Basic Algorithms, Assignment ω

Due by 8 a.m. May 5.
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The universe is not only queerer than we suppose but queerer than we *can* suppose. – J.B.S. Haldane

1. `Please try both of these but submit only one -- your choice!` For the following pairs L_1, L_2 of problem classes show that $L_1 \leq_P L_2$. That is, given a "black box" that will solve any instance of L_2 in unit time, create a polynomial time algorithm that will solve any instance of L_1 in polynomial time.

   (a) Let L_2 be `TRAVELLING-SALESMAN DESIGNATED PATH`. The input here would be a graph G, two designated vertices, a source v_1 and a sink v_n, together with a positive integer weight $w(e)$ for each edge e and an integer B. Yes would be returned iff there was a Hamiltonian Path (i.e., one goes through all the vertices $v_1, \ldots, v_n$ in some order (starting and ending at the designated vertices) but does *not* return from v_n back to v_1) which had total weight at most B. L_1 is `TRAVELLING-SALESMAN` as described in Assignment 12.

   (b) Let L_2 be `CLIQUE`. The input here would be a graph G together with a positive integer B. Yes would be returned iff there was a clique with at least B vertices. (A set of vertices in a graph G is a clique if *every* pair of them are adjacent.) Let L_1 be `INDEPENDENT-SET`. The input here would be a graph G together with a positive integer B. Yes would be returned iff there was a independent set with at least B vertices. (A set of vertices in a graph G is an independent set if *no* pair of them are adjacent.)

2. Suppose that we are doing Dijkstra's Algorithm on vertex set $V = \{1, \ldots, 500\}$ with source vertex $s = 1$ and at some time we have $S = \{1, \ldots, 100\}$. What is the interpretation of $\pi[v], d[v]$ for $v \in S$? What is the interpretation of $\pi[v], d[v]$ for $v \notin S$? Which v will have $\pi[v] = NIL$ at this time. For those v what will be $d[v]$?

3. (Extra from last week!) You may use Agarwal/Kayal/Saxena but, if so, mark clearly how it is used.

   (a) Call a positive integer n `XINYU` if it has at least one prime divisor p of the form $p = 10k + 7$. Show $XINYU \in NP$.

(b) (harder!) Call a positive integer n `YUCHEN` if it has exactly one prime divisor p of the form $p = 10k + 7$. Show $YUCHEN \in NP$.

4. Let G be a `DAG` on vertices $1, \ldots, n$ and suppose we are *given* that the ordering $1 \cdots n$ is a Topological Sort. Let `COUNT[i,j]` denote the number of paths from i to j. Let s, a "source vertex" be given. Give an efficient algorithm to find `COUNT[s,j]` for all j.

A clever man commits no minor blunders. – Goethe