Basic Algorithms, Assignment 11

Due by 8 a.m. Wednesdy, April 20.
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Every block of stone has a statue inside it, and it is the task of the sculptor to discover it. – Michelangelo

1. Here is a variant implementation `EasyPrim` that doesn't use the parent π. With a given set S the data structure will be a minheap Q consisting of *all* crossing edges $\{x, y\}$ with $key[x, y] = w(x, y)$. The initial step, the minheap of all edges $\{s, x\}$ is the same. Now $V - 1$ times we EXTRACT-MIN getting a crossing edge $\{x, y\}$, $x \in S, y \notin S$ of minimal weight. We add edge $\{x, y\}$ to T and y to S. The UPDATE is different. For each $z \in Adj[y]$ we examine $\{y, z\}$. If $z \in S$ then $\{y, z\}$ is no longer a crossing edge so we delete it (caution: this takes time!) from the minheap. If $z \notin S$ then $\{y, z\}$ has become a crossing edge so we add it to the minheap, using its weight as its key.

   Analyze the time for `EasyPrim`. Show that the total time is $O(E \lg E)$. (Key idea: How often – upper bound – overall is an edge added to the minheap and how often – again upper bound – overall is an edge removed from the minheap.)

2. Consider Prim's Algorithm for MST on the complete graph with vertex set $\{1, \ldots, n\}$. Assume that edge $\{i, j\}$ has weight $|j - i|^3$. Let the root vertex $r = 1$. Show the pattern as Prim's Algorithm is applied. In particular, Let $n = 500$ and consider the situation when the tree created has 211 vertices and π and *key* have been updated.

   (a) What are these 211 vertices and what are the edges.

   (b) What are $\pi[309]$ and $key[309]$.

3. Find $d = \gcd(144, 89)$ and x, y with $144x + 89y = 1$. [Remark: This is part of a pattern with two consecutive numbers from the Fibonacci sequence $0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$]

4. Find $\frac{311}{507}$ in Z_{1000}. (This means $311 \cdot 507^{-1}$. Like the real numbers in Z_n we can add, subtract, multiply and divide – but the prohibition against dividing by zero becomes a prohibition against dividing by any b not relatively prime to n.)

5. Solve the system
   $x \equiv 34 \bmod 101$
   $x \equiv 59 \bmod 103$.

Math is natural. Nobody could have invented the mathematical universe. It was there, waiting to be discovered, and its crazy; its bizarre.

– John Conway 1937-2020