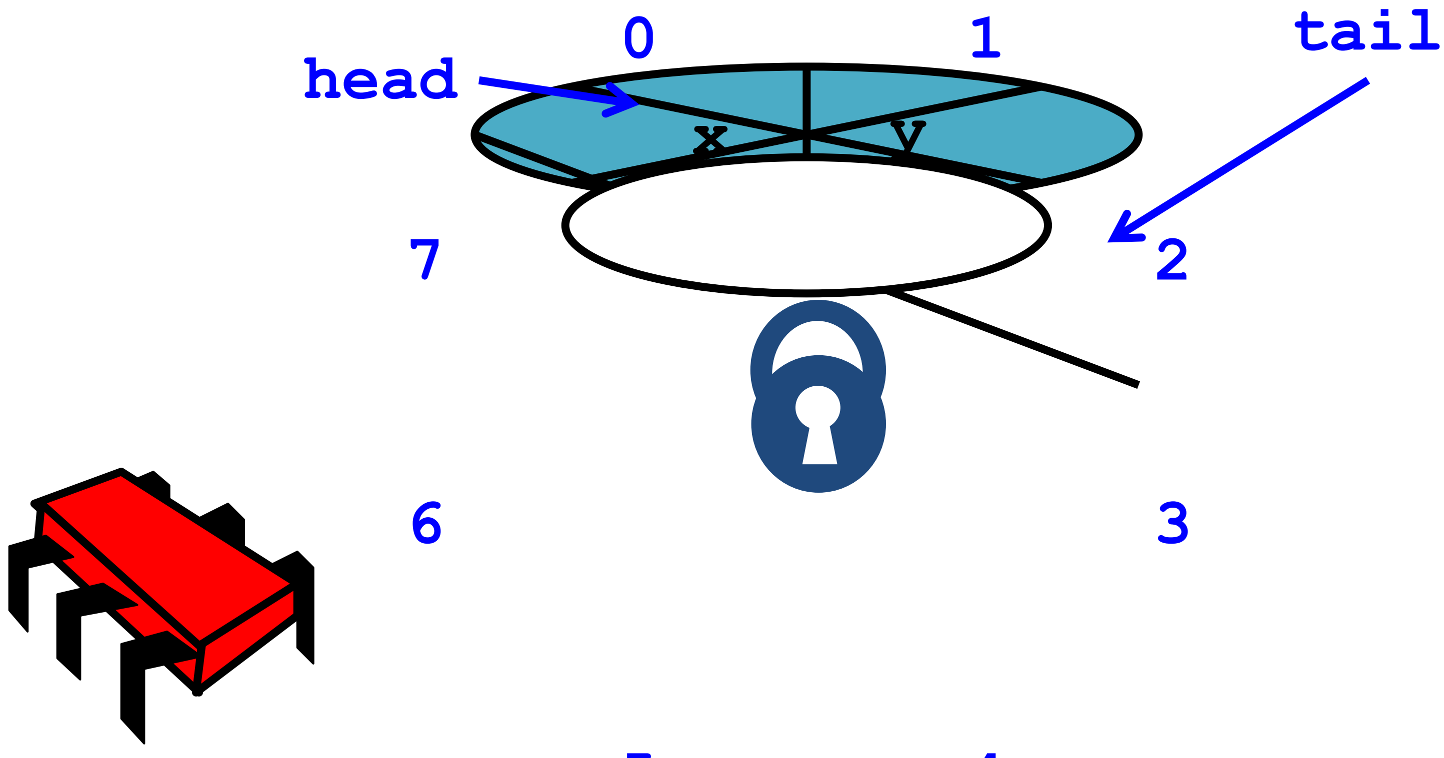
Fields protected by single shared lock

Lock-Based Queue

Initially head = tail



Lock-Based deq()



Implementation: deq()

```
int deq(queue_t q, void **elem) {
  int res;
  pthread_mutex_lock(&q->lock);
  if (q->tail == q->head) res = 0;
  else {
    *elem = q->items[q->head % CAPACITY];
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    res = 1;
  }
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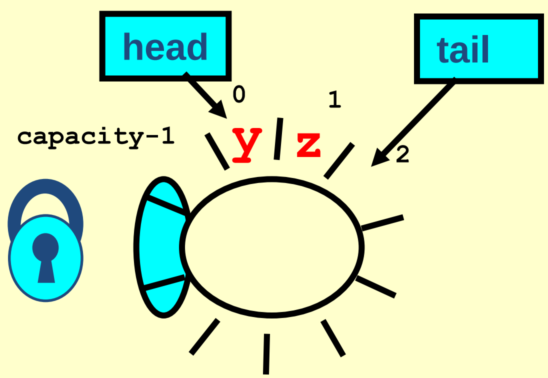
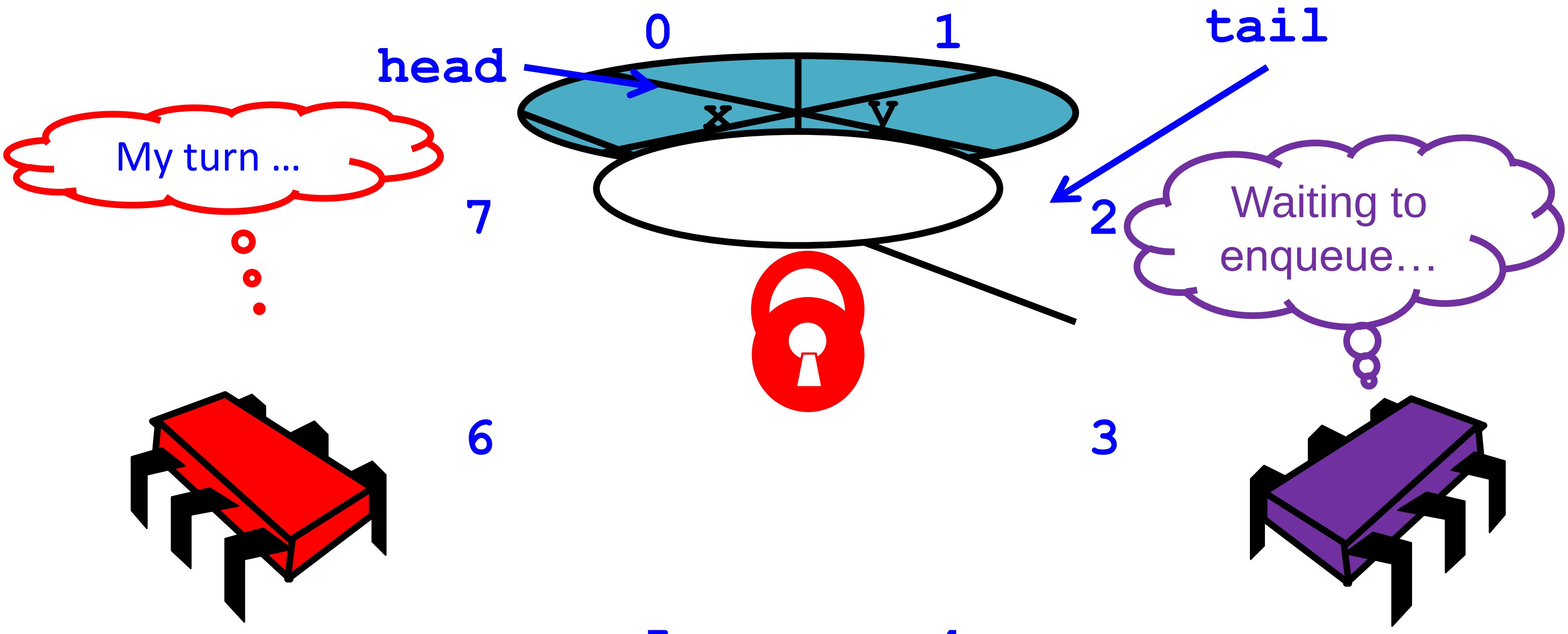
  return res;
}
```



Acquire Lock



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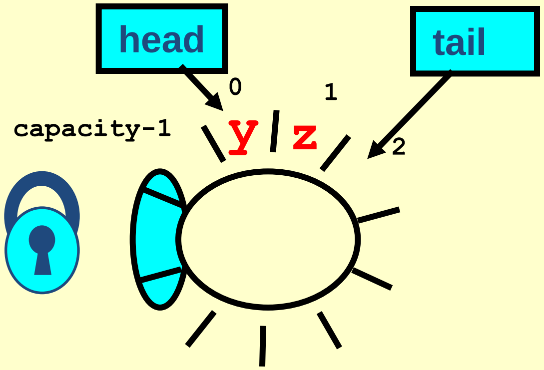
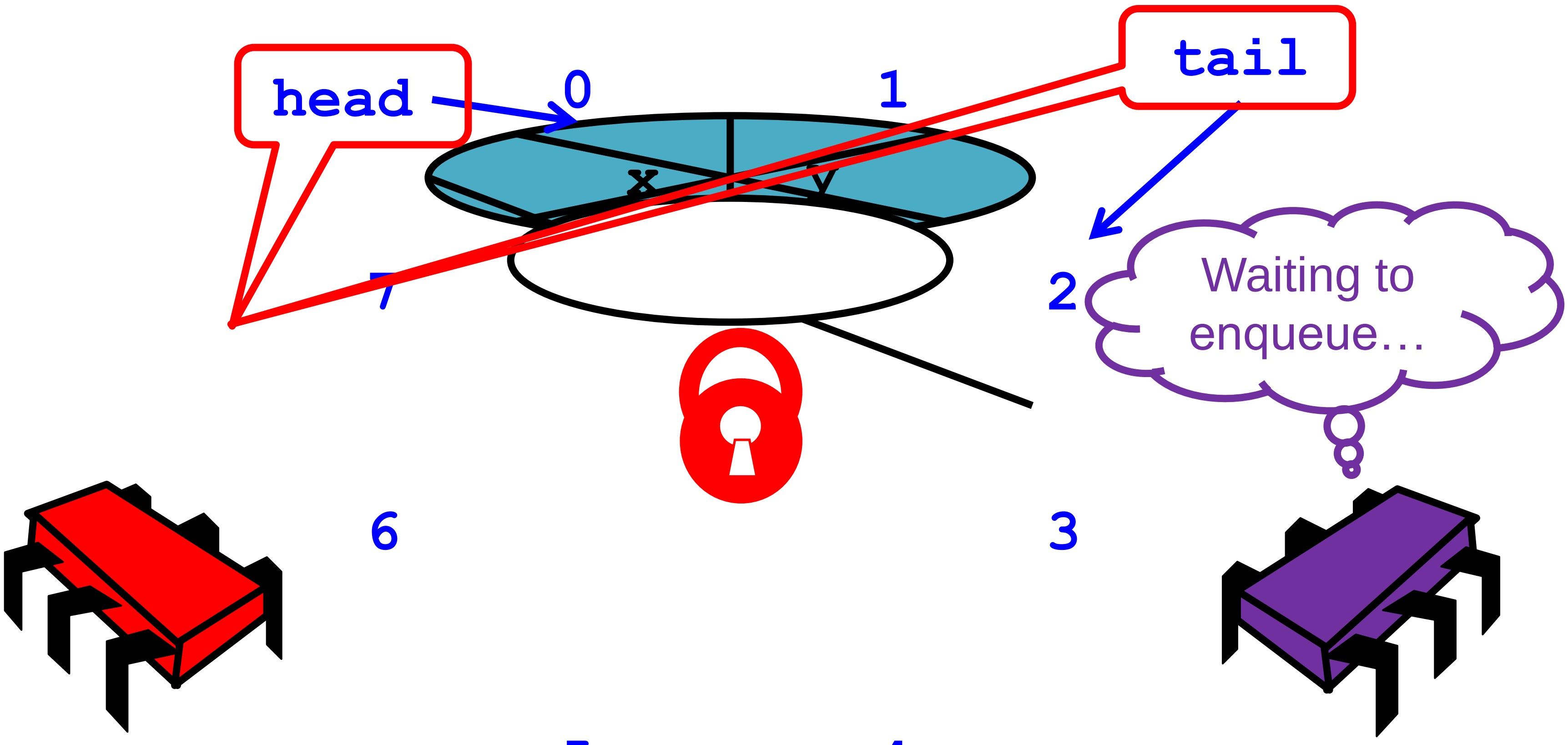
```

Acquire lock at method start



Check if Non-Empty



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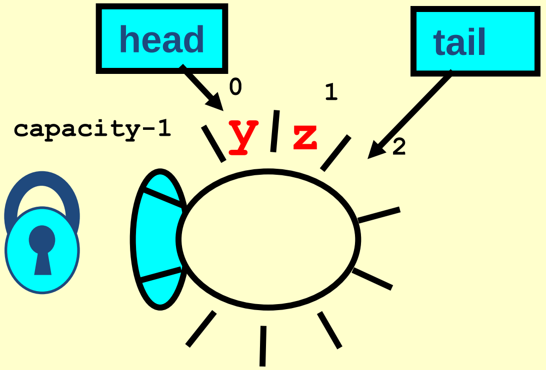
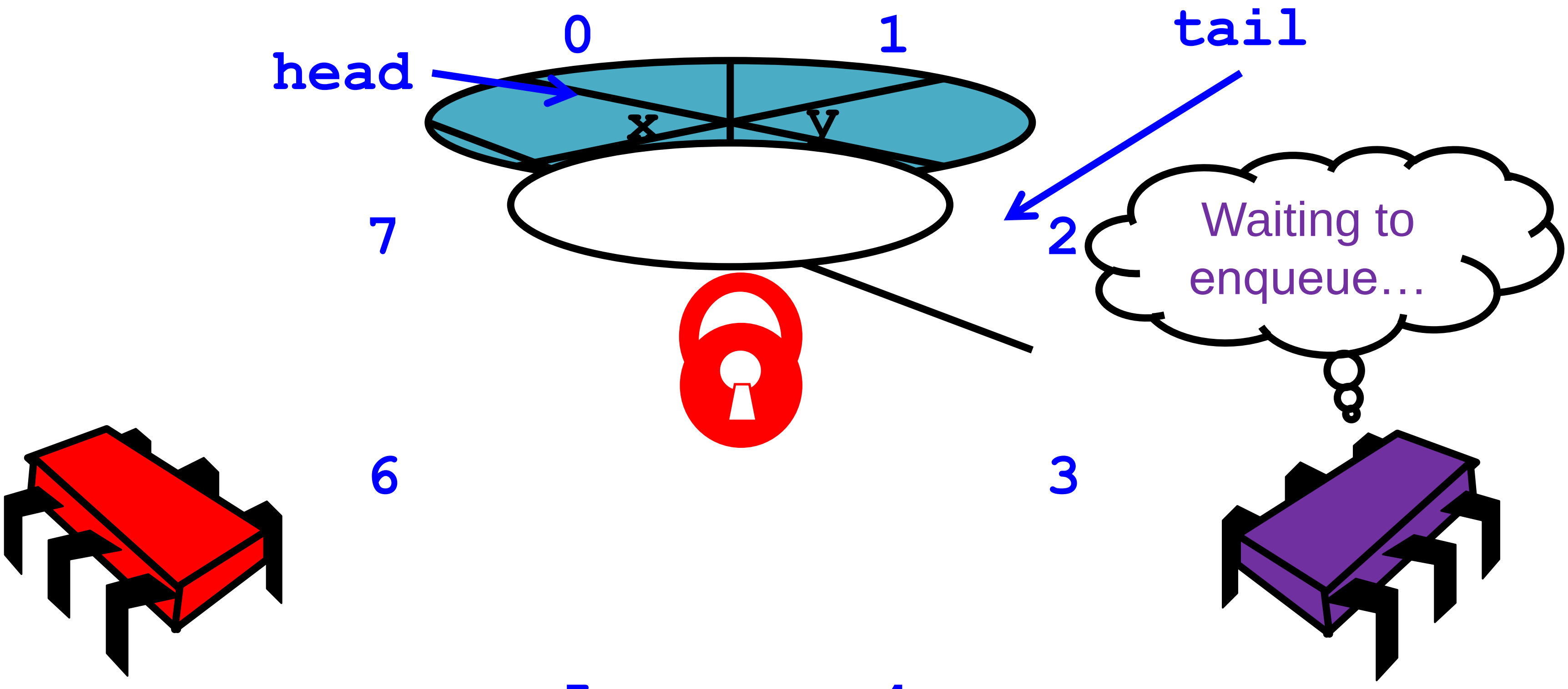
```

If queue empty
return "failure"



Modify the Queue



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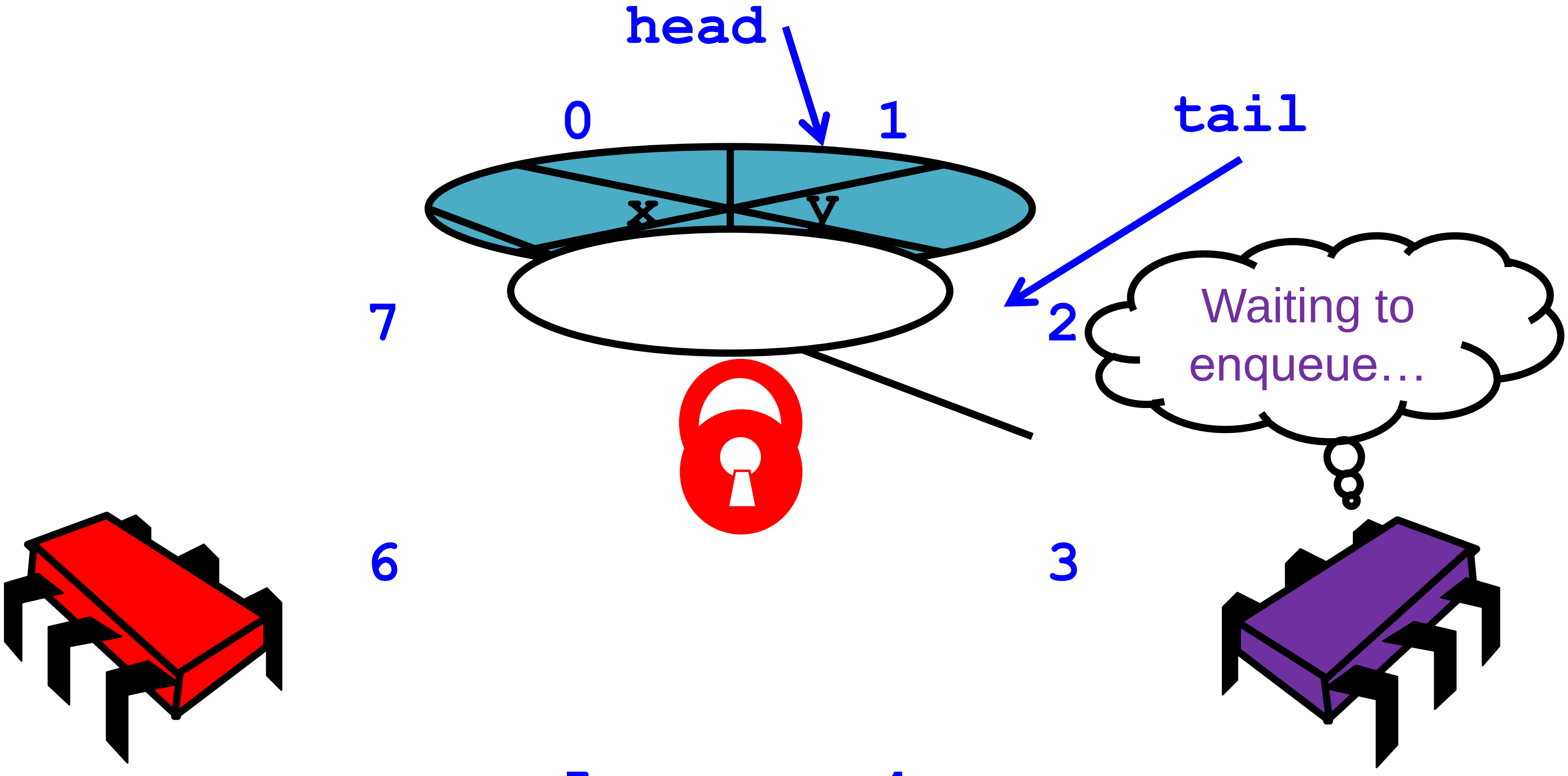

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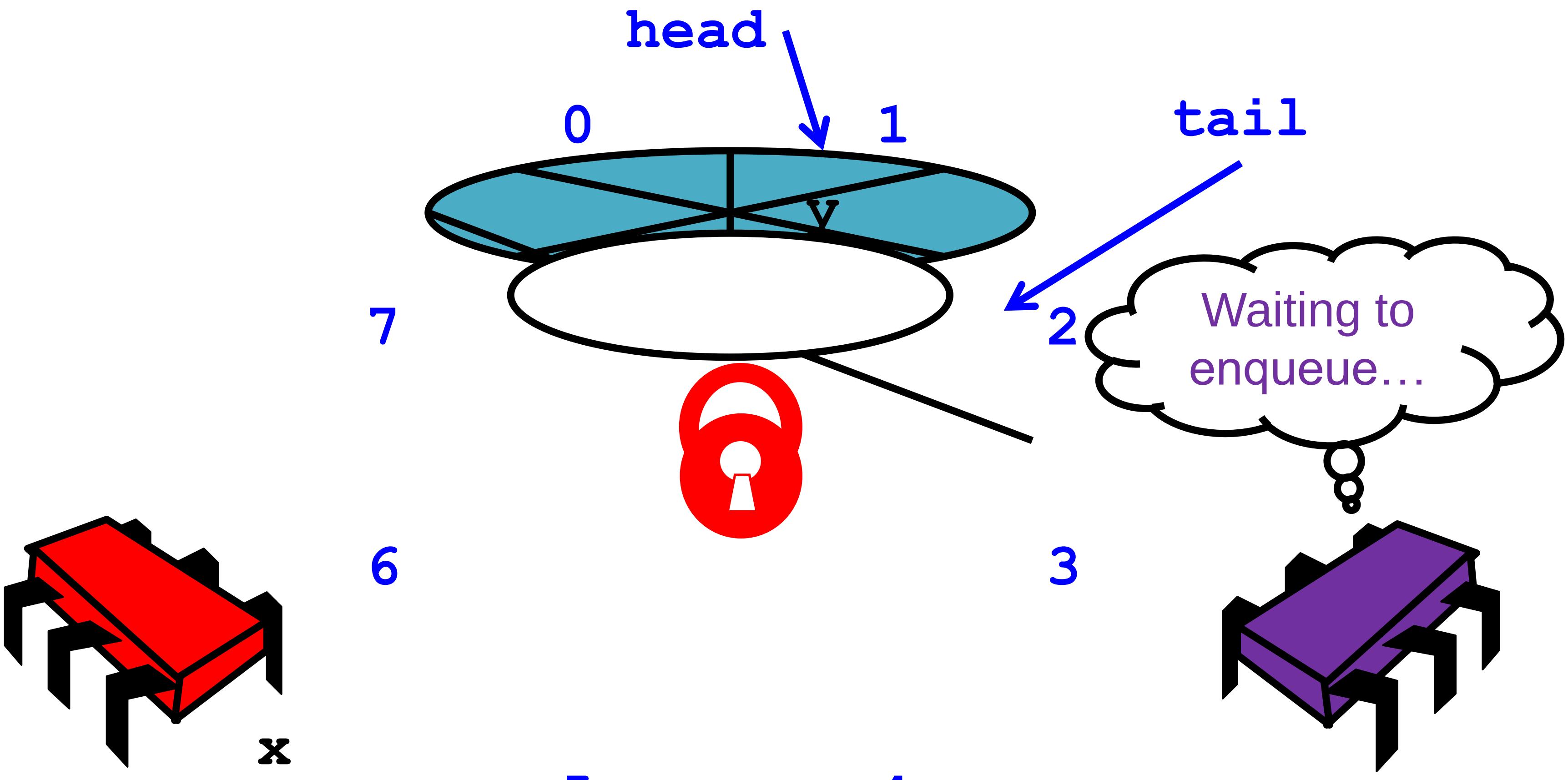

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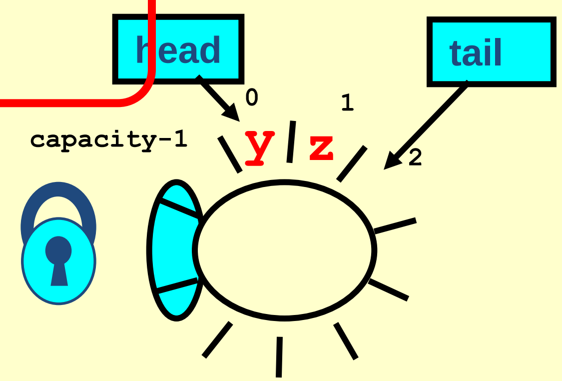
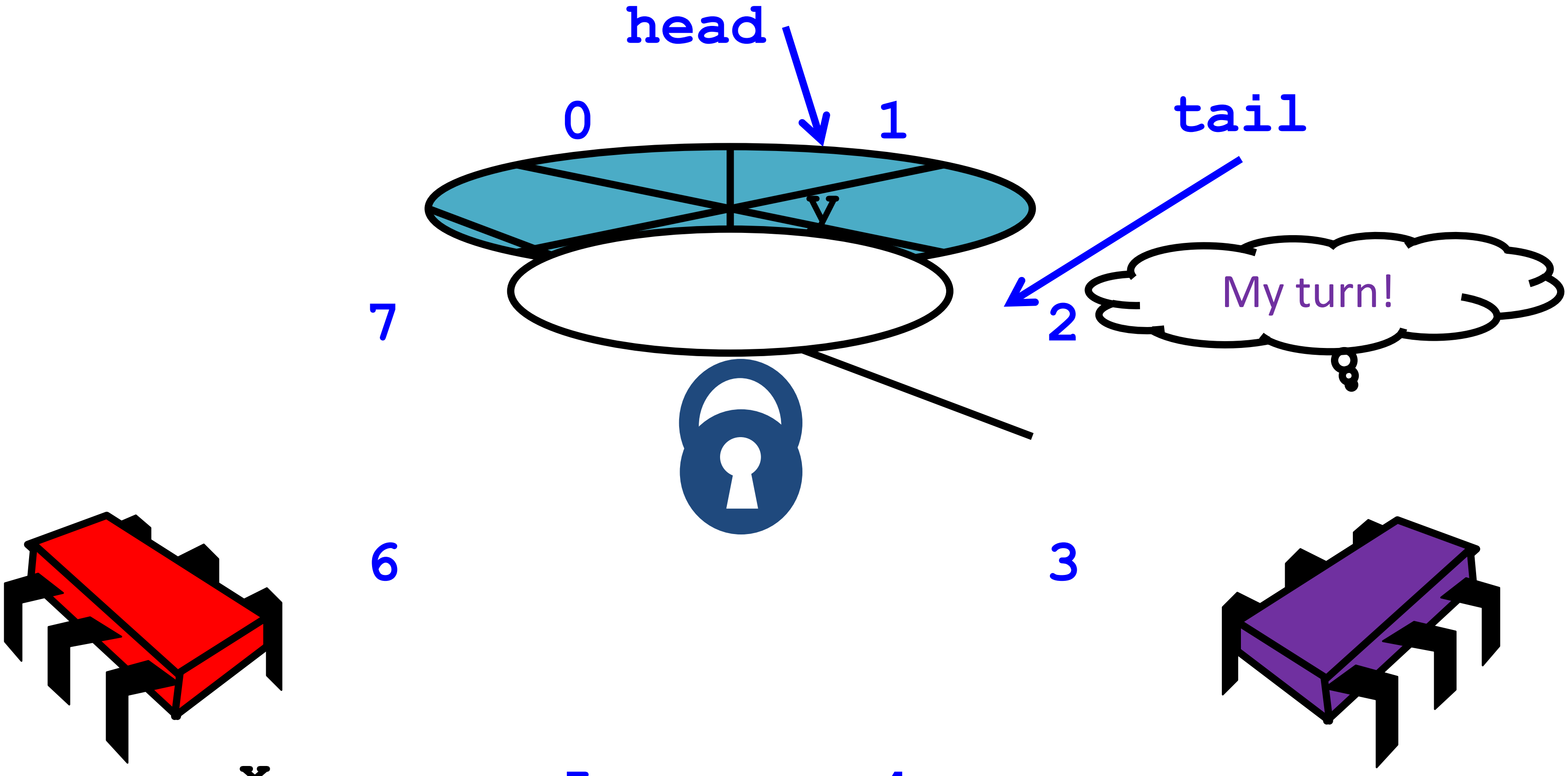
Queue not empty?
Remove item and update head



Release the Lock



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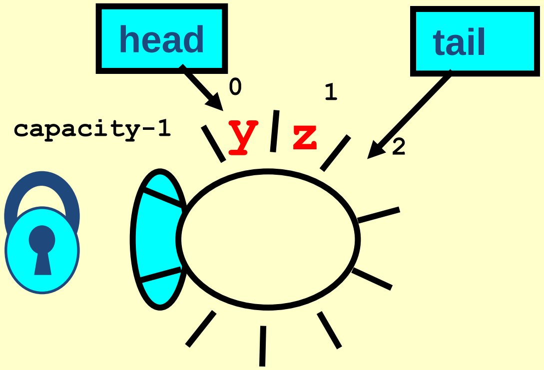
```

Release lock no matter what!



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  }
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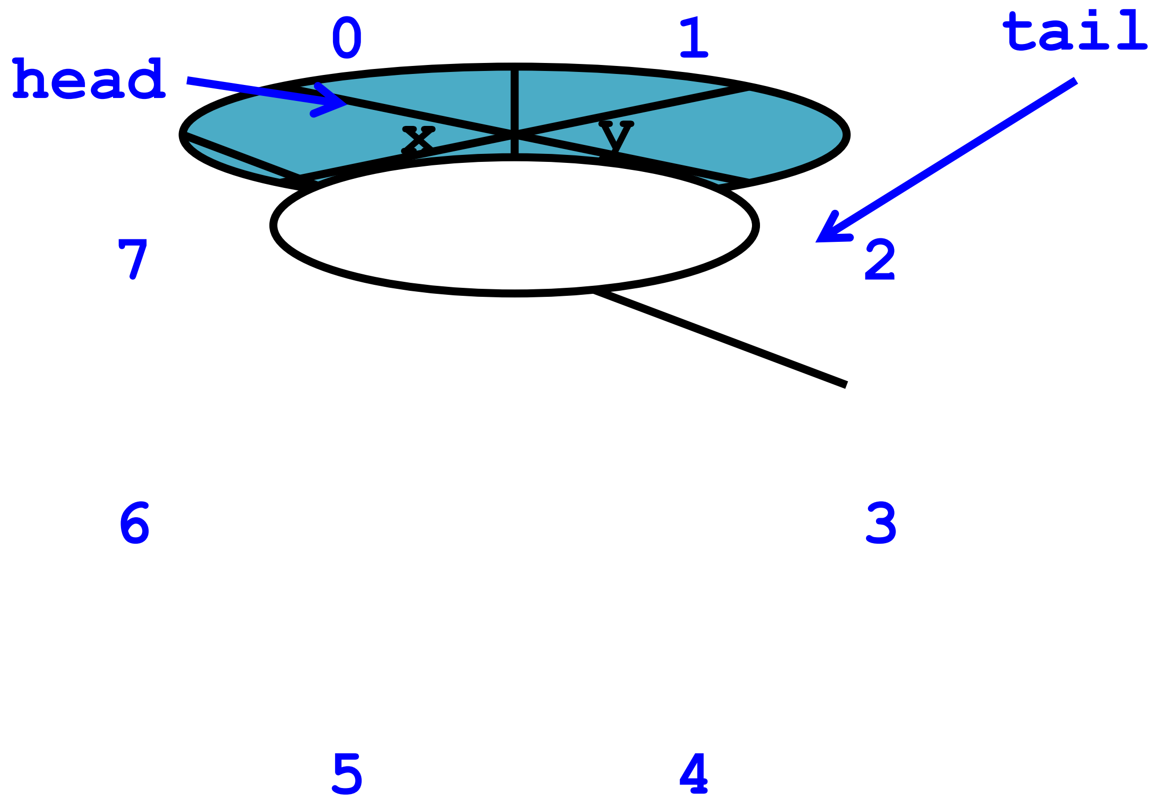
  return res;
}
```

Should be correct because modifications are mutually exclusive…

Now consider the following implementation

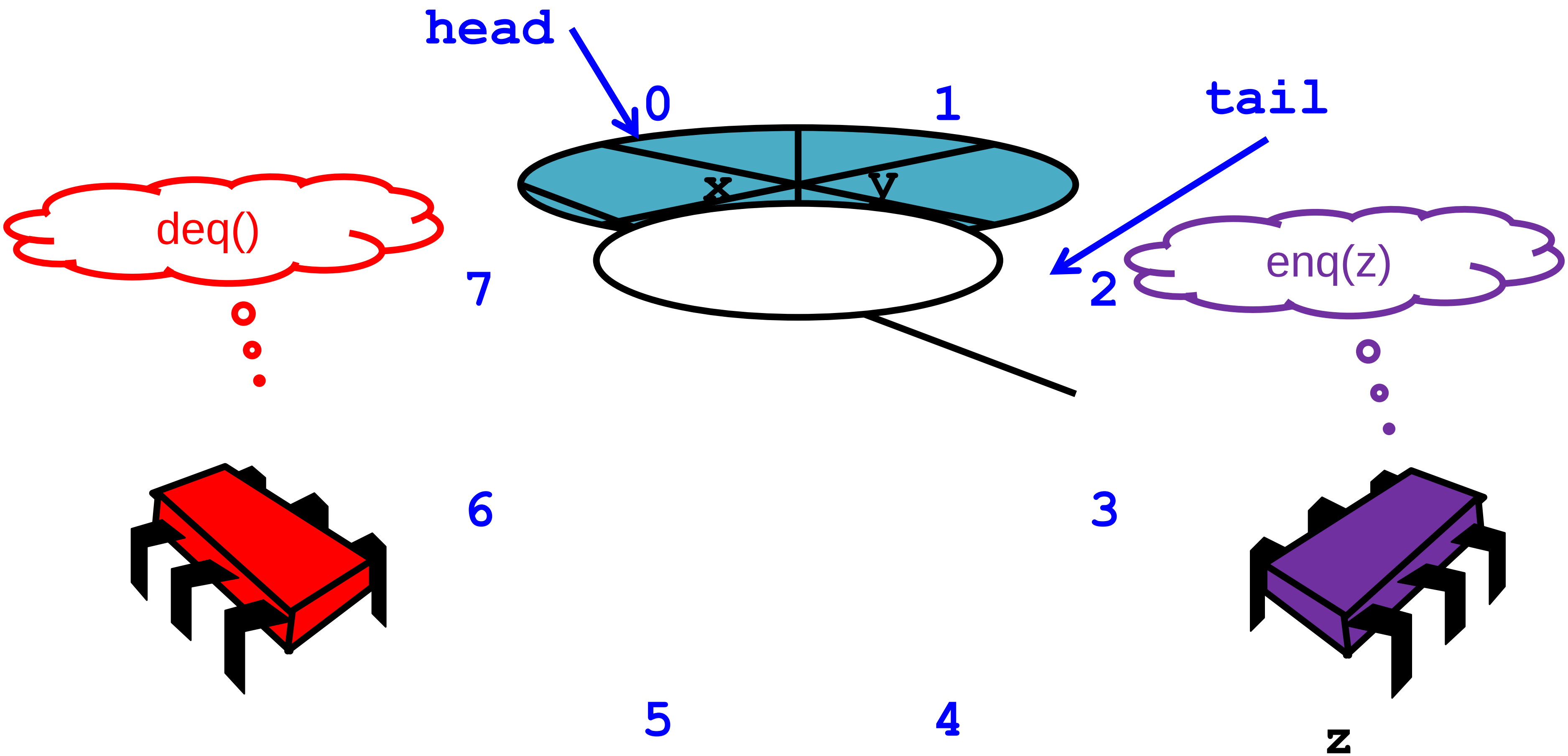
- The same thing without mutual exclusion
- For simplicity, only two threads
  - One thread enq only
  - The other deq only

Wait-free 2-Thread Queue



CAPACITY = 8

Wait-free 2-Thread Queue



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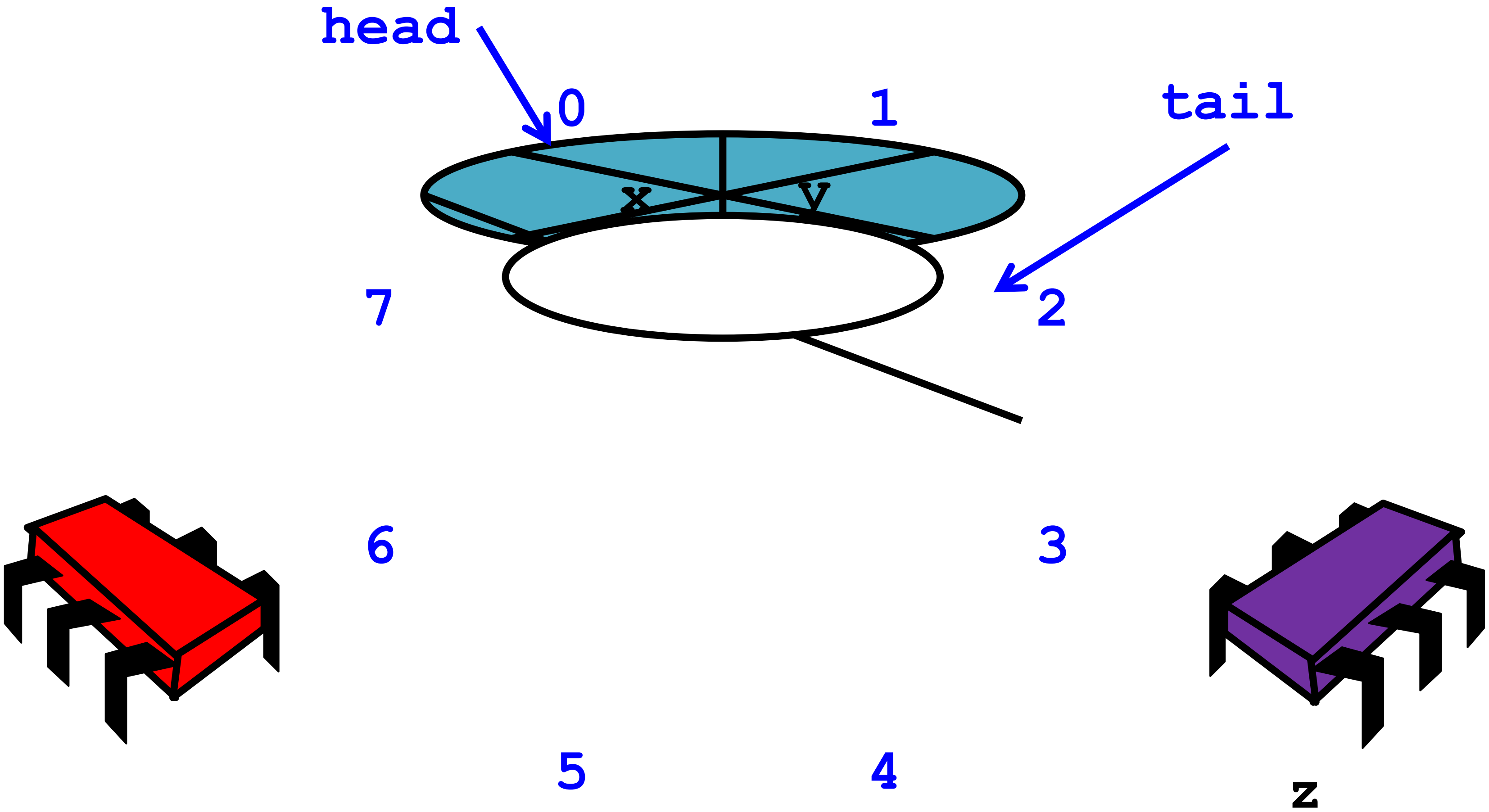

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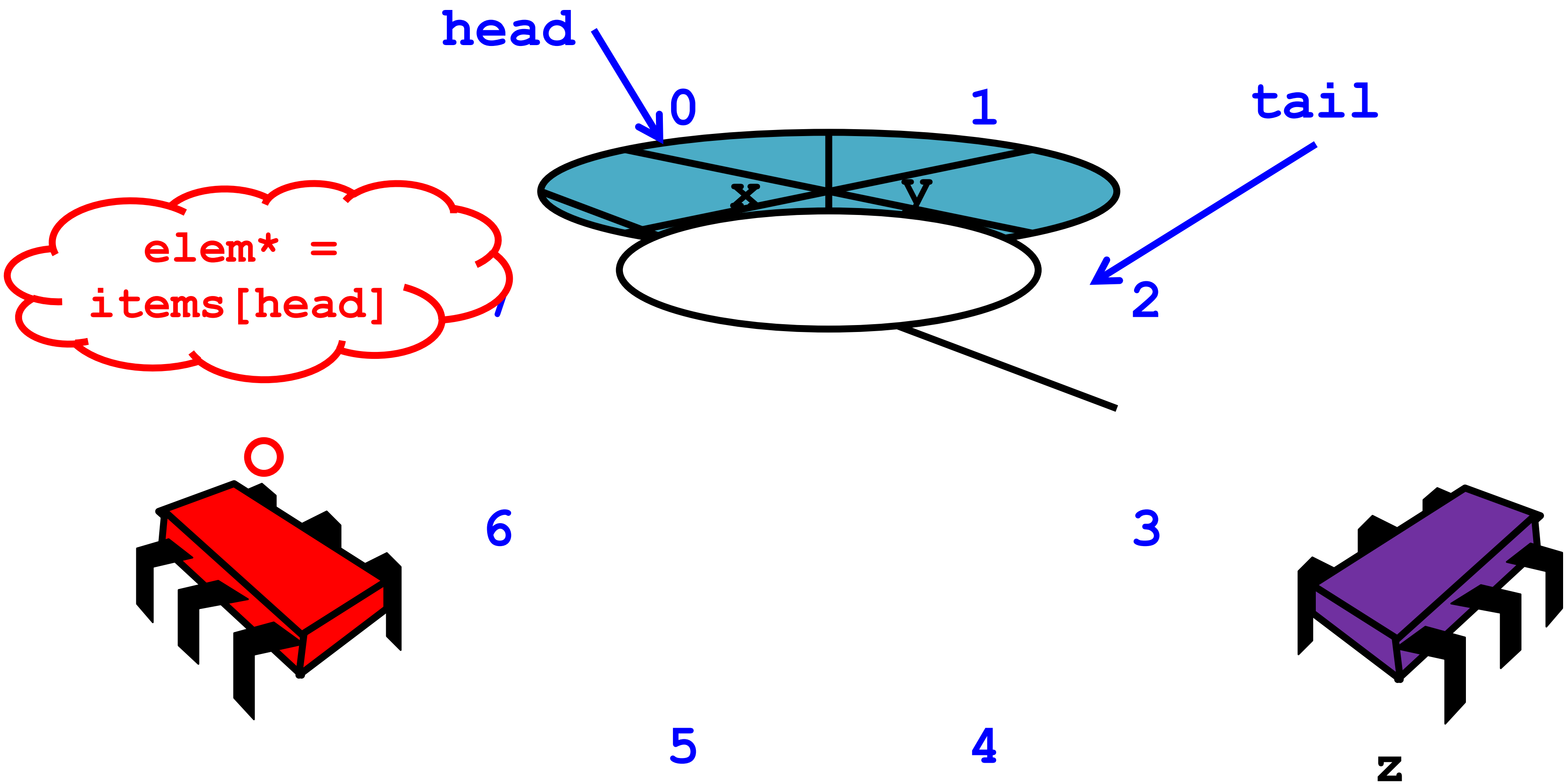

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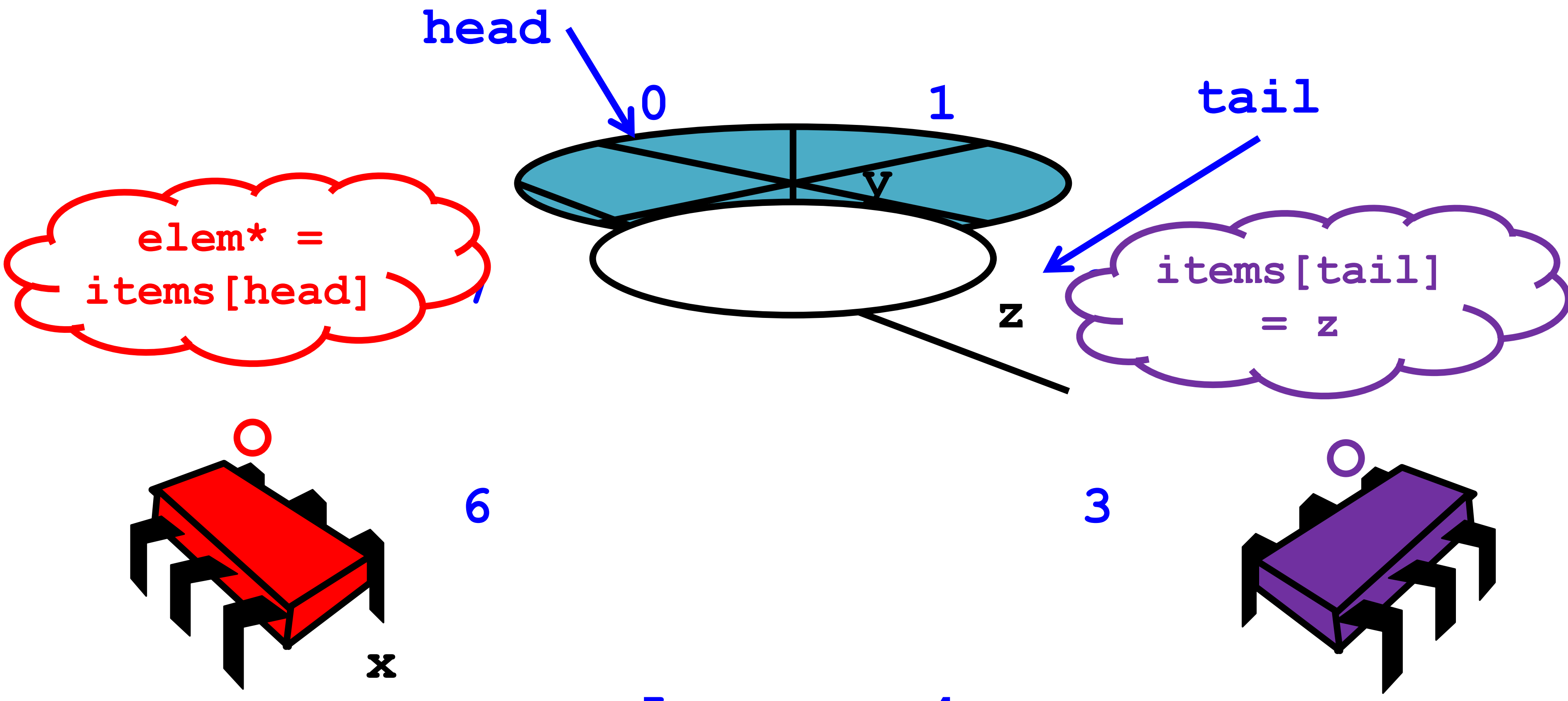

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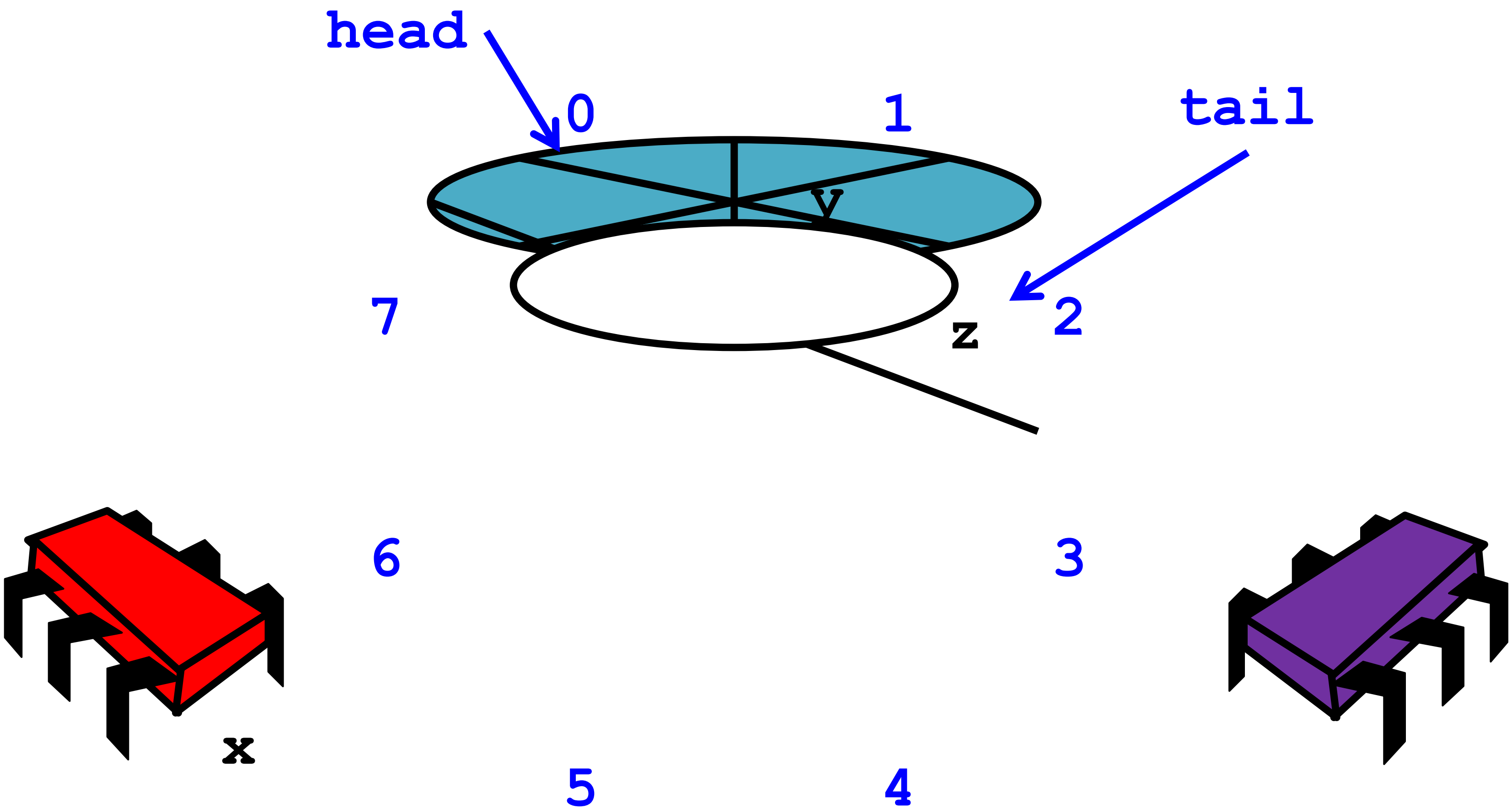

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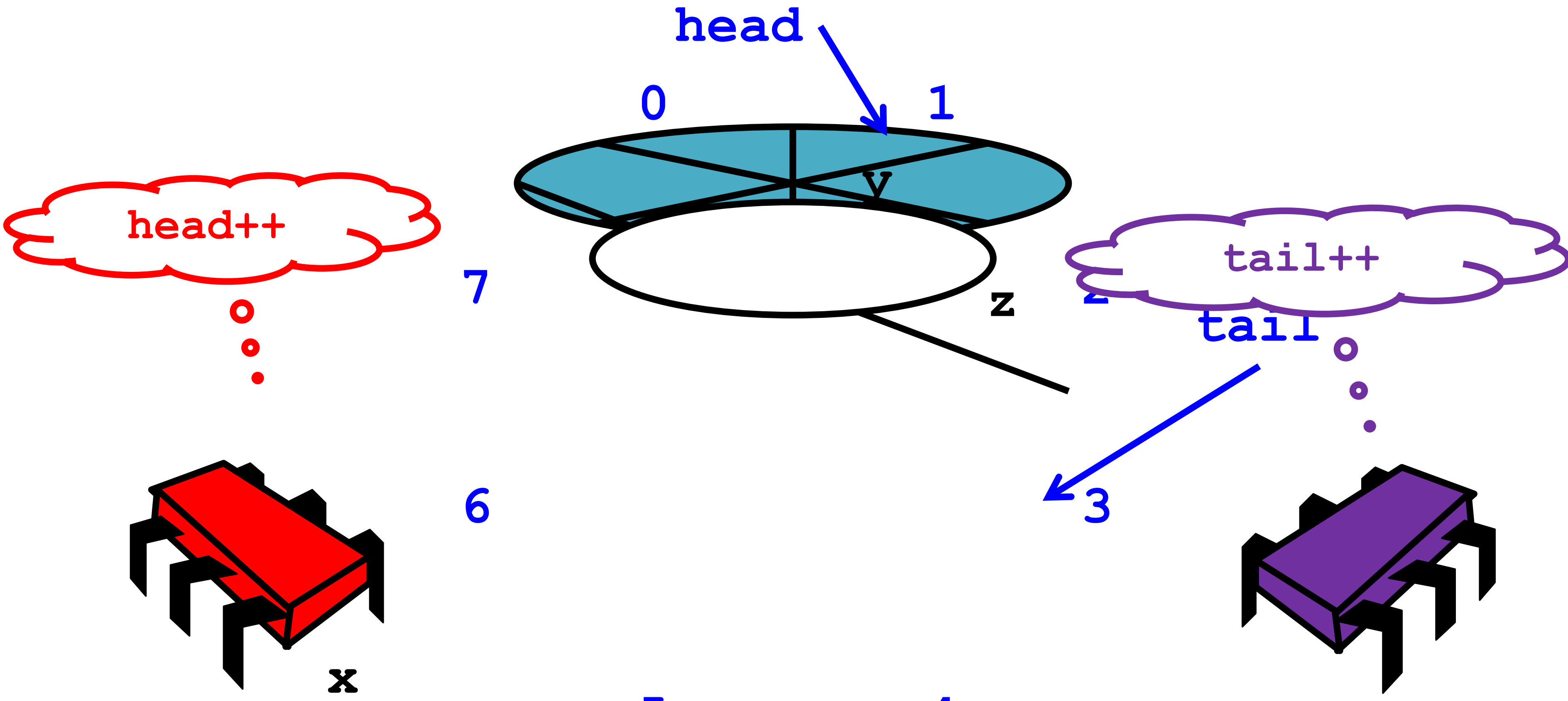

Wait-free 2-Thread Queue

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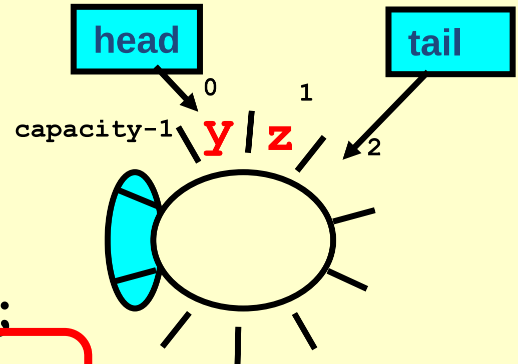
int deq(queue_t q, void **elem) {
  if (q->tail == q->head) return = 0;
  *elem = q->items[q->head % CAPACITY];
  q->head++;
  return 1;
}

int enq(queue_t q, void *x) {
  if (tail-head == CAPACITY) return 0;
  q->items[q->tail % CAPACITY] = x;
  q->tail++;
  return 1;
}
```



No lock needed !

Wait-free 2-Thread Queue

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}
```

How do we define "correct" when modifications are not mutually exclusive?

What *is* a Concurrent Queue?

- Need a way to specify a concurrent queue object
- Need a way to prove that an algorithm implements the object's specification
- Let's talk about object specifications …

Correctness and Progress

- In a concurrent setting, we need to specify both the safety and the liveness properties of an object
- Need a way to define
  - when an implementation is correct
  - the conditions under which it guarantees progress

Correctness and Progress

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  - when an implementation is correct
  - the conditions under which it guarantees progress

Let's begin with correctness

Sequential Objects

- Each object has a ***state***
  - Usually given by a set of ***fields***
  - Queue example: `items`, `head`, `tail`
- Each object has a set of ***methods***
  - Only way to manipulate state
  - Queue example: **enq** and **deq** methods

Sequential Specifications

- If (precondition)
  - the object is in such-and-such a state
  - before you call the method,
- Then (postcondition)
  - the object will be in some other state
  - and the method will return a particular value

Pre and Postconditions for Dequeue

- Precondition:
  - Queue is non-empty
- Postcondition:
  - Returns 1
- Postcondition:
  - Removes first item in queue

Pre and Postconditions for Dequeue

- Precondition:
  - Queue is empty
- Postcondition:
  - Returns 0
- Postcondition:
  - Queue state unchanged

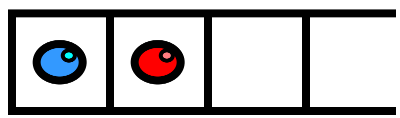
Why Sequential Specifications Totally Rock

- Interactions among methods captured by side-effects on object state
  - State meaningful between method calls
- Documentation size linear in number of methods
  - Each method described in isolation
- Can add new methods
  - Without changing descriptions of old methods

What About Concurrent Specifications ?

- Methods?
- Documentation?
- Adding new methods?

Methods Take Time



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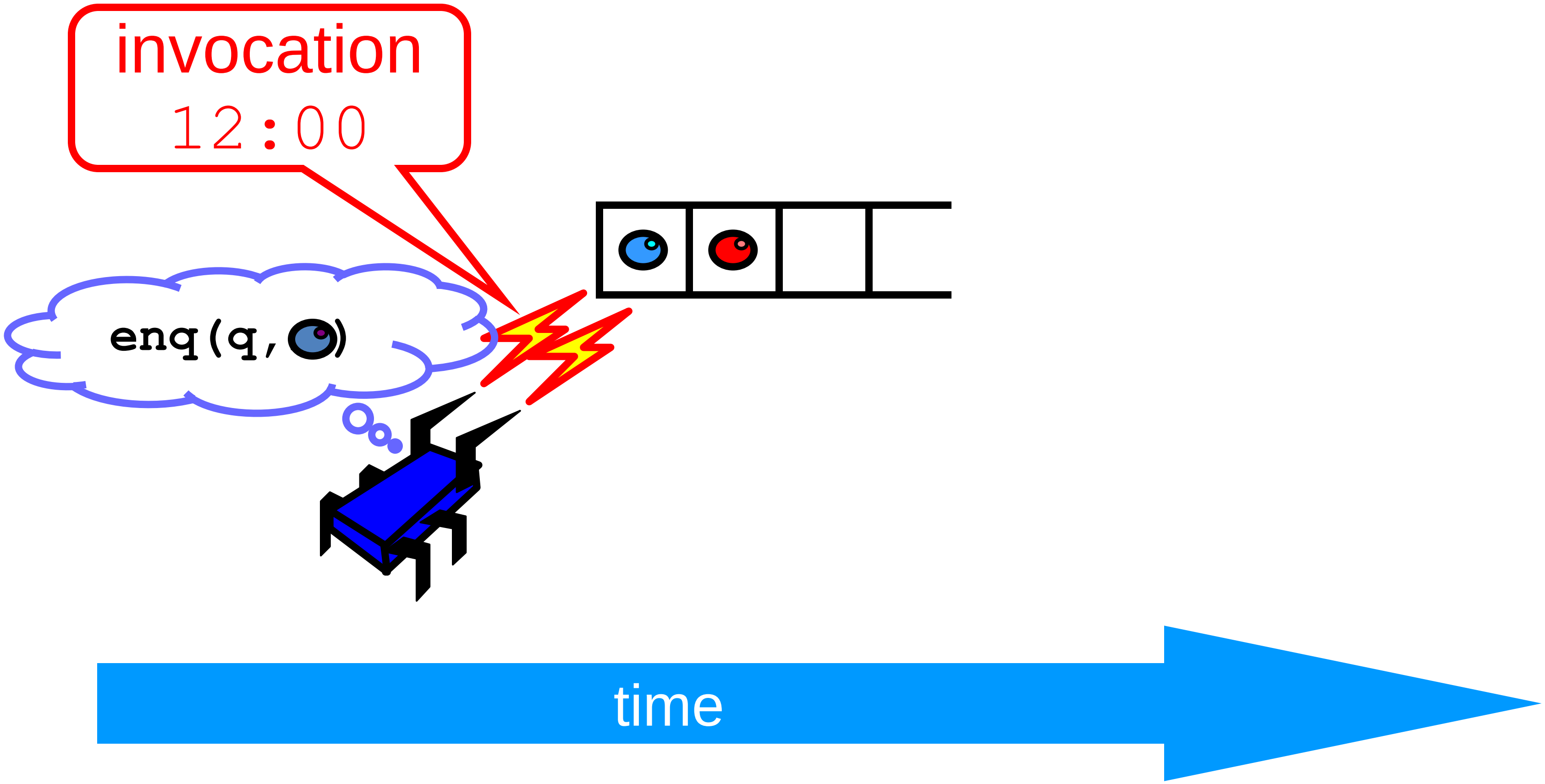

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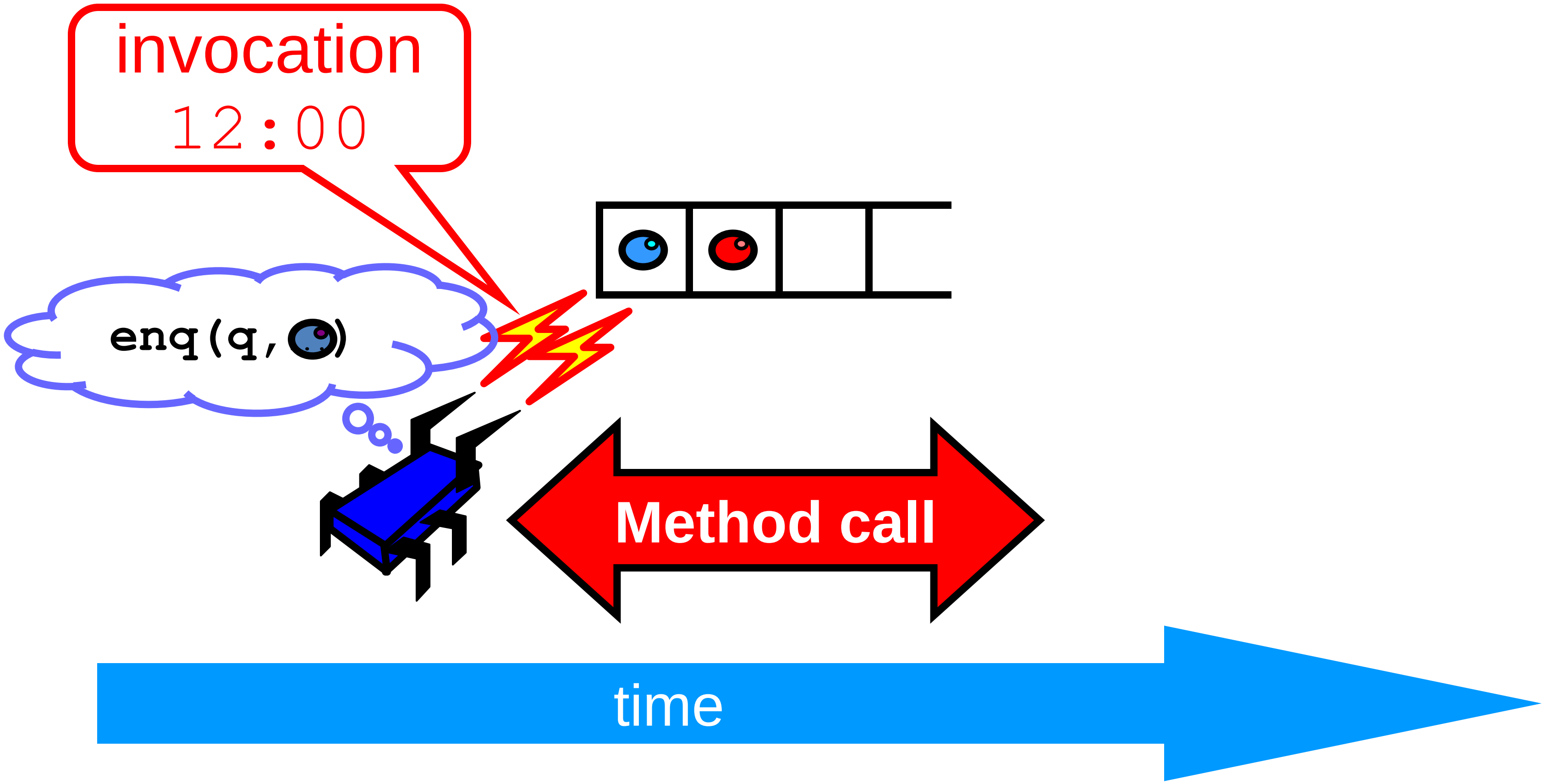

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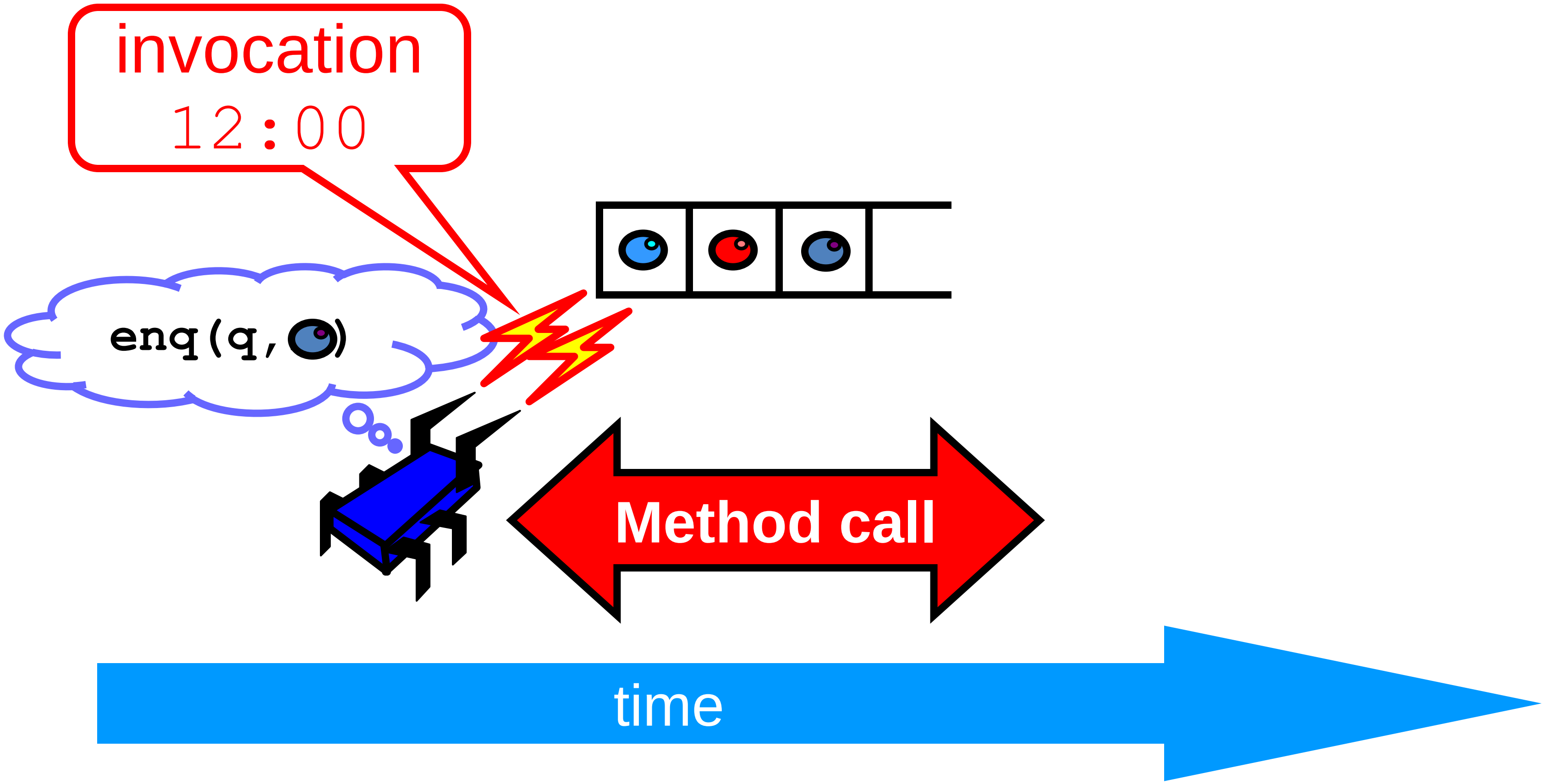

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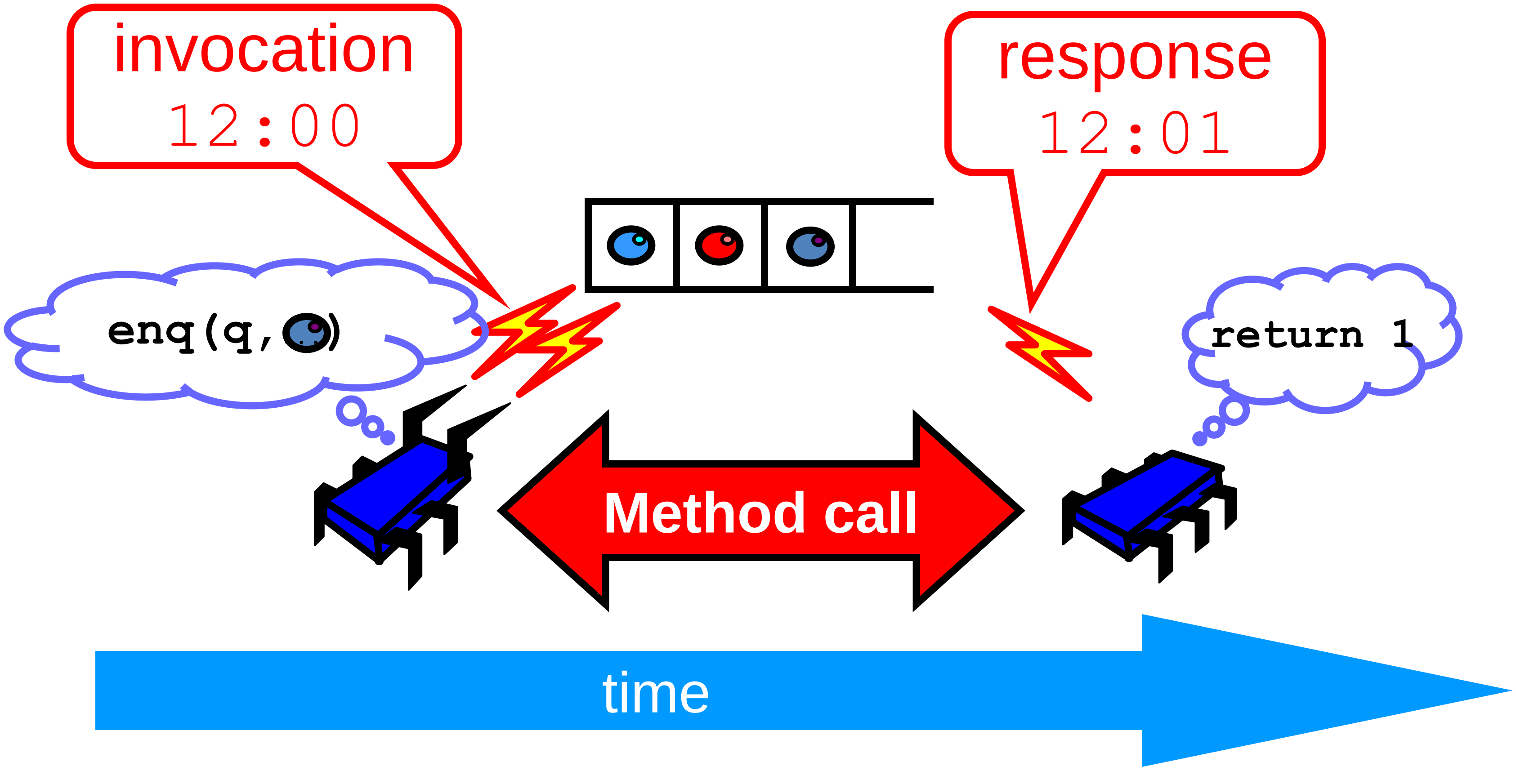
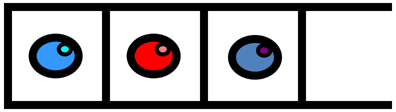


Sequential vs Concurrent

- Sequential
  - Methods take time? Who knew?
- Concurrent
  - Method call is not an event
  - Method call is an interval.

Concurrent Methods Take Overlapping Time



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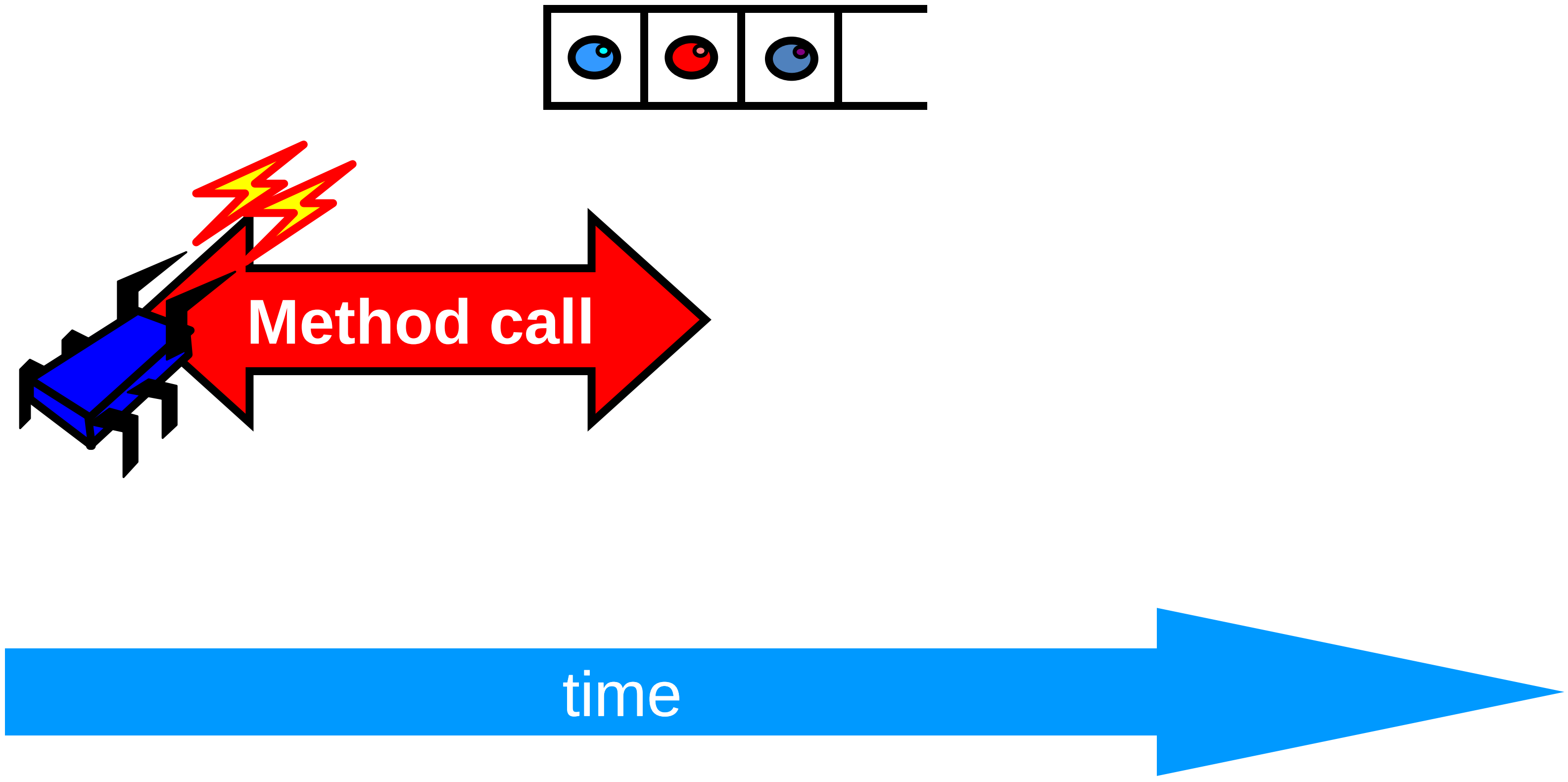

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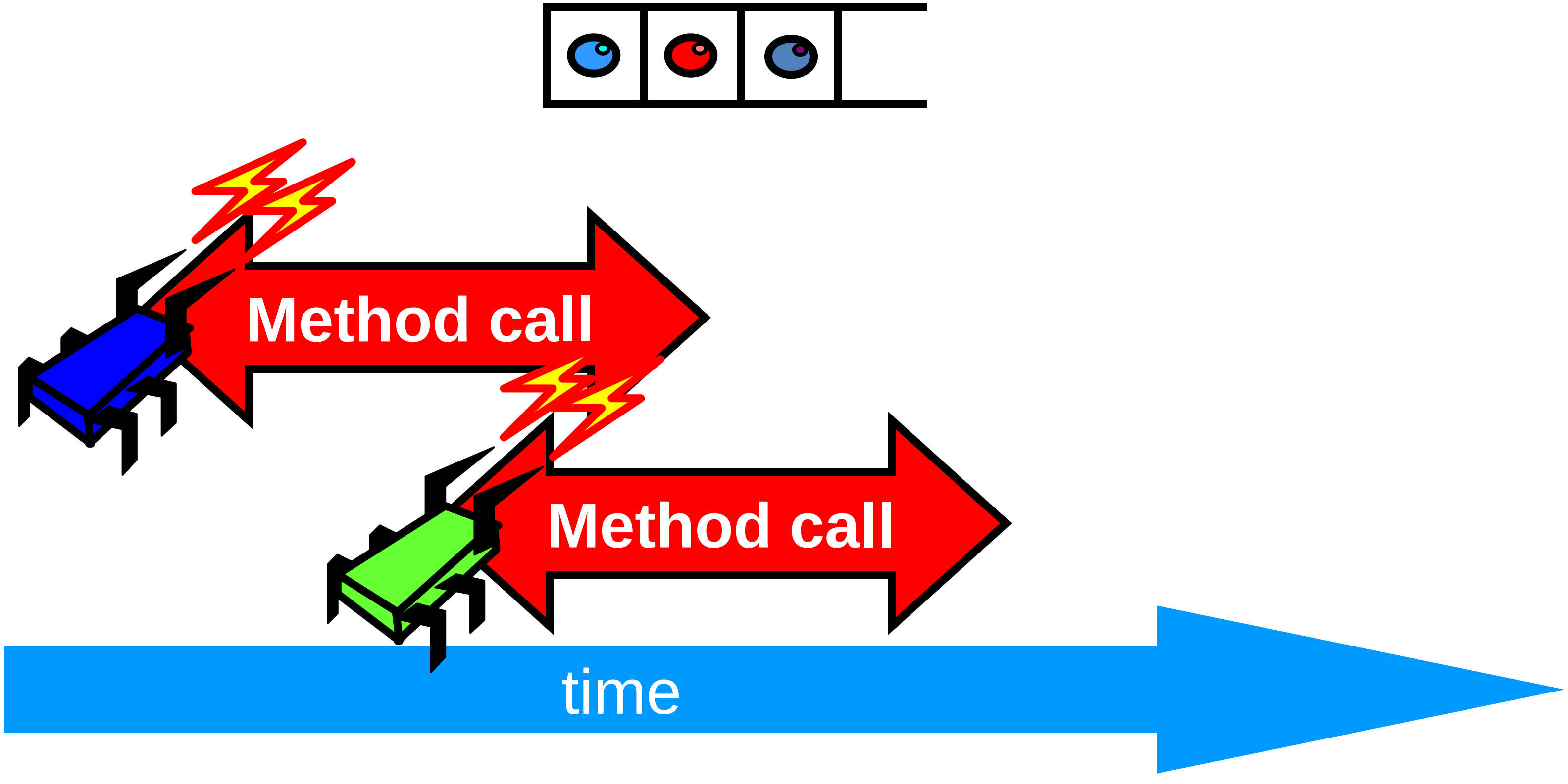

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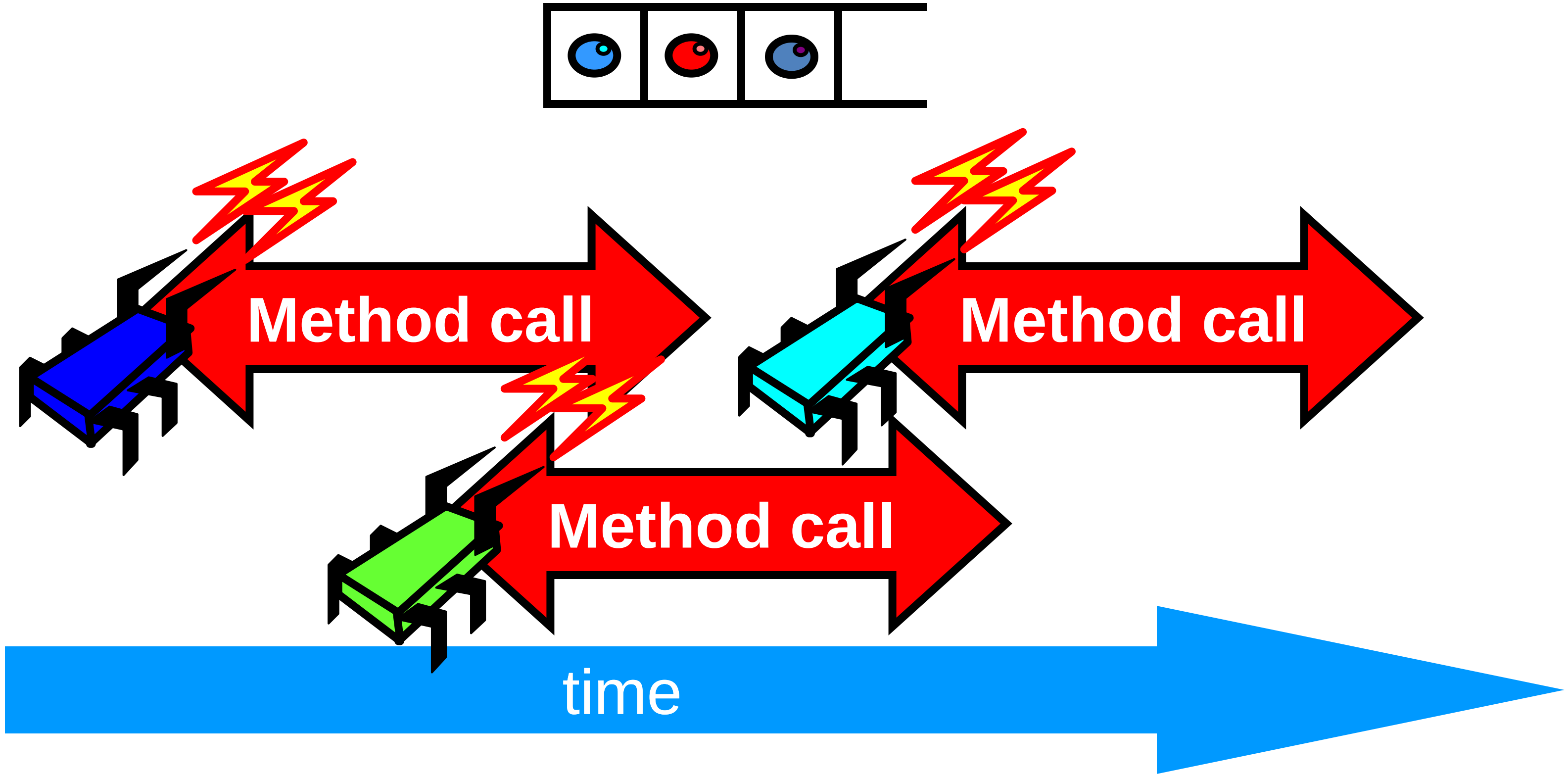

Sequential vs Concurrent

- Sequential:
  - Object needs meaningful state only ***between*** method calls
- Concurrent
  - Because method calls overlap, object might ***never*** be between method calls

Sequential vs Concurrent

- Sequential:
  - Each method described in isolation
- Concurrent
  - Must characterize ***all*** possible interactions with concurrent calls
    - What if two enqs overlap?
    - Two deqs? enq and deq? …

Sequential vs Concurrent

- Sequential:
  - Can add new methods without affecting older methods
- Concurrent:
  - Everything can potentially interact with everything else

Sequential vs Concurrent

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  - Can add new methods without affecting older methods
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Panic!

The Big Question

- What does it mean for a *concurrent* object to be correct?
  - What *is* a concurrent FIFO queue?
  - FIFO means strict temporal order
  - Concurrent means ambiguous temporal order

Intuitively…

```
int deq(queue_t q, void **elem) {
  int res;
  pthread_mutex_lock(&q->lock);
  if (q->tail == q->head) res = 0;
  else {
    *elem = q->items[q->head % CAPACITY];
    q->head++;
    res = 1;
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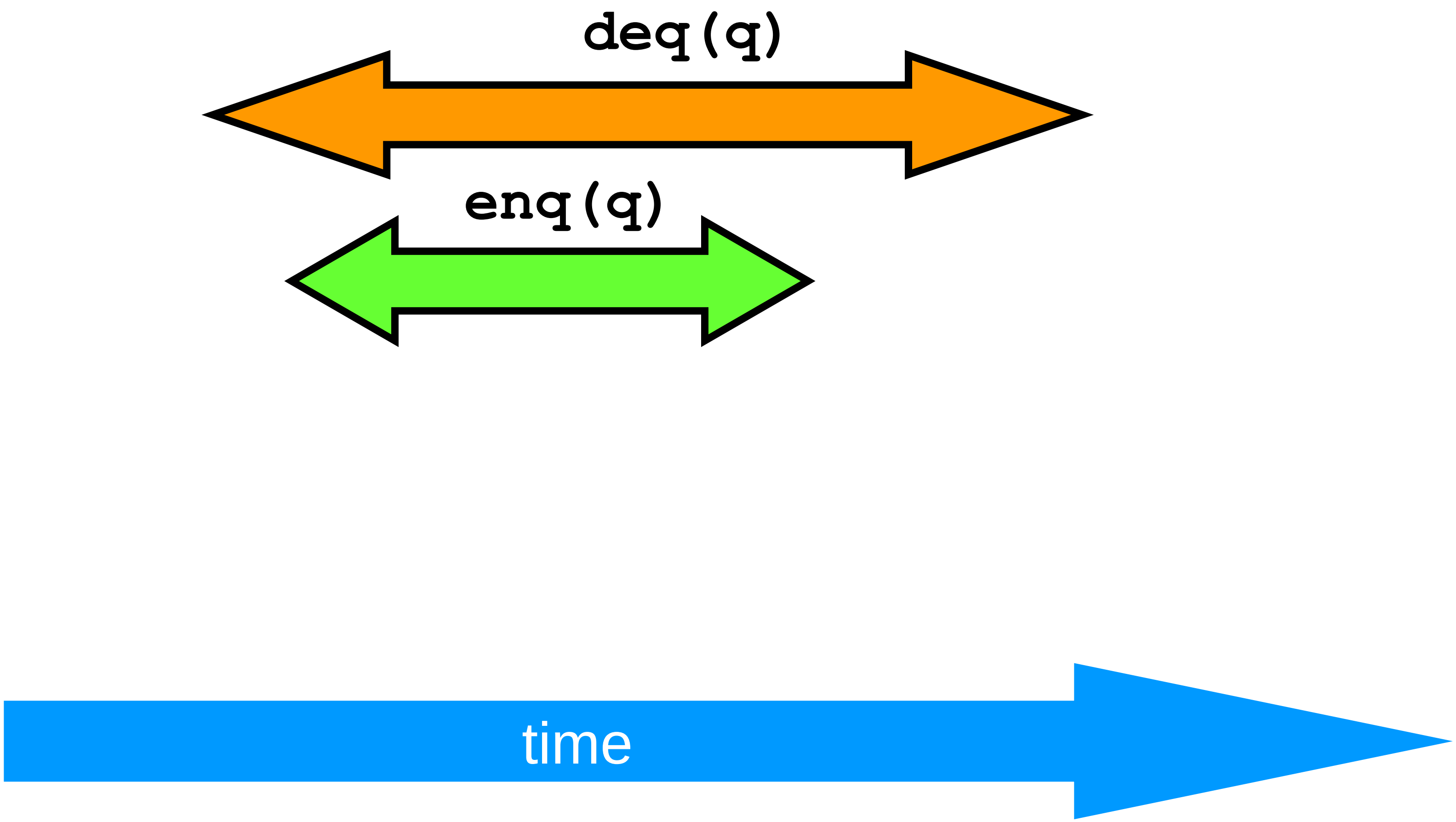
```

All queue modifications are mutually exclusive

Intuitively



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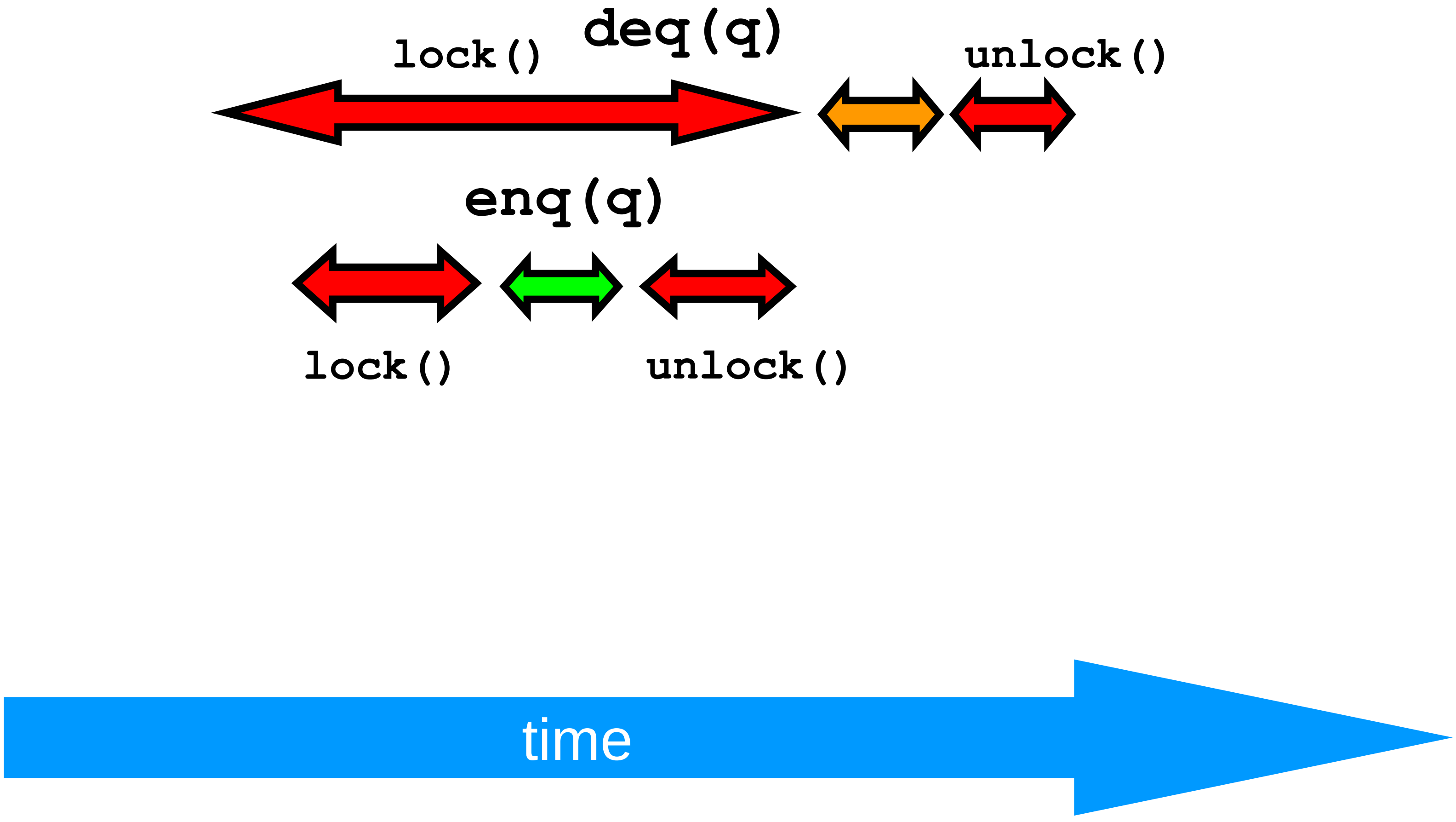

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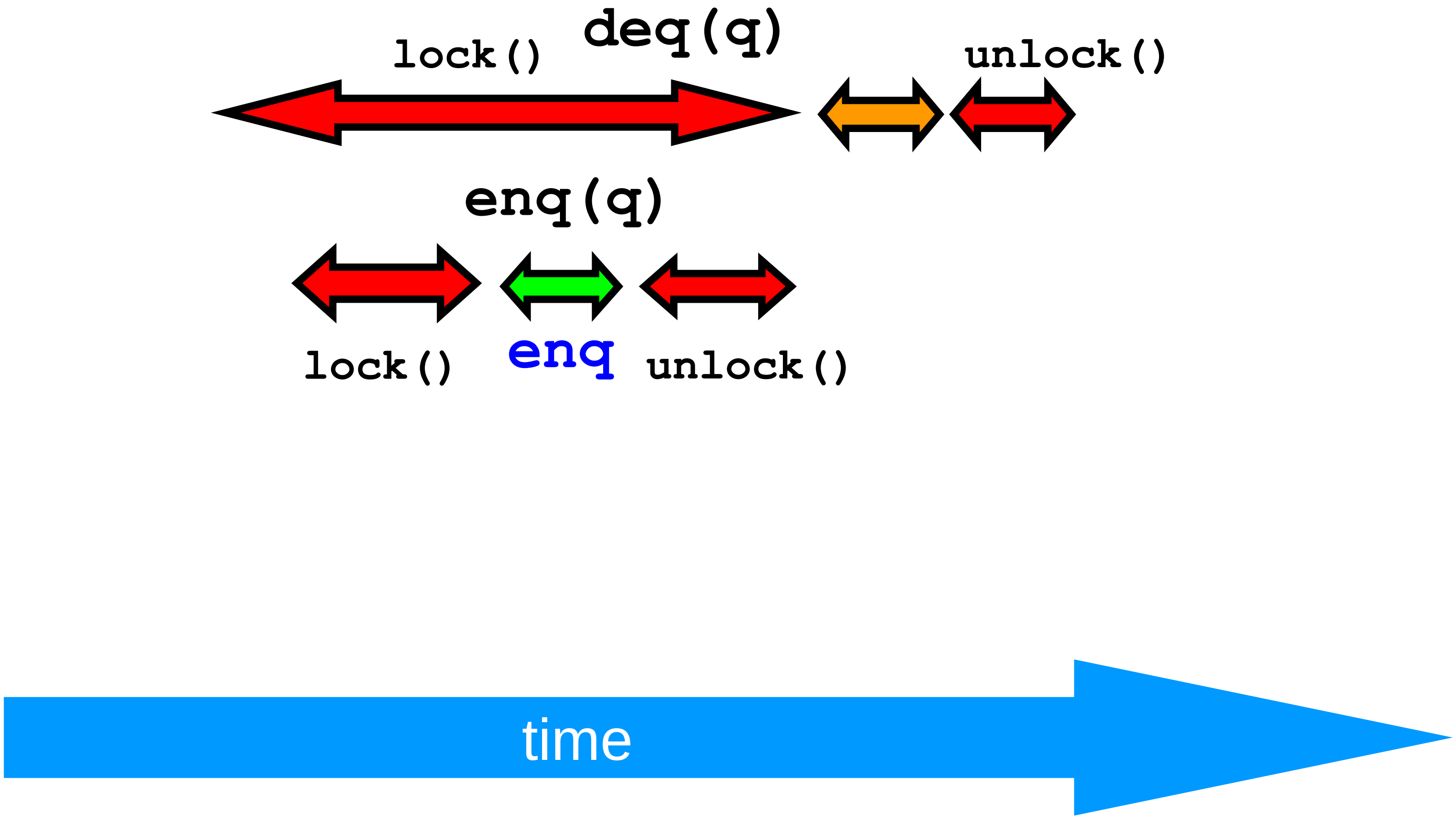

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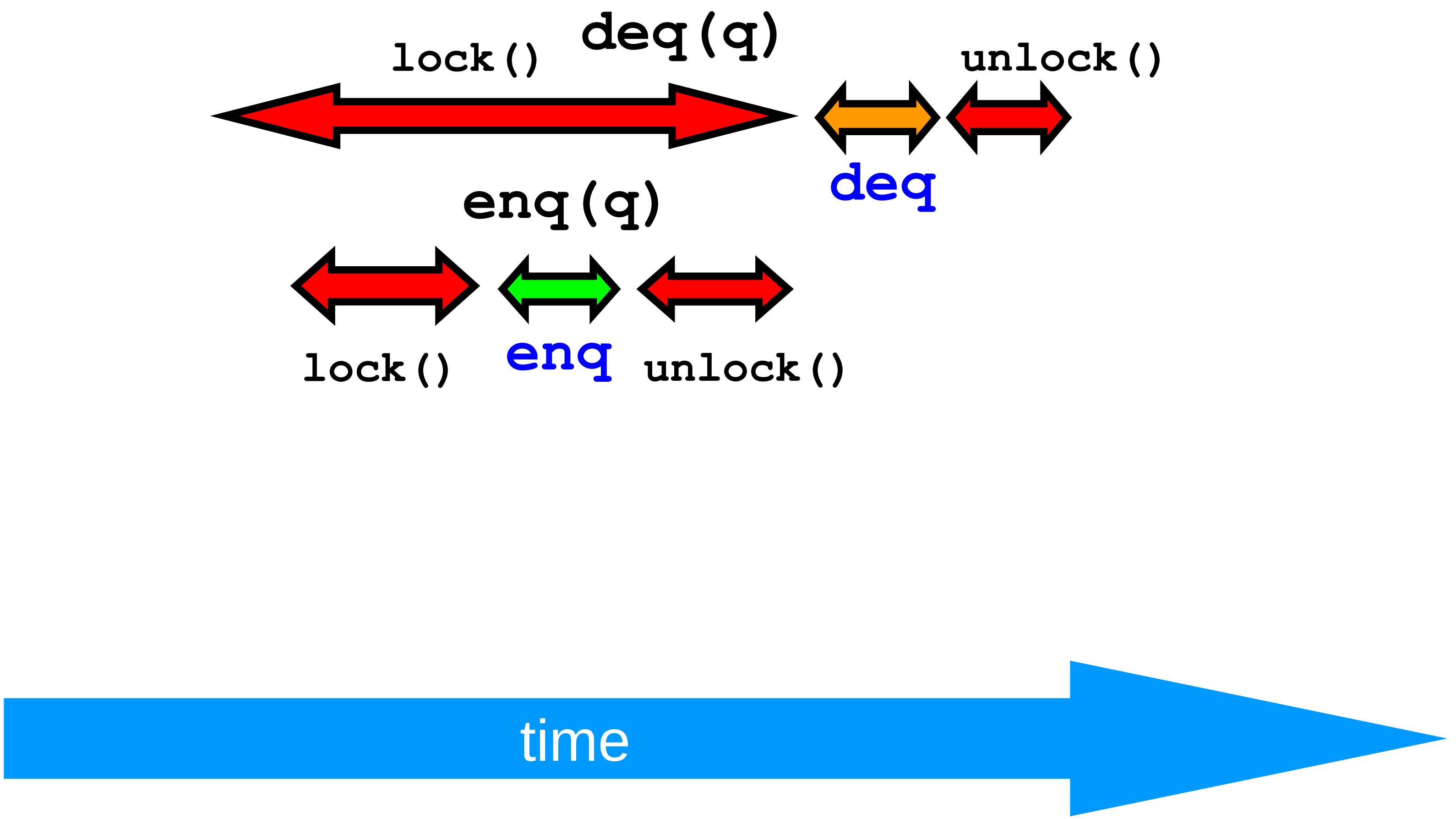

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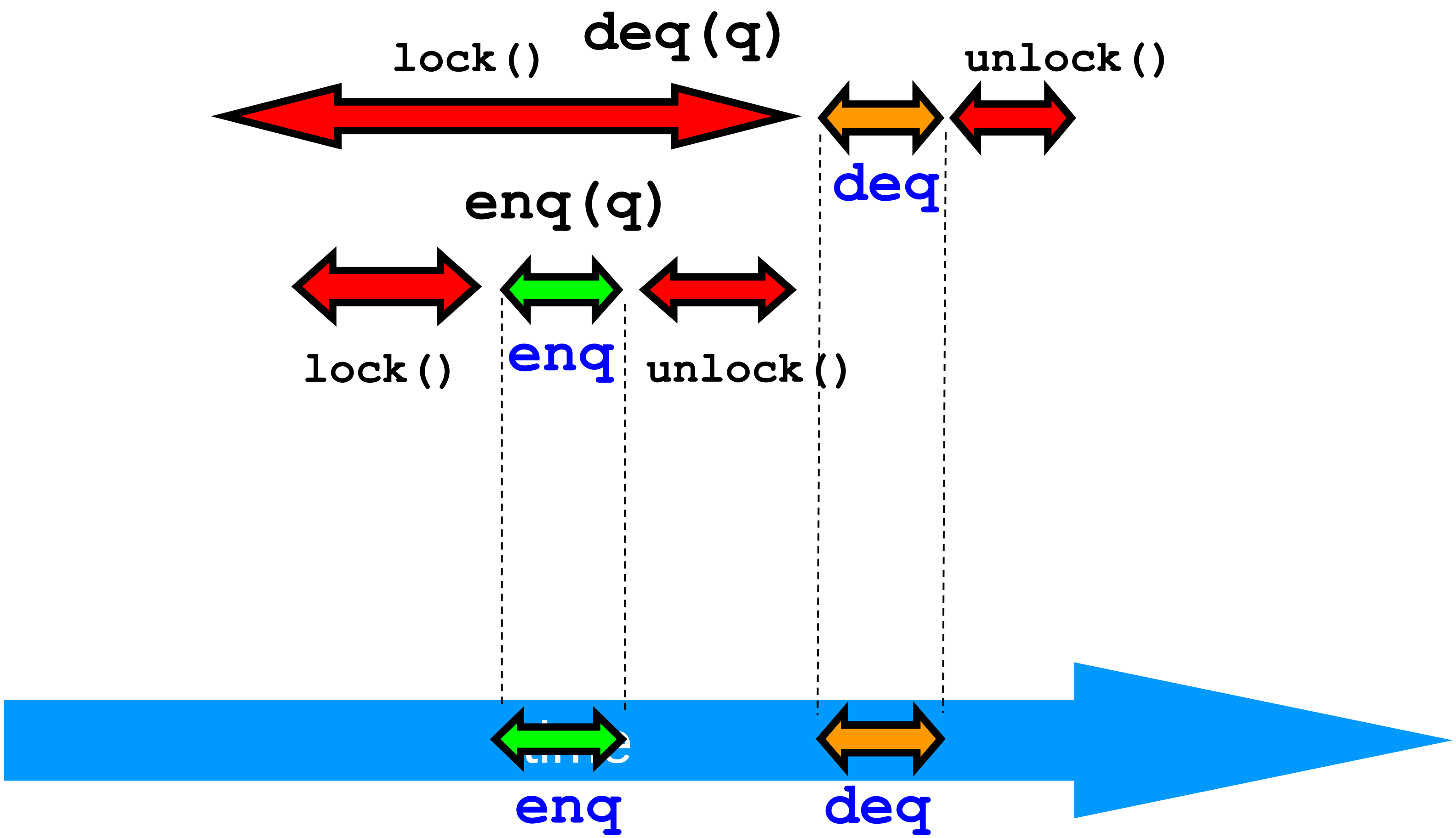
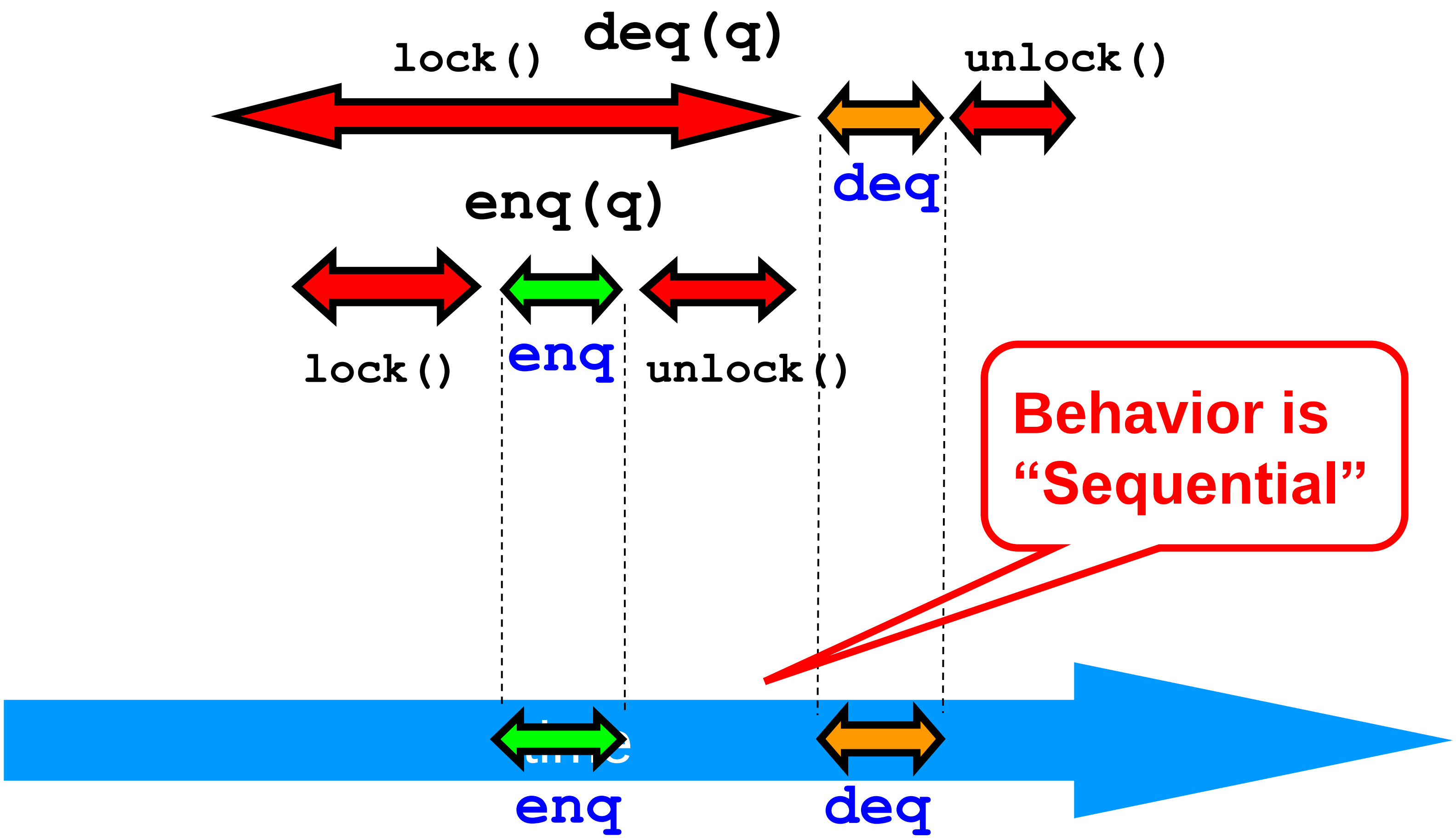
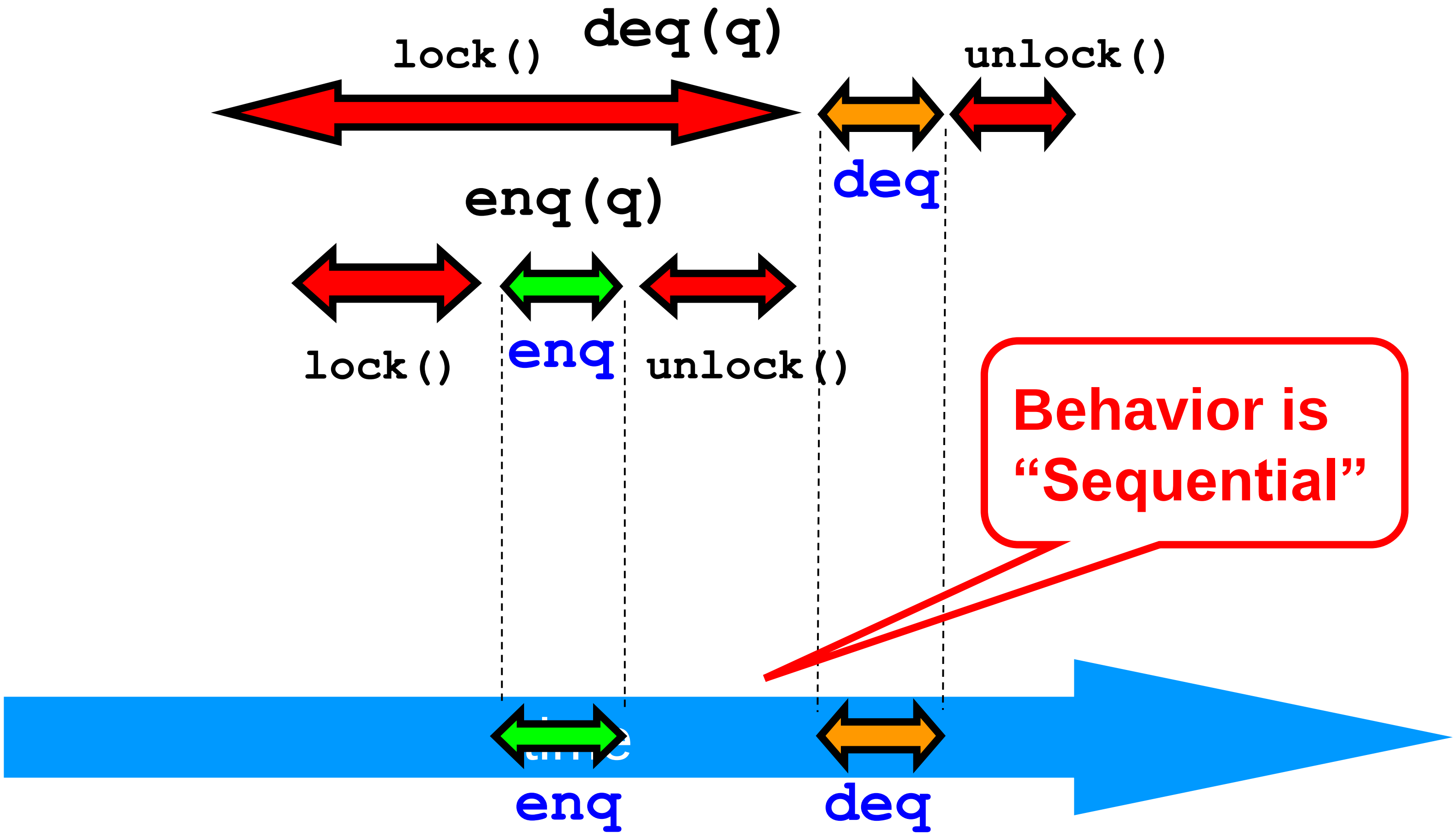


Intuitively



Intuitively

Lets capture the idea of describing the concurrent via the sequential



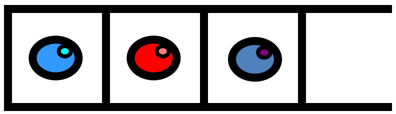
Linearizability

- Each method should
  - “take effect”
  - instantaneously
  - between invocation and response events
- Object is correct if this “sequential” behavior is correct
- Any such concurrent object is called
  - **Linearizable**

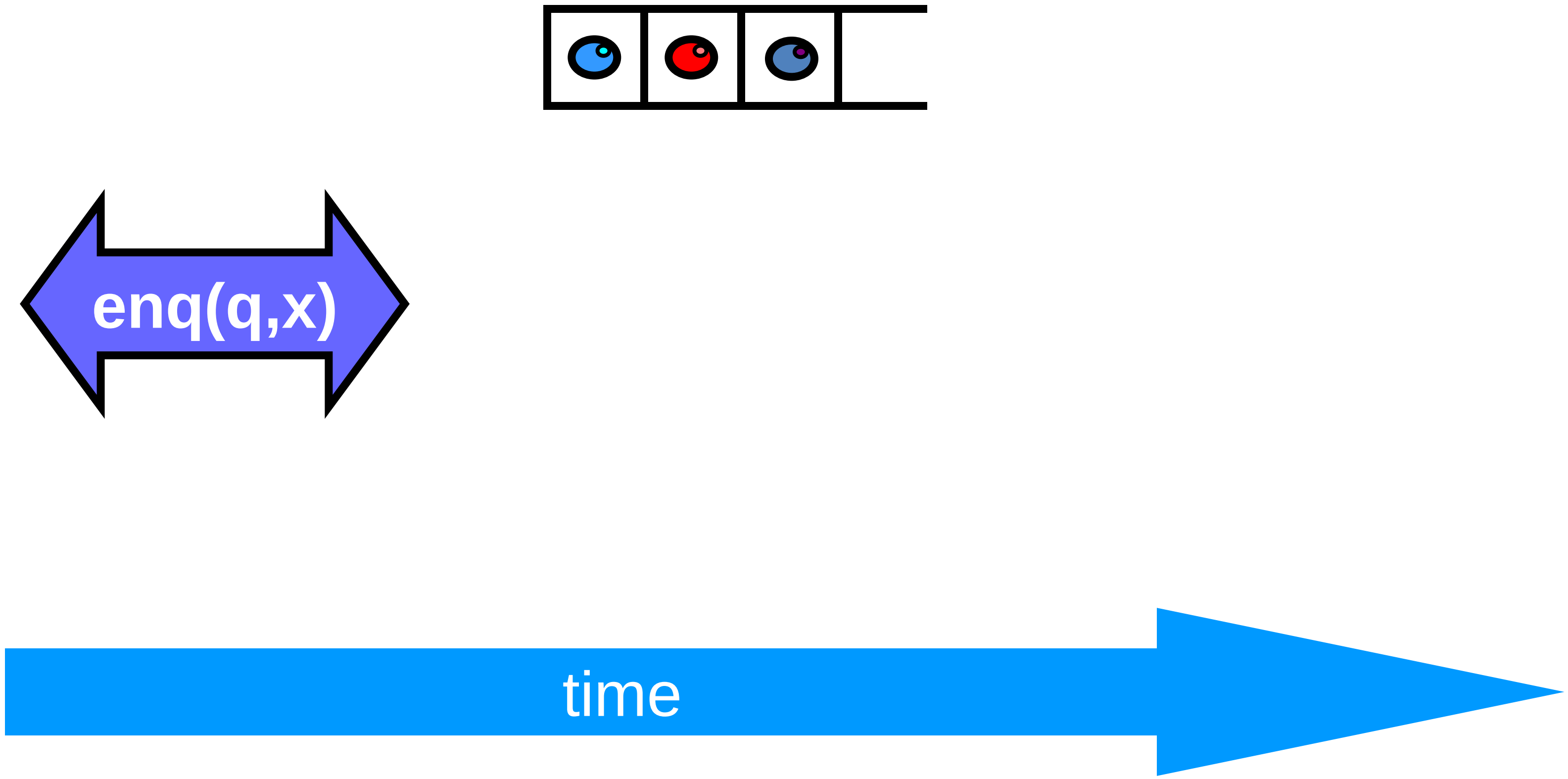
Is it really about the object?

- Each method should
  - “take effect”
  - instantaneously
  - between invocation and response events
- Sounds like a property of an execution…
- A linearizable object: one all of whose possible executions are linearizable

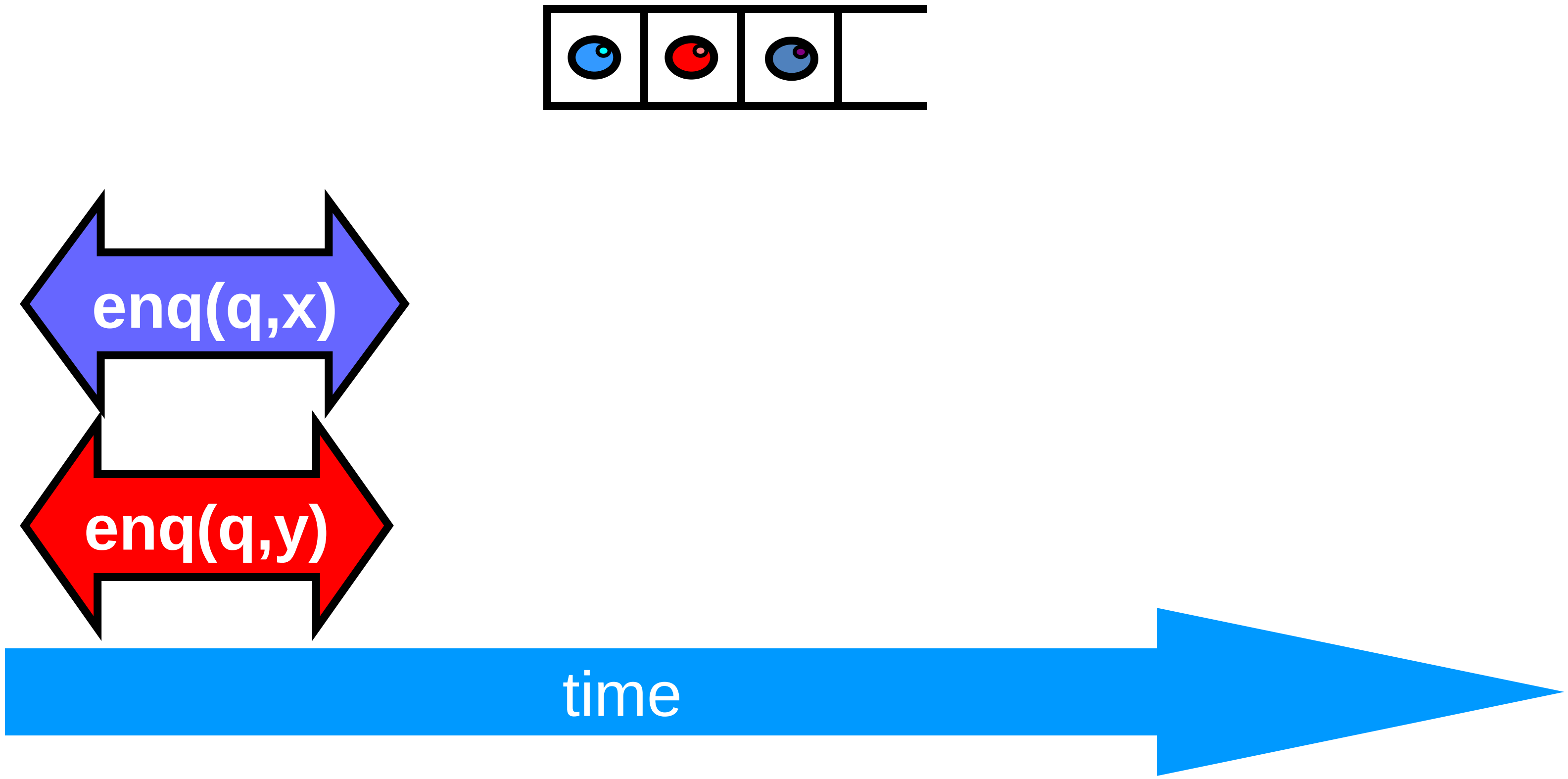
Example



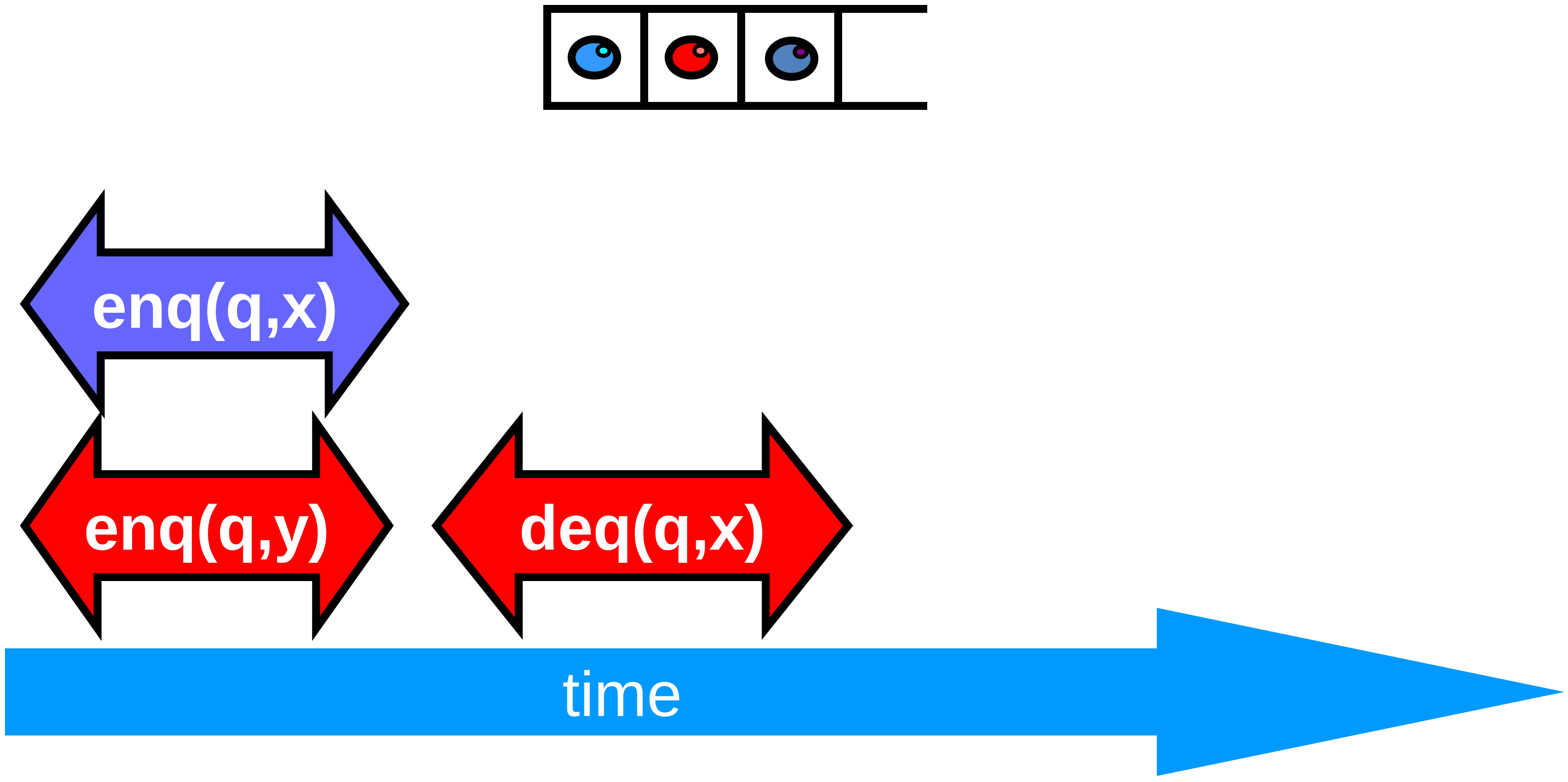
Example



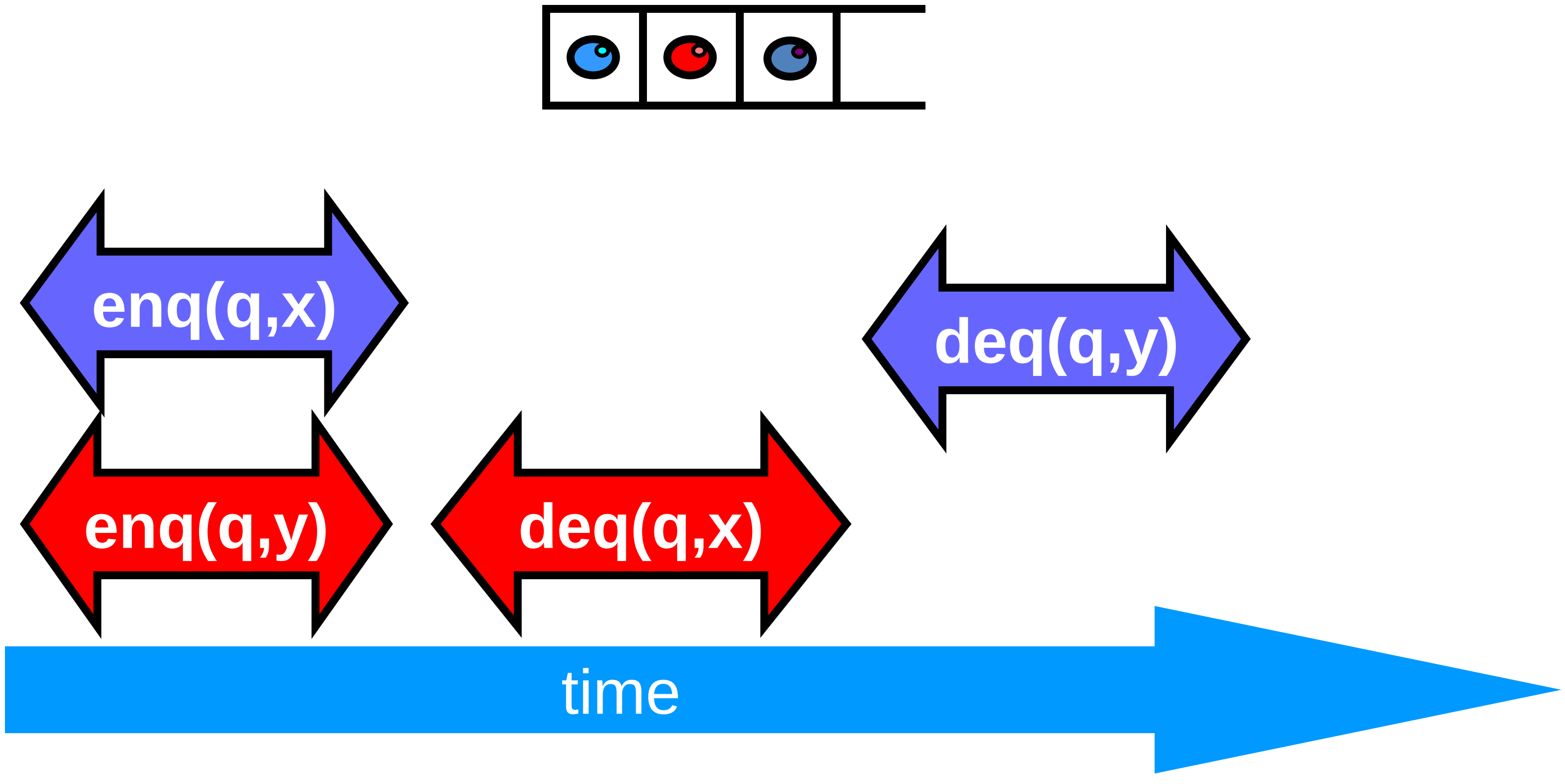
Example



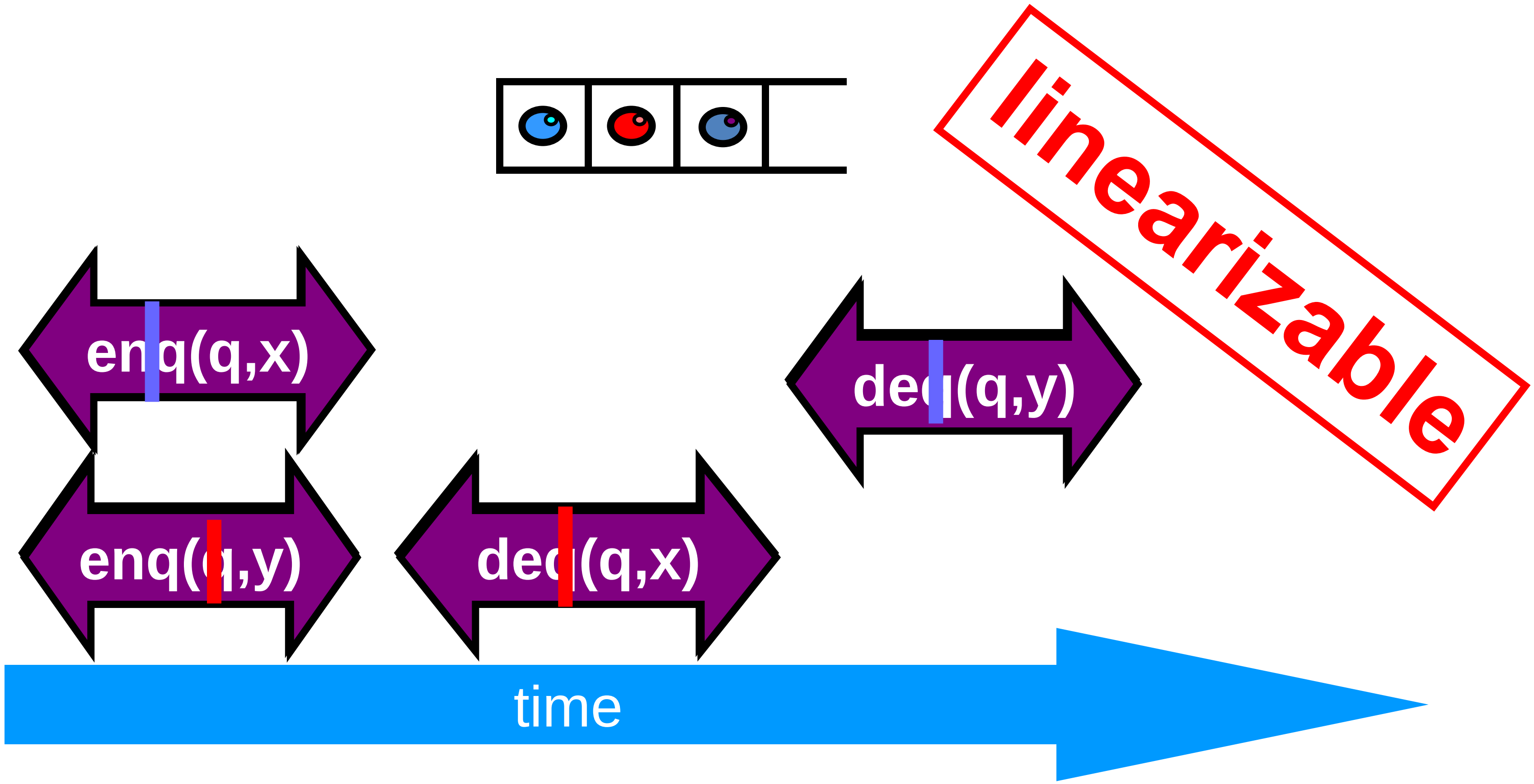
Example



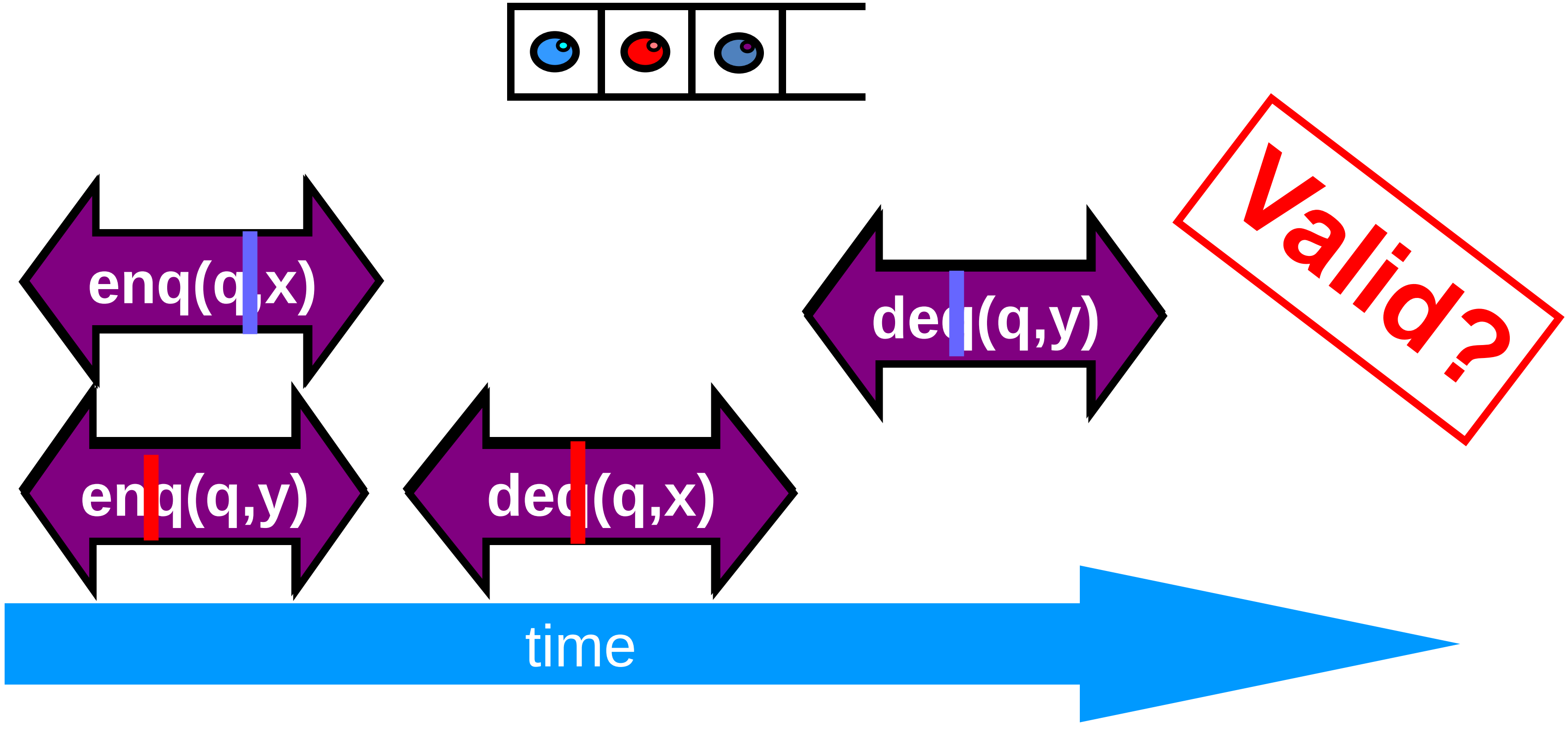
Example



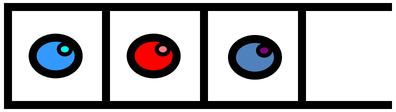
Example



Example



Example



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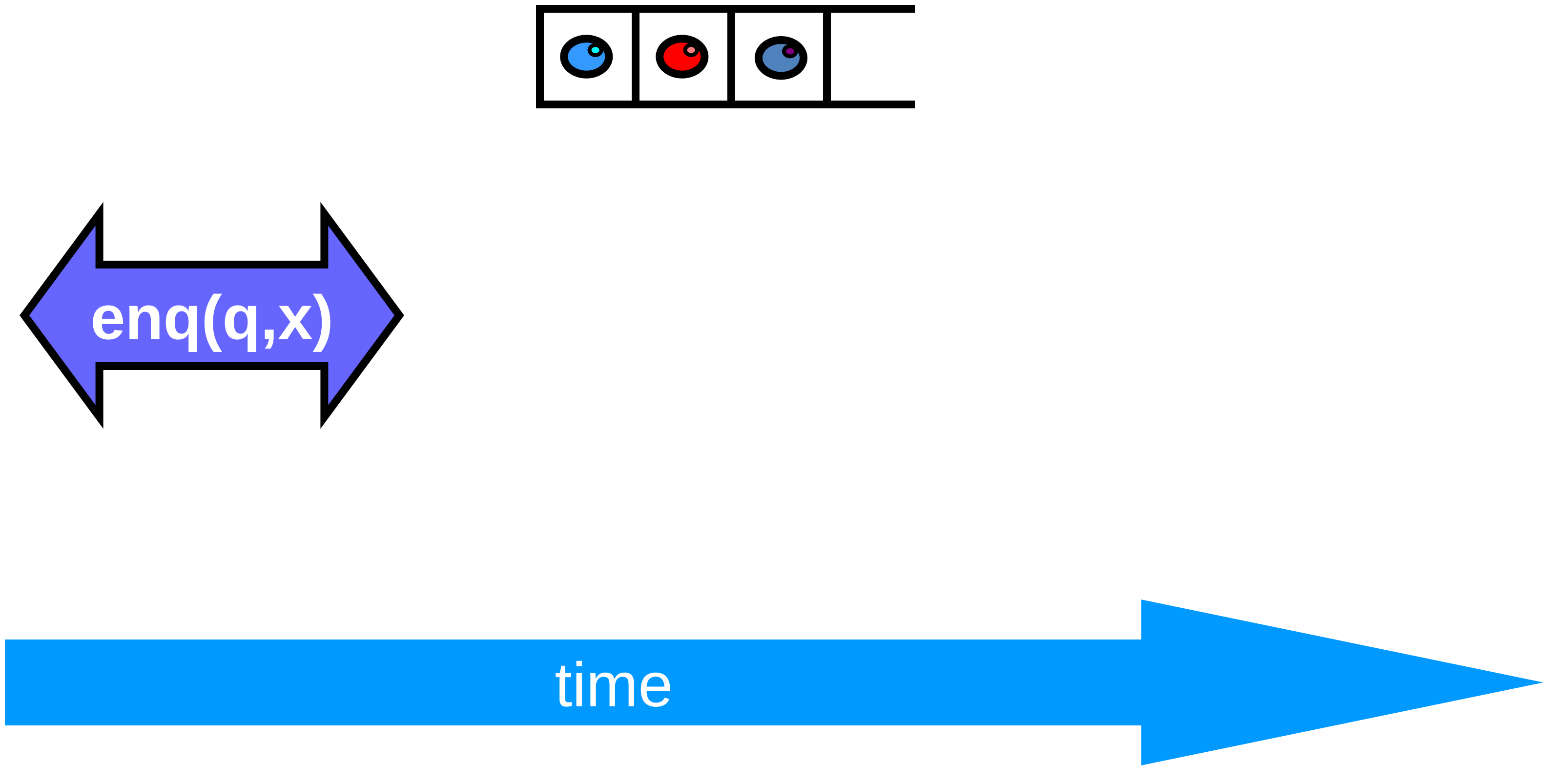

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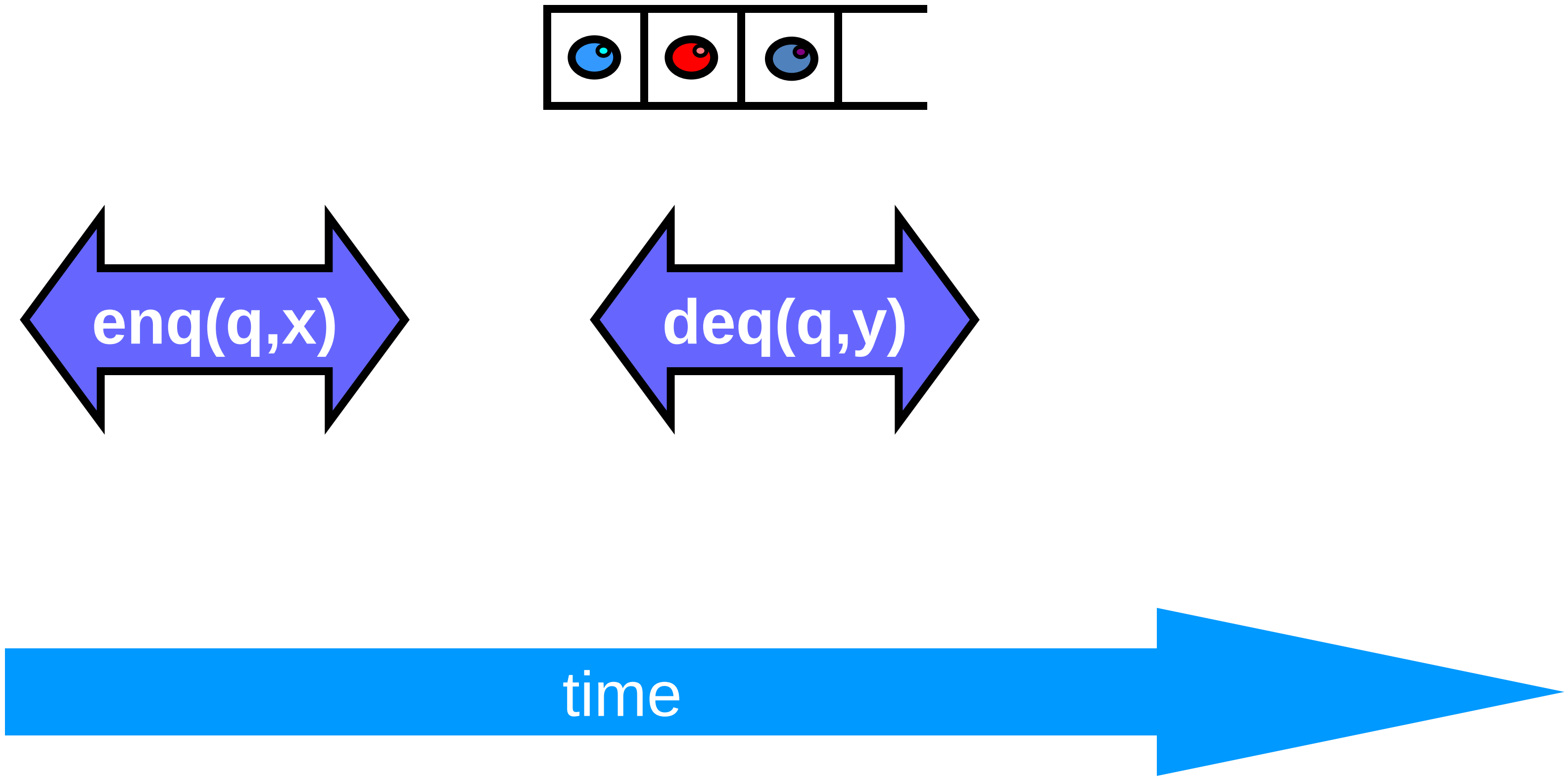

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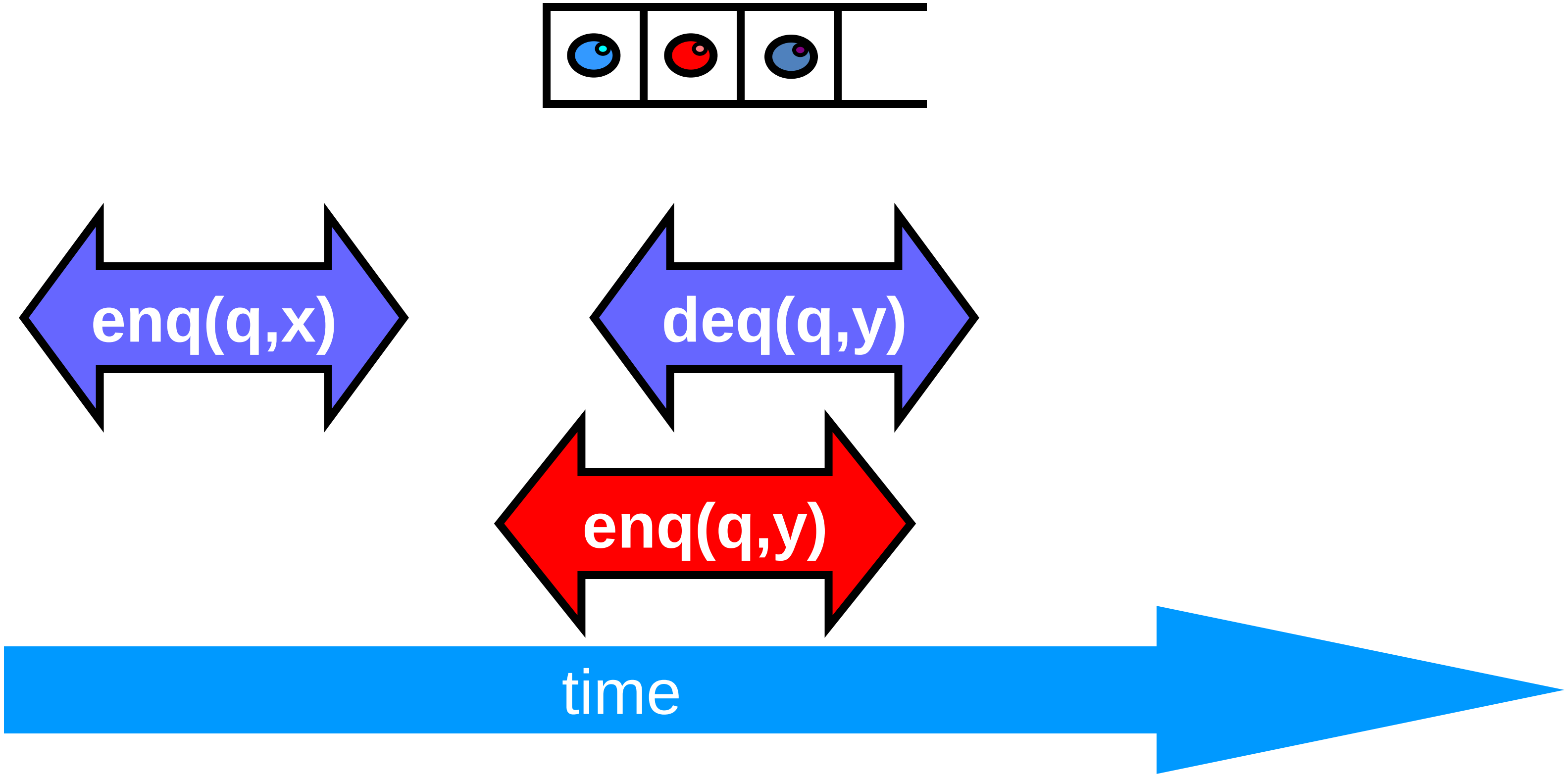

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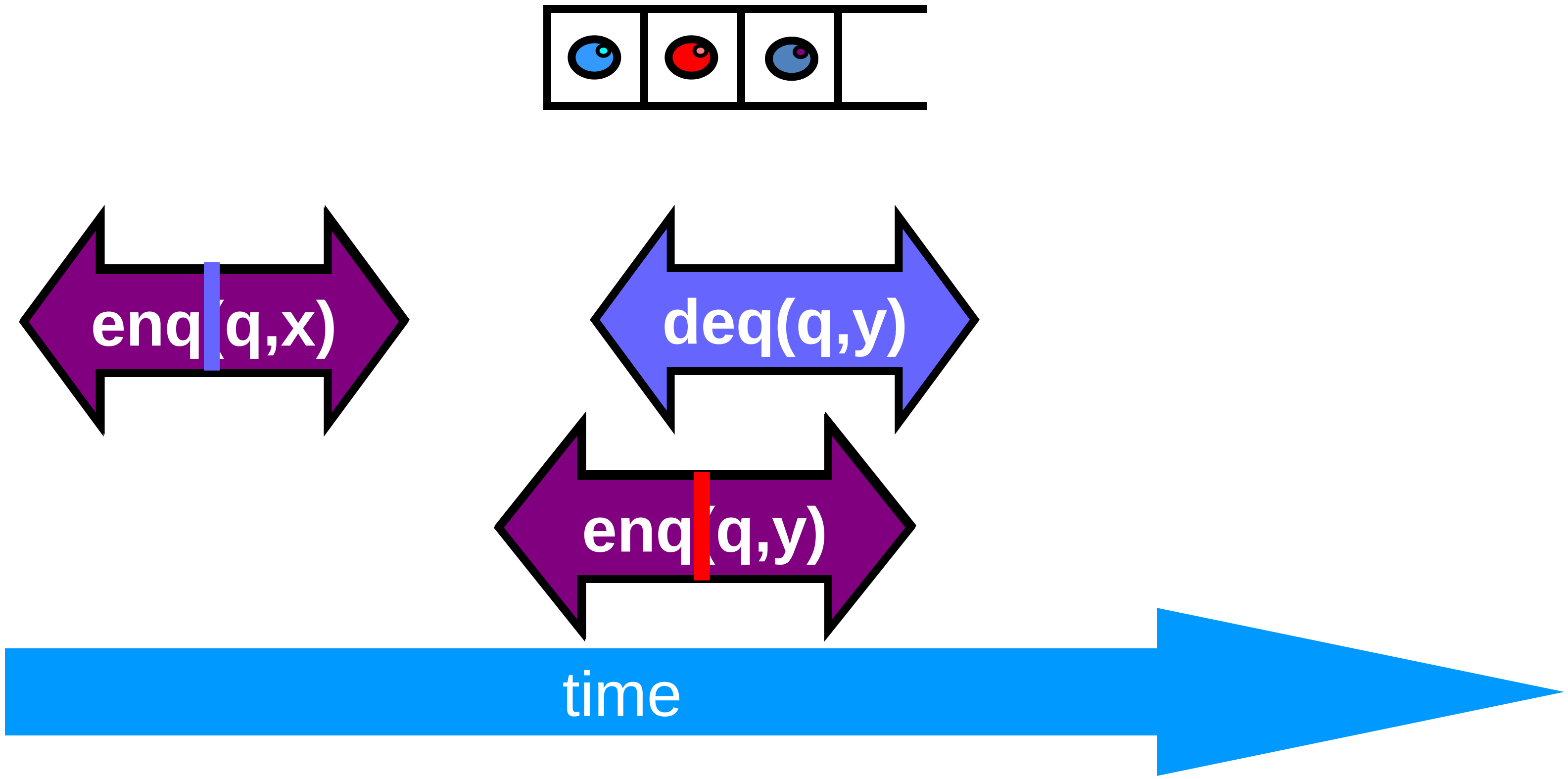

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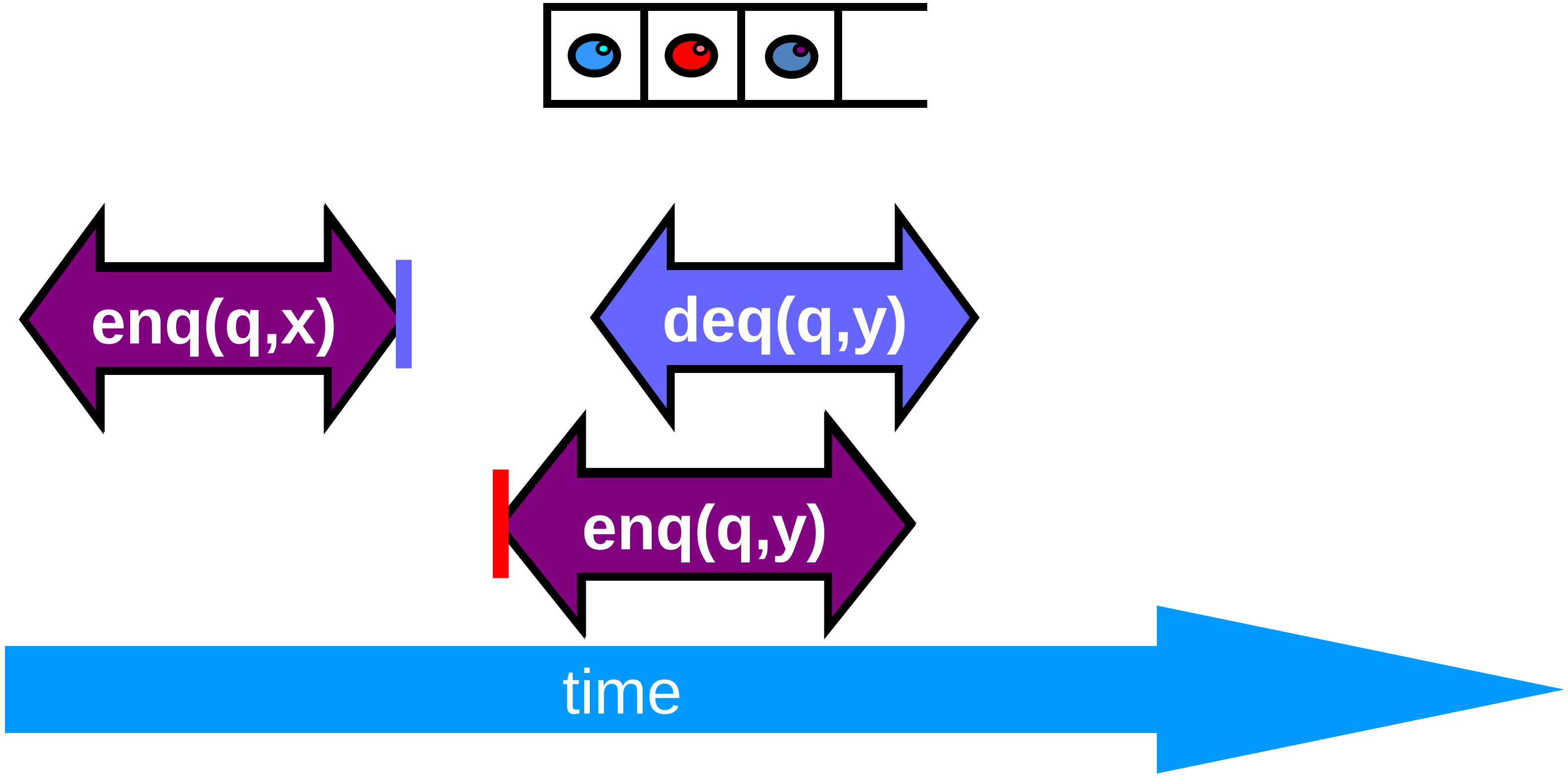

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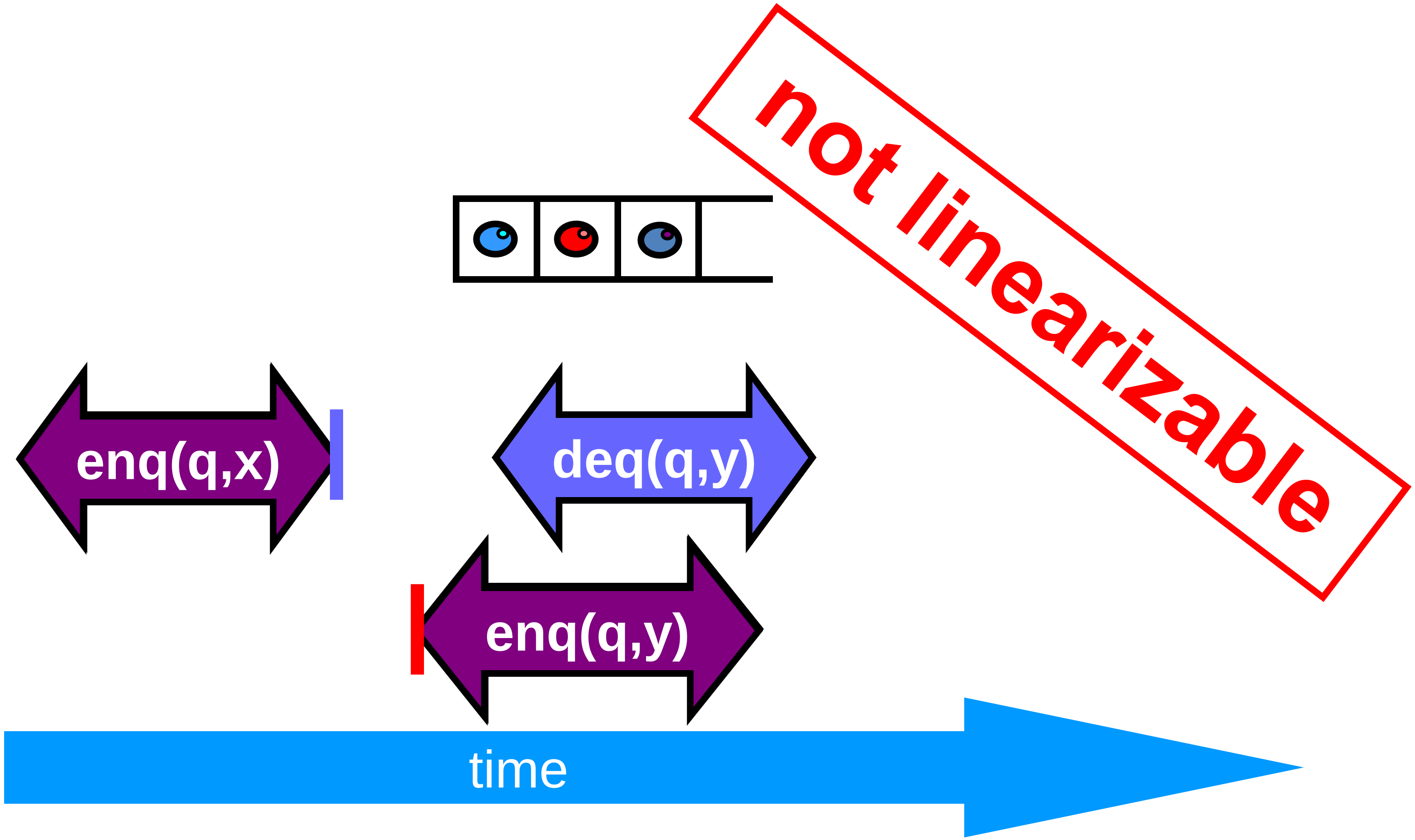

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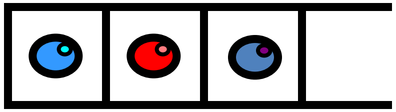

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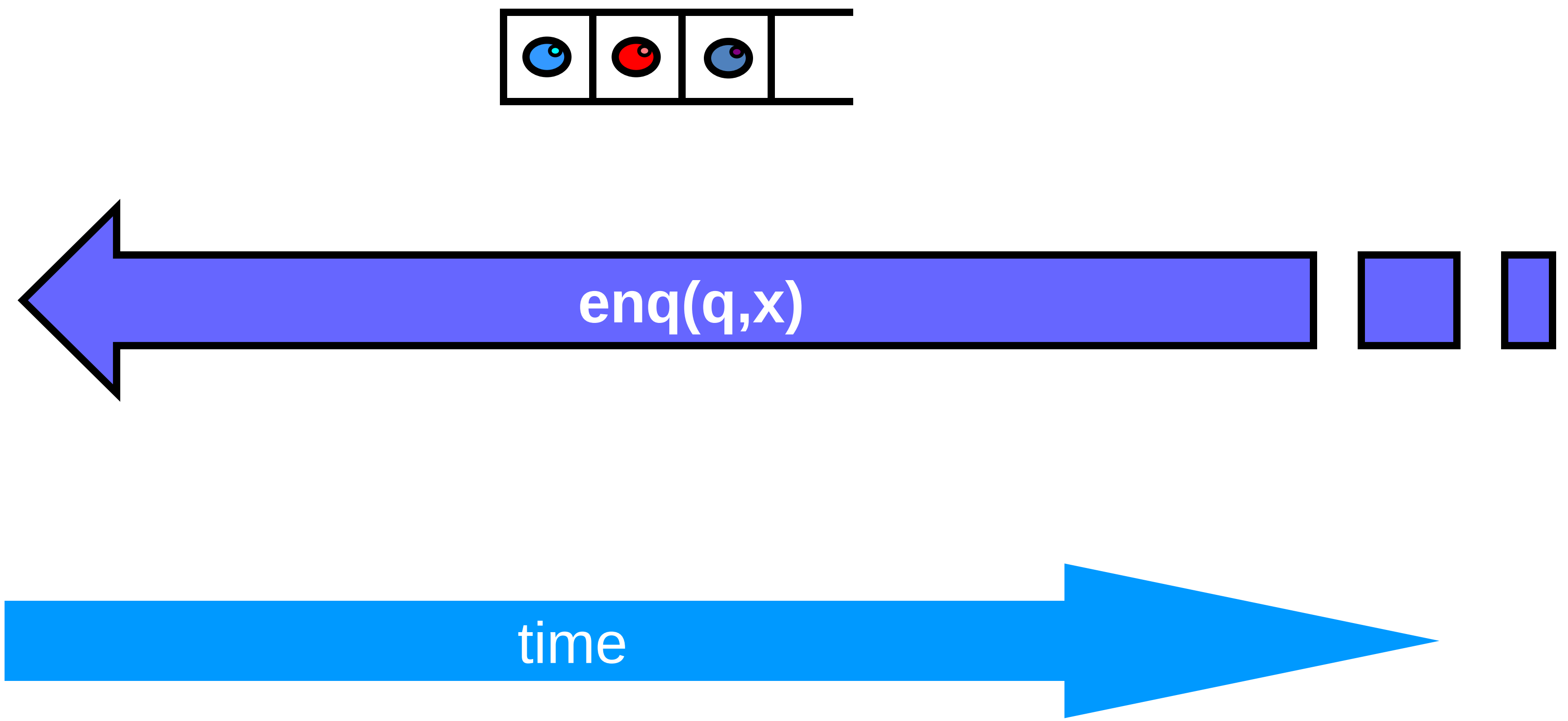

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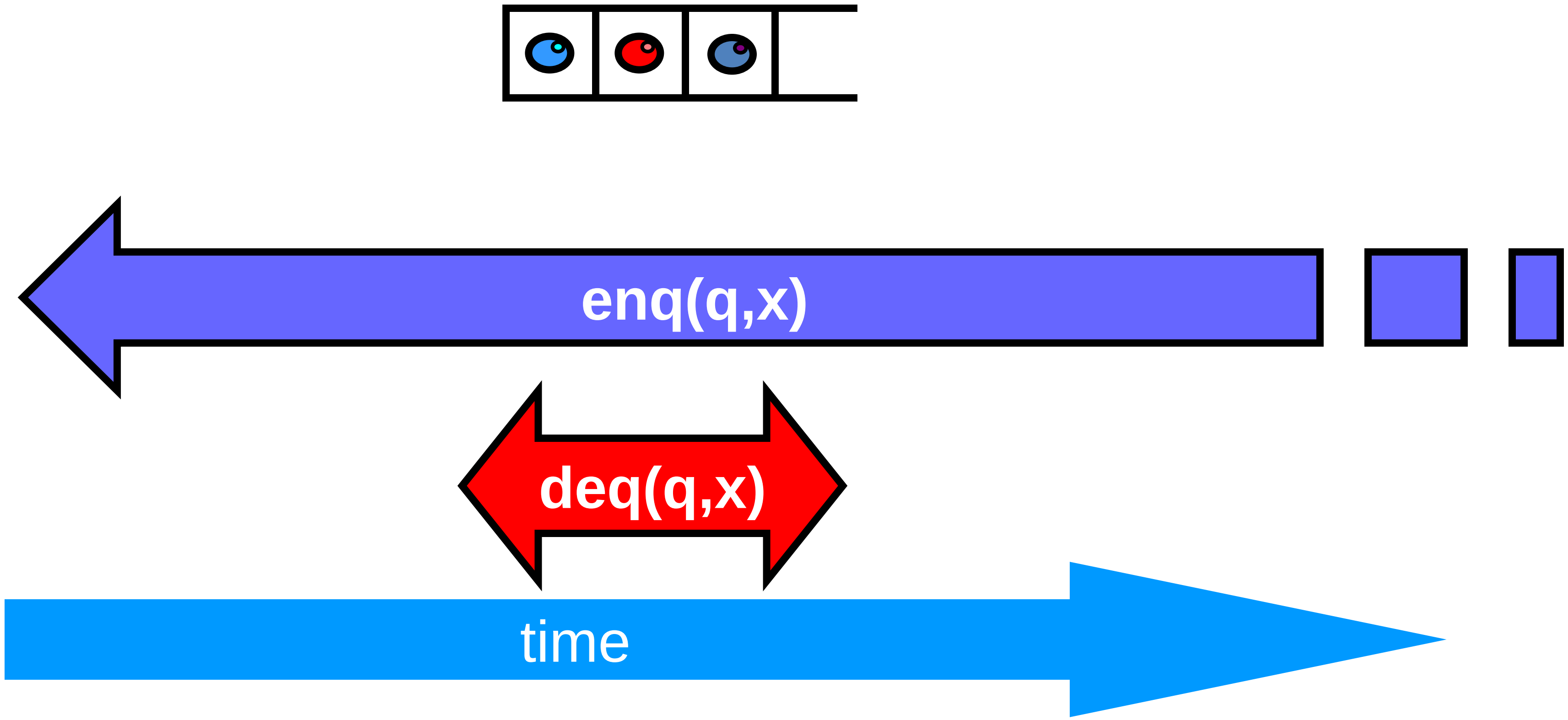

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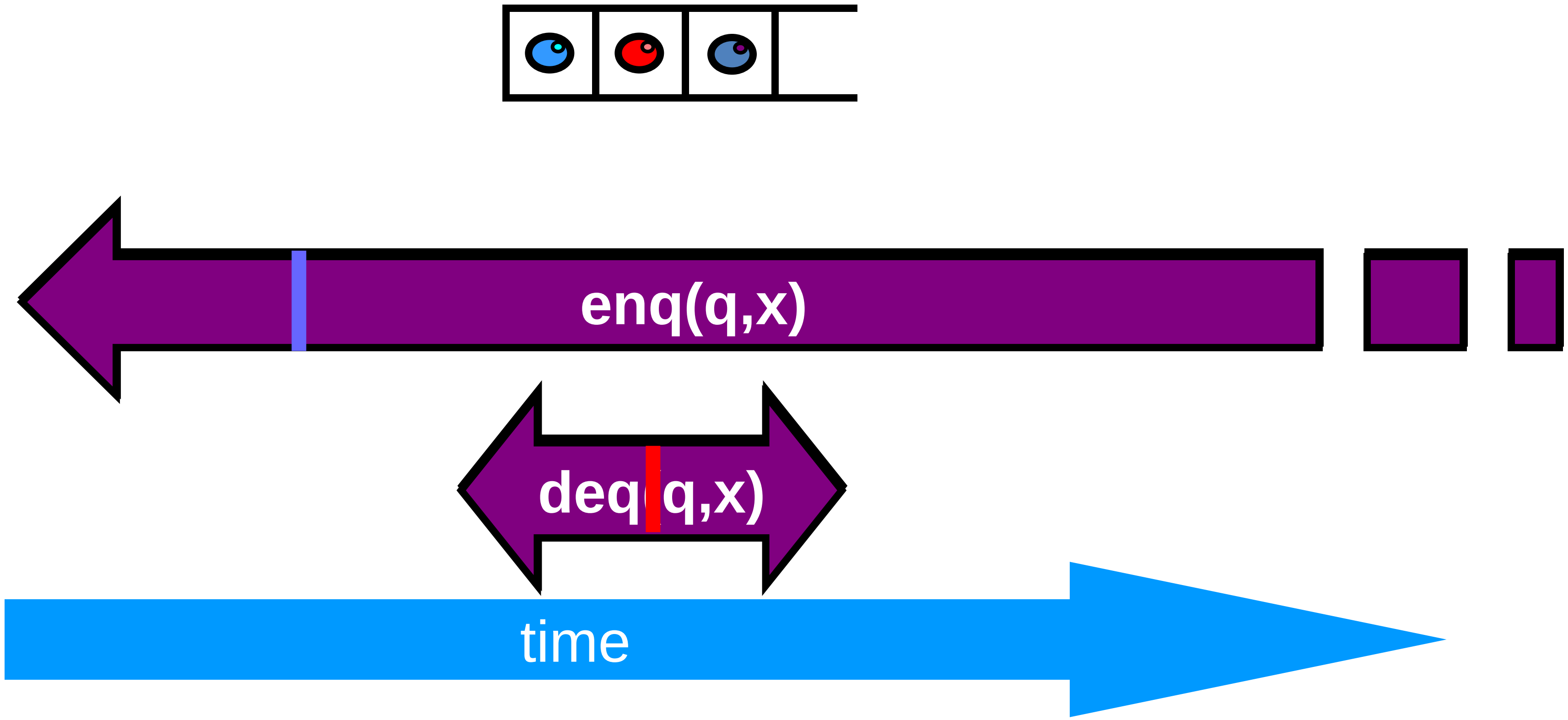

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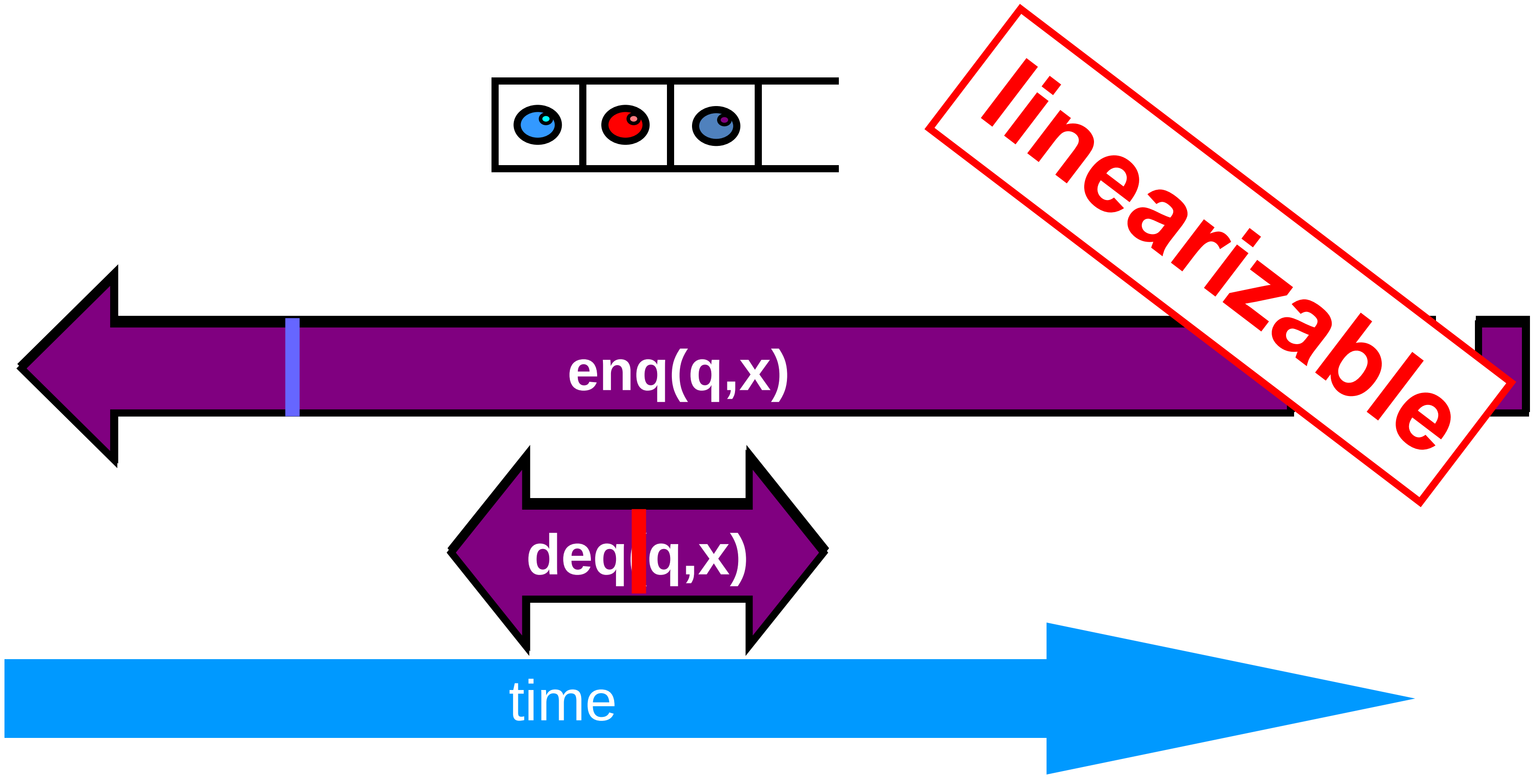

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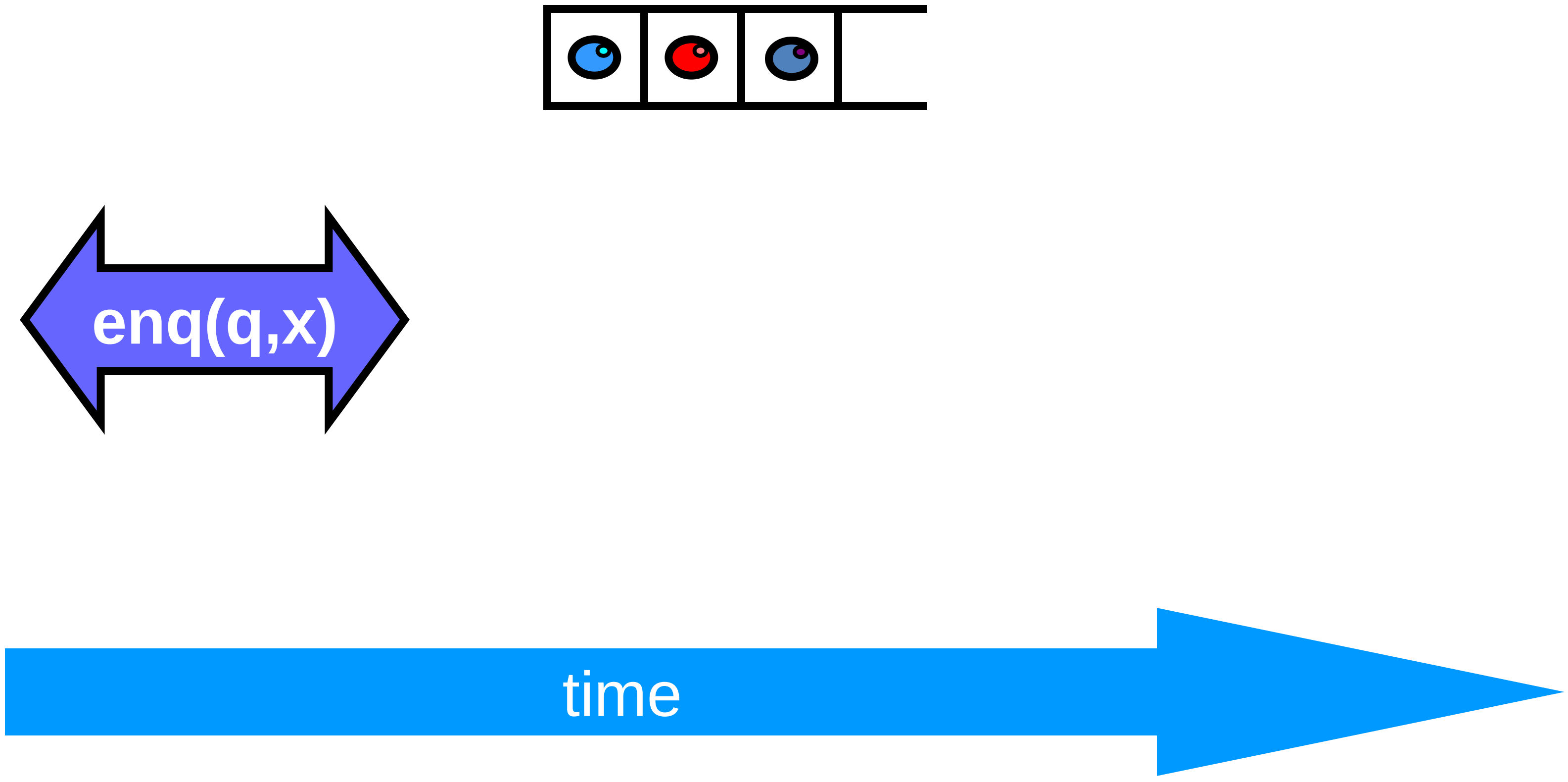

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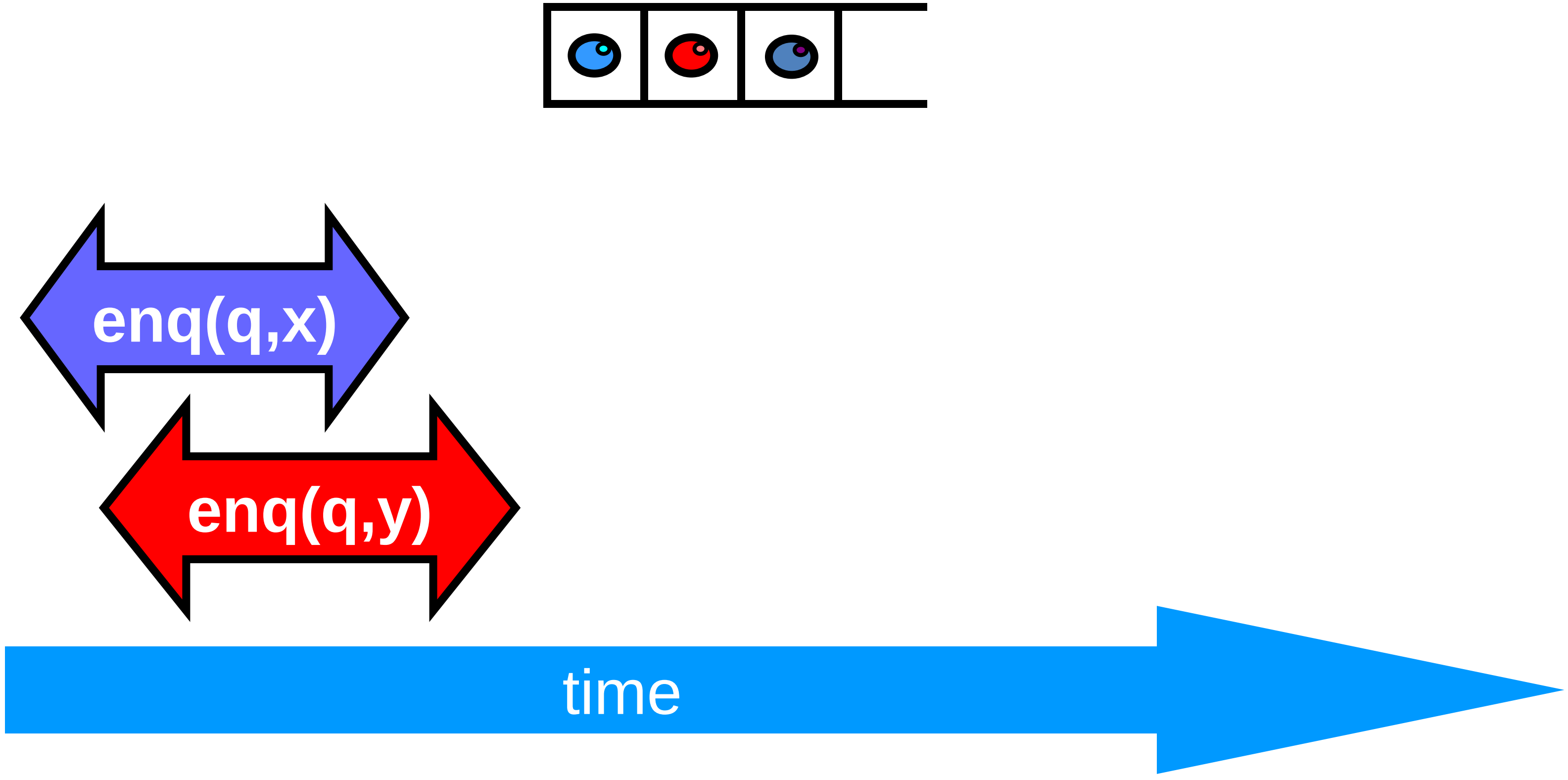

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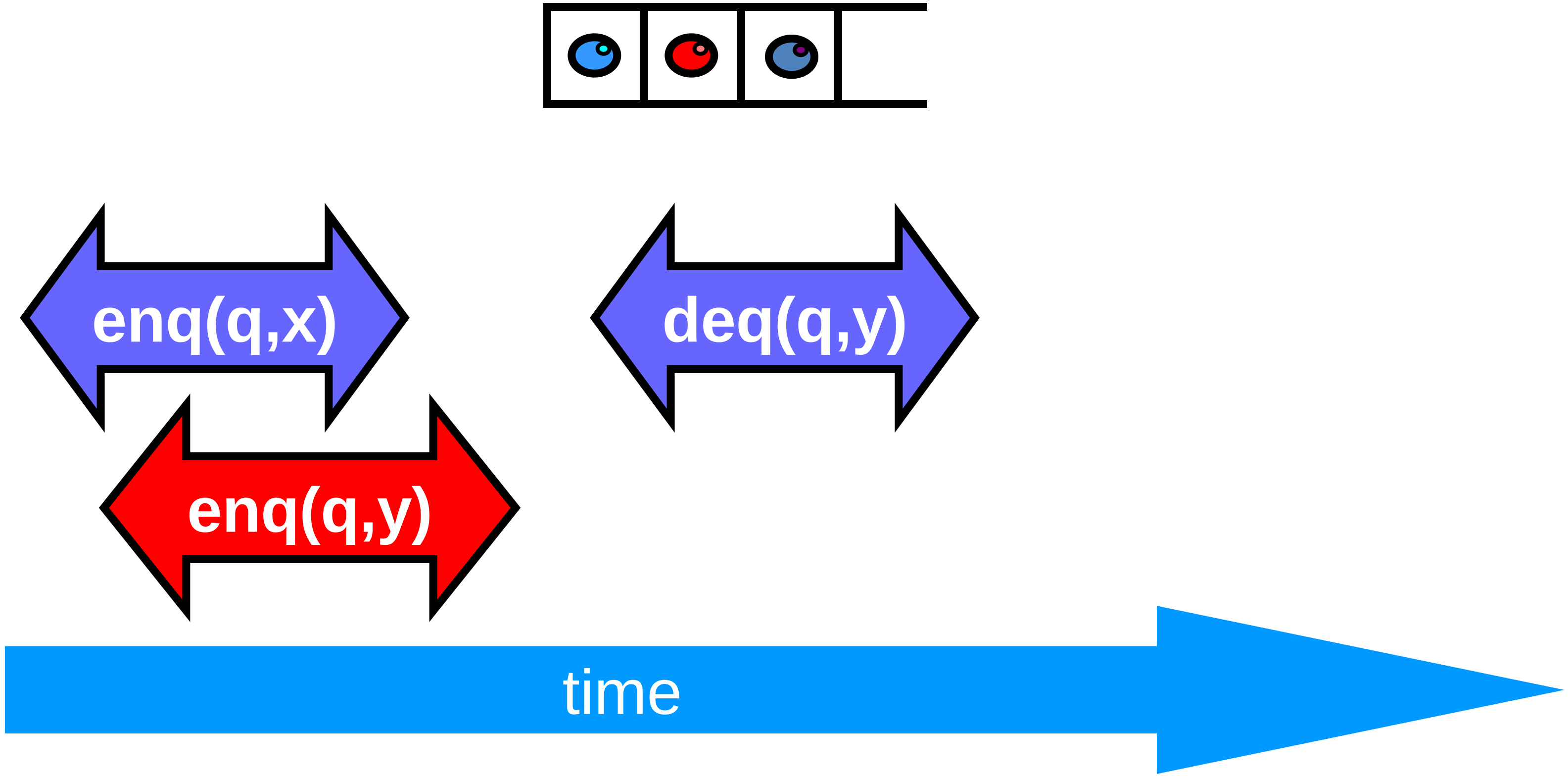

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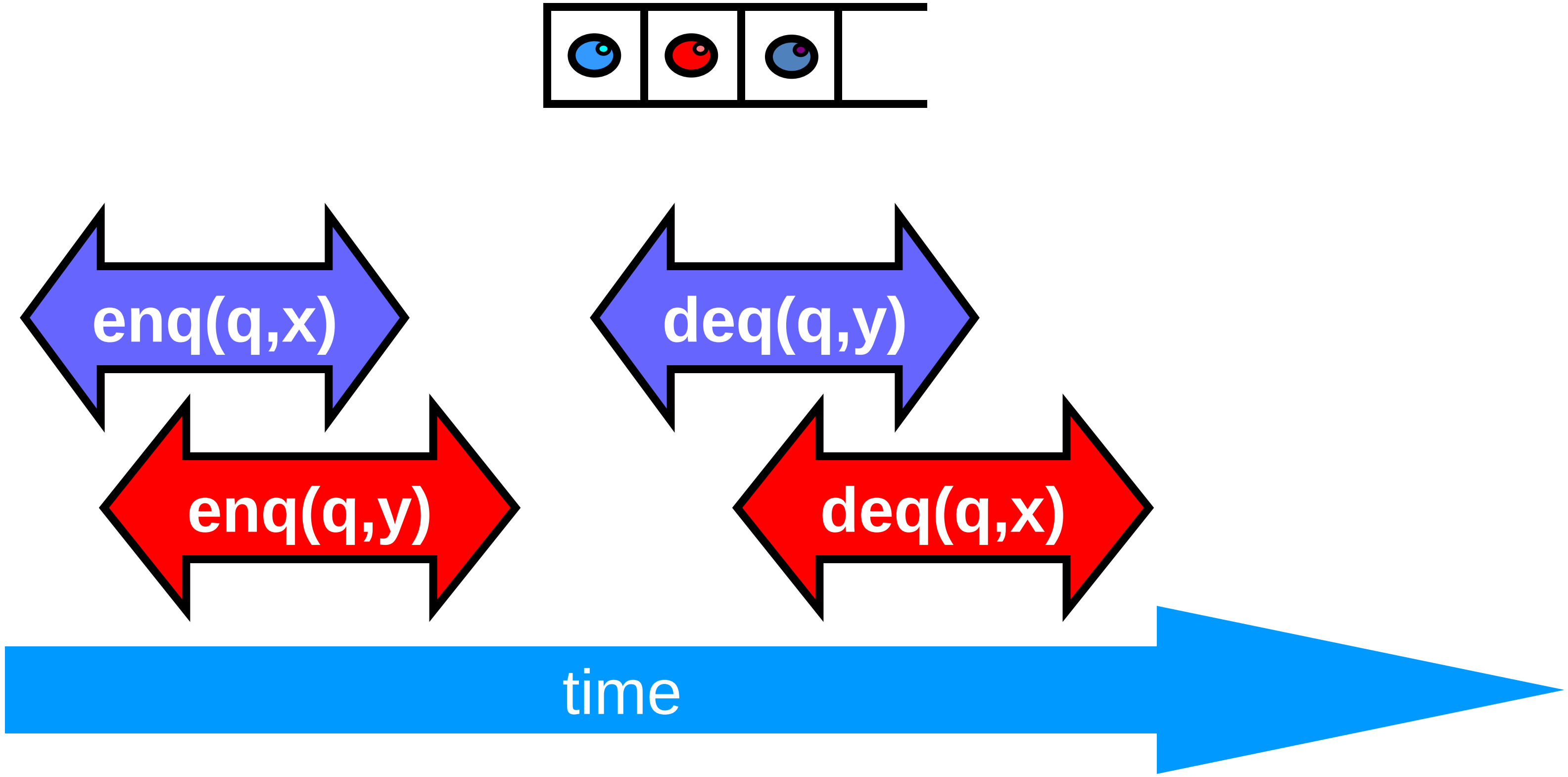
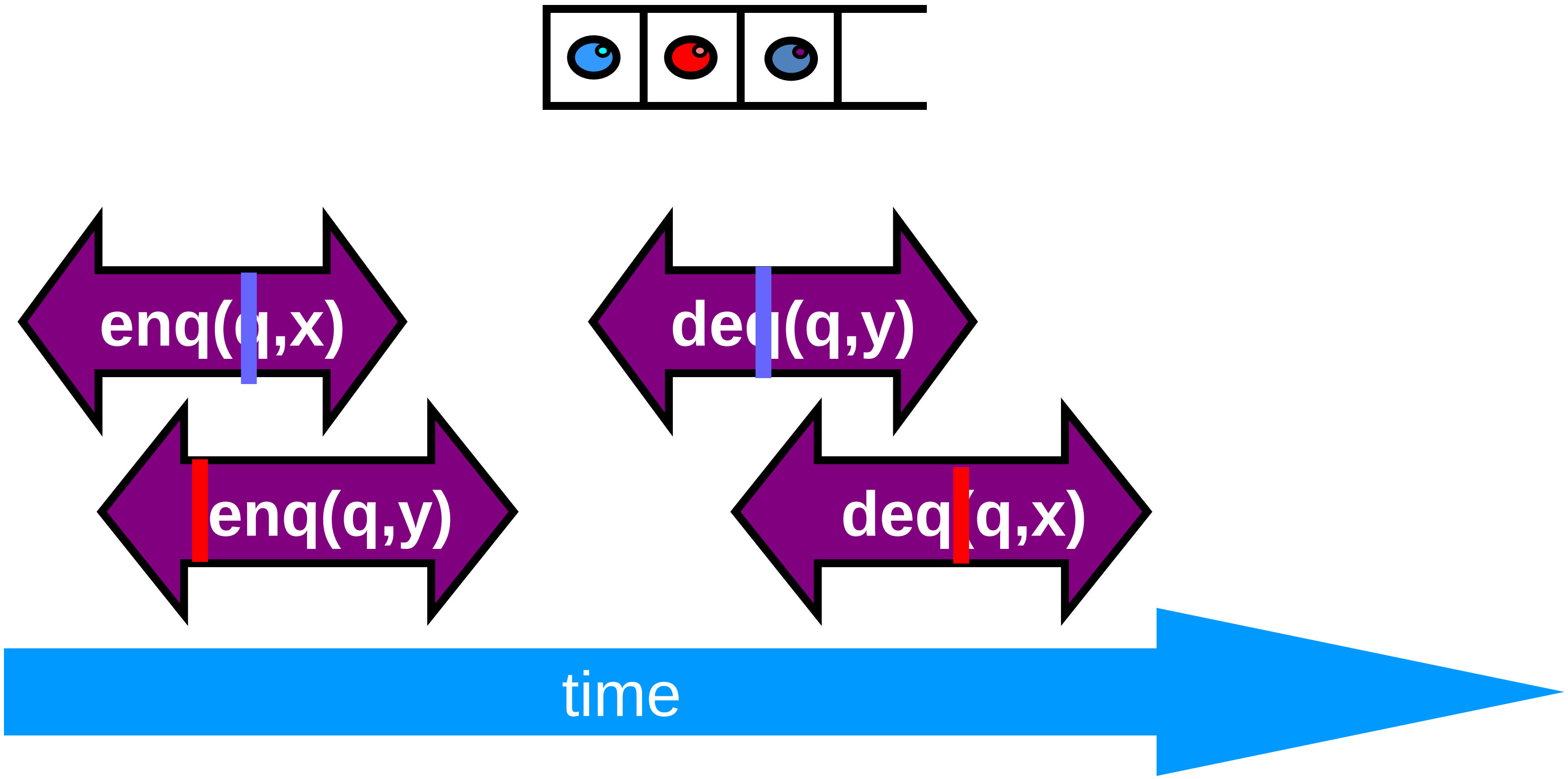


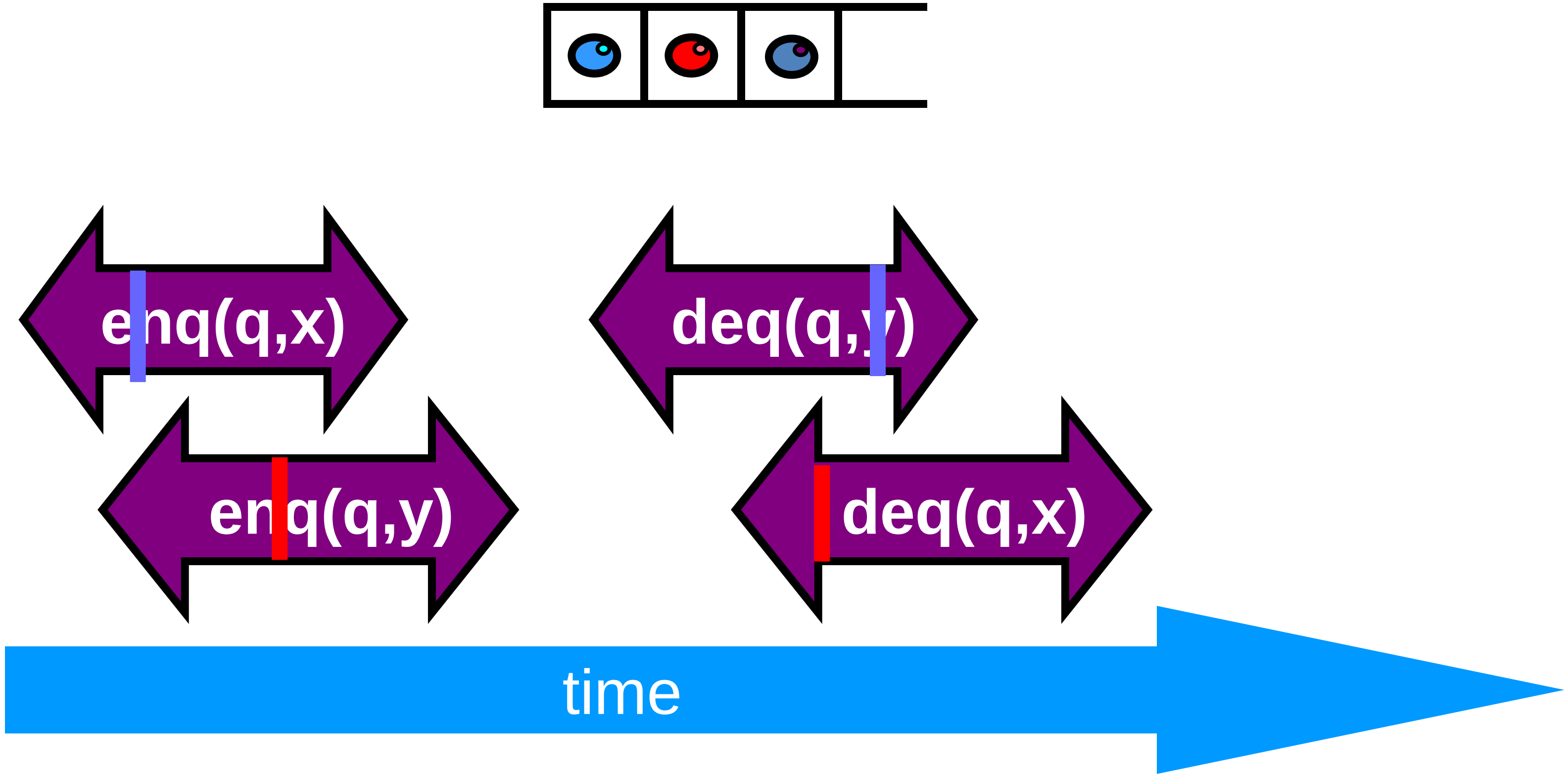
Example

Comme ci



Example

Comme ci
Comme ça



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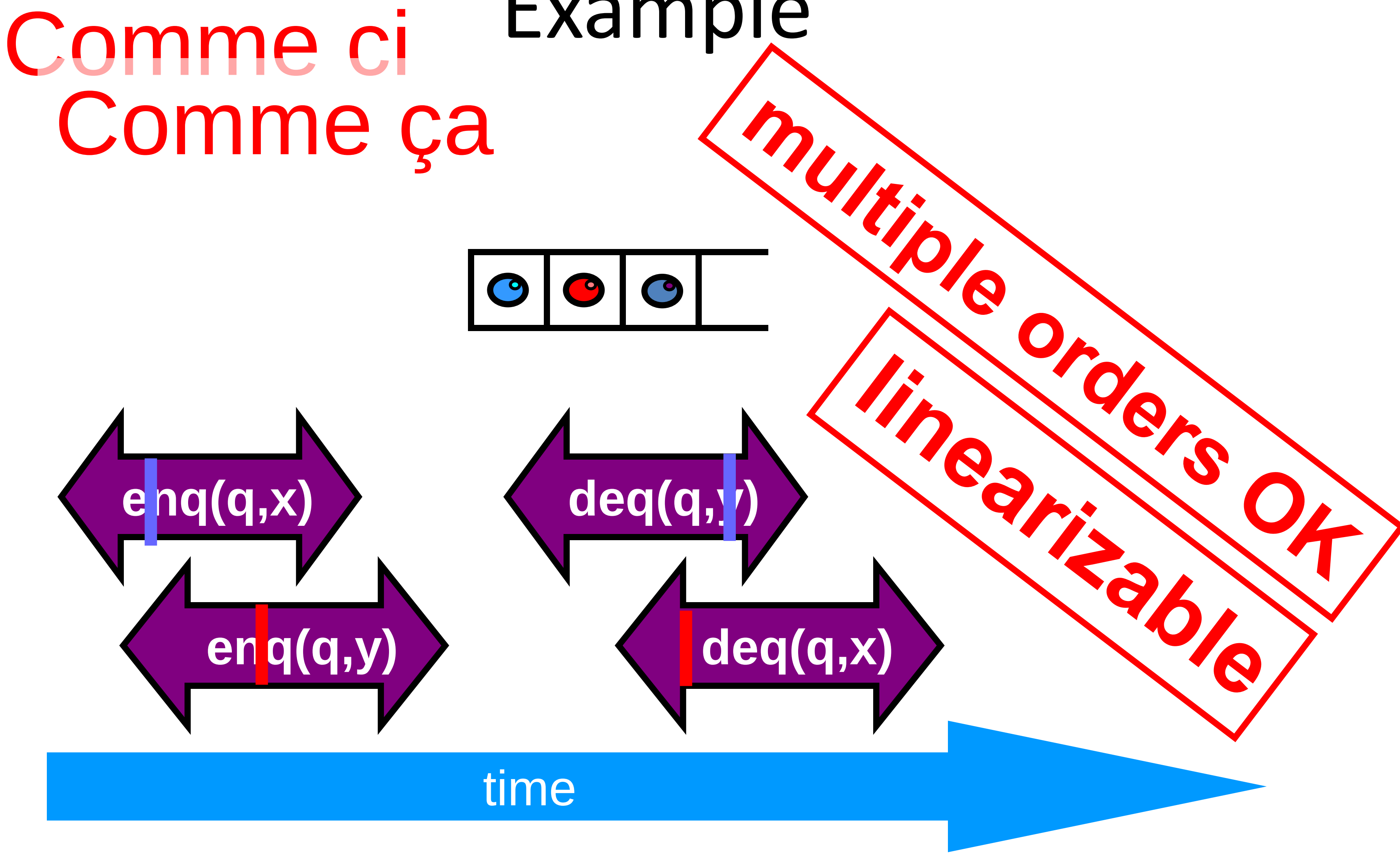


Comme ci
Comme ça
multiple orders OK
linearizable
enq(q,x)
enq(q,y)
deq(q,y)
deq(q,x)
time

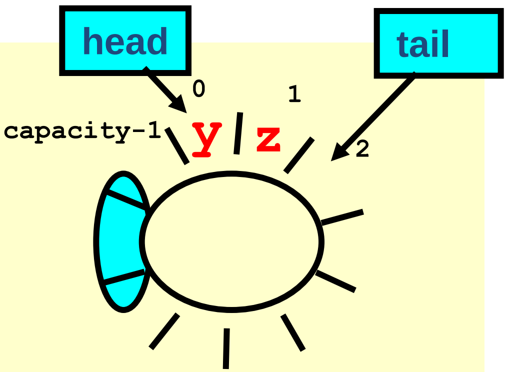
Talking About Executions

- Why executions?
  - Can't we specify the linearization point of each operation without describing an execution?
- Not Always
  - In some cases, linearization point depends on the execution

Linearizable Objects are Composable

- Modularity
- Can prove linearizability of objects in isolation
- Can compose independently-implemented objects

Reasoning About Linearizability: Locking




```
int deq(queue_t q, void **elem) {
  int res;
  pthread_mutex_lock(&q->lock);
  if (q->tail == q->head) res = 0;
  else {
    *elem = q->items[q->head % CAPACITY];
    q->head++;
    res = 1;
  }
  pthread_mutex_unlock(&q->lock);
  return res;
}
```

Reasoning About Linearizability: Locking

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int deq(queue_t q, void **elem) {
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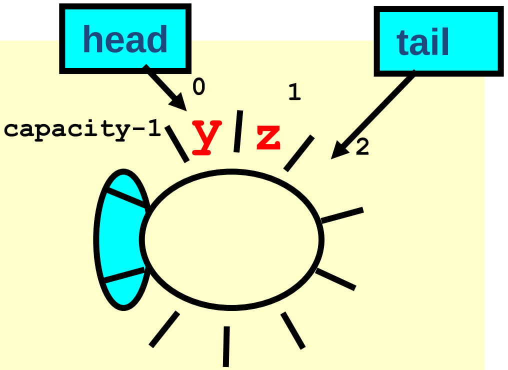
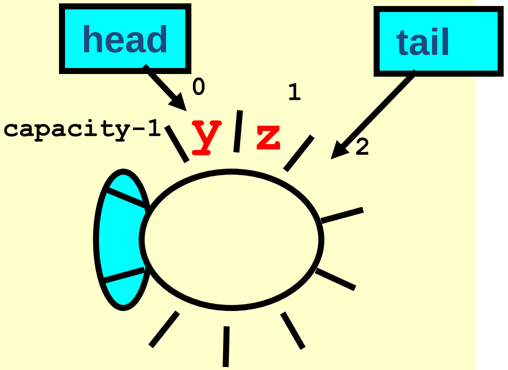


Linearization points are when locks are released

More Reasoning: Wait-free

```
int deq(queue_t q, void **elem) {
  if (q->tail == q->head) return = 0;
  *elem = q->items[q->head % CAPACITY];
  q->head++;
  return 1;
}

int enq(queue_t q, void *x) {
  if (tail-head == CAPACITY) return 0;
  q->items[q->tail % CAPACITY] = x;
  q->tail++;
  return 1;
}
```



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Linearization order is order head and tail fields modified

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  q->tail++;
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}
```

head

tail

Linearization order is order head and tail fields modified

Remember that there is only one enqueuer and only one dequeuer

Strategy

- Identify one atomic step where method "happens"
  - Critical section
  - Machine instruction
- Doesn't always work
  - Might need to define several different steps for a given method

Linearizability: Summary

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being "atomic"
- Don't leave home without it

Progress

- We saw an implementation whose methods were lock-based (deadlock-free)
- We saw an implementation whose methods did not use locks (lock-free)
- How do they relate?

Progress Conditions

- *Deadlock-free:* some thread trying to acquire the lock eventually succeeds.
- *Starvation-free:* every thread trying to acquire the lock eventually succeeds.
- *Lock-free:* some thread calling a method eventually returns.
- *Wait-free:* every thread calling a method eventually returns.

Progress Conditions

	Non-Blocking	Blocking
Everyone makes progress	**Wait-free**	**Starvation-free**
Someone makes progress	**Lock-free**	**Deadlock-free**