Monte Hall Problem:

There are 10 doors. One of the doors has gold (which you want). The others have paper wrappers (which you probably don’t want). Monte asks you: “Choose a door”

You choose a door.

Then Monte opens up 8 other doors (not the one that you pointed to) that all have wrappers.

“Do you want to stick with your original door or switch doors?”

GWWWWWWWWW ☺

WGWWWWWWWW

WWGWWWWWWW

WWWGWWWWWW

….

A bunch of medical doctors were asked the following question. There is a test for some disease D such that if someone has the disease, then the test will be positive 90% of the time. Prob(test will be positive GIVEN THAT have disease) = 0.9. Note that this is not the same as Prob(have disease GIVEN THAT test will be positive). That’s like saying a horse chestnut is the same as a chestnut horse. If someone doesn’t have the disease the test will be negative 90% of the time. P(test negative GIVEN THAT don’t have disease) = 0.9. This disease is fairly rare and only 1% of the population is likely to have it. P(have disease) = 0.01

Random testing is done and a person comes in and tests positive. What is the probability that the person has the disease? P(has disease GIVEN THAT test is positive).

Imagine 100 people. One person has the disease. For how many will there be a positive test? The person with the disease – test is likely to be 1 (contribution of .9). For the people who don’t have the disease, test will be negative 90% of the time. So it will be positive about 10% of the time even for non-diseased people. So, there will be about 11 people who test positive, but only one has it. Odds are something 1 in 11.

If you test negative, what is the probability that you have the disease? Prob(disease | test negative) . Imagine 1000 people. How many have the disease? 10. 9 of those people will have positive test outcome. 1 will have a negative test outcome. So, 1 in 1000 times will you have the disease with a negative test.

A more formal development:

Bayesian light: In general, from probability theory, P(X GIVEN THAT Y) = P(X and Y)/P(Y).

Y = test positive. X means have the disease.

With 100 people, how many will test positive? The number that test positive given that they have the disease + the number that test positive given that they don’t have the disease: 1 \* 0.9 + 99 \* 0.1 = 10.8. How many will both have the disease and test positive: 0.9. So, Number(X and Y)/Number(Y) = 0.9 / 10.8.

Full flown Bayesian analysis:

prob(have D| Test is positive) = Prob(have D and test is positive)/Prob(test is positive).

Given is that Prob(test is positive | have disease). Prob(test is negative | don’t have the disease).

Prob(test is positive | don’t have the disease) = 1 – Prob(test is negative| don’t have the disease).

Prob(test is positive) = prob(have disease)\* Prob(test is positive | have disease) + prob(don’t have disease)\* prob(test is positive|don’t have disease) = 0.01\*0.9 + 0.99\*0.1 = 0.009 + 0.099 = 0.108.

Prob(have disease and test is positive) = prob(have disease)\*prob(test is positive | have disease) = 0.01 \* 0.9 = 0.009.

So, prob(have D|Test is positive) = Prob(have D and test is positive)/Prob(test is positive) = 0.009/0.108 = 0.0833

3. Lucky Roulette. Two bullets in a six shooter that are adjacent (next to one another). Initially, what is my chance of survival?

B = bullet; E= empty

Equi-probable settings:

BBEEEE

EBBEEE

EEBBEE

EEEBBE

EEEEBB

BEEEEB

Didn’t die the first time. Initial settings had to be:

EBBEEE

EEBBEE

EEEBBE

EEEEBB

If you don’t spin, you will live the next click with probability ¾. If you spin, then you will live with probability 2/3 .

If the bullets had to be two away, then what are the possibilities:

BEBEEE

EBEBEE

EEBEBE

EEEBEB

BEEEBE

EBEEEB

If you survive the first time, which possibilities remain?

EBEBEE

EEBEBE

EEEBEB

EBEEEB

Initially, two doors have gold. 2 out of 10 is 1/5, so initial guess has 1/5 chance of being correct. Then Monte opens up seven doors having wrappers. So now we have three doors left. Either your door has gold or it doesn’t in which case both of the other doors have gold. Your chance on your door is 1/5 and 4/5 for the other.

Prob(event A)

If A is mutually exclusive with B, then

Prob(A or B) = Prob(A) + Prob(B)

Single die: prob(2) = 1/6. Prob(3) = 1/6

So prob(2 or 3) = 1/6 + 1/6 = 1/3

First choice has gold.

First choice doesn’t have gold.

Prob(final door has gold and first choice has gold and we switch) + prob(final has gold and first choice doesn’t have gold and we switch) = 0.1 + 0.8 = prob(final door has gold and we switch).

Two doors have gold. Choose a door. Suppose Monte opens six doors. Then do the decision tree. You get 1/15 + 8/15.

Bait and Switch:

Expected winning: prob of winning \* value of winning.

Example: I give you $100. Or I give you a 1/10 chance of winning $100,000. Expected gain of taking the chance is 1/10 \* 100,000 = 10,000. If this is your last $100, then maybe don’t take the chance even though the expected gain of taking the chance is good.

Two doors:

One door has 100 coins; the other has 200 coins. Half the time.

One door has 200 and the other has 400.

¼ of the time

One door has 400 and the other has 800. 1/4 of the time.

100 200

200 100

100 200

200 100

200 400

400 200

400 800

800 400

Given that you a door and it has 400, then you are in these situations:

400 200

400 800

expected winning after switching given that you see 400 is:

½ \* 200 + ½ \* 800 = 100 + 400 = 500

If you see 200 and you switch:

1/3 \* 100 + 1/3\*100 + 1/3\*400 = 200

Fair Dice hypothesis:

One die: 1 through 6.

Each outcome has a 1/6 probability per role.

Two dice (one green and one red): range of values is 2 through 12. How do you get two?

1, 1

How do you get 3

1, 2

2,1

How do we get 4:

1,3

2,2

3,1

How do we get 7:

1,6

2,5

3,4

4,3

5,2

6,1

Total number of possibilities for two dice:

6 for the green and 6 for the red. 6\*6 possibilities or 36.

How likely to get a 7? 6/36 = 1/6.

Game where Dennis wins on 5, Paula wins on 7? What are the odds that Paula will win?

Paula’s solution: 4 ways of getting a 5. 6 ways of getting a 7. Therefore Paula will win 6/10 times when one of us wins.

If Dennis gives Paula $60 if she wins, then what Paula give Dennis if he wins to be fair.

Leighton and Paula agree: $90.

Paula’s expected winning: 6/10 \* 60 – 4/10\*90 = 0

so that must be correct to be fair.

Den’s expected winning: 4/10 \* 90 – 6/10\*60 = 0

“Pure” Craps: First roll, roller wins if he/she gets 7 or 11. Loses on 2, 3, 12. If first roll is any other number X, then roller wins if he/she gets X before getting 7. “Real” Craps, roller loses on 2, 3, 12 on first roll but the better against the roller gets nothing on 12.

Cynthia wins if we roll a 7. Dennis wins on a 6. What are Cynthia’s odds of winning? How many ways to get 7? 6. How many ways to get 6? 5. What are the odds that Cynthia wins with probability 6/11. If Dennis pays Cynthia $50 if he loses, what should Cynthia pay Dennis to make this a fair bet? So Cynthia should pay Dennis $60.

Cynthia’s expected winning: 6/11 \* 50 – 5/11 \* 60 = 0

B W B W B

W B W B W

B W B W B

Each turn you will go up diagonally either or to the right. Moreover, if you aim left, then you will go left with prob 0.9. If you aim right, you will go right with prob 0.9.

Central planner without feedback: go right (up and to the right) then go left. Prob of success = prob(go right the first time|want to go right)\* prob(go left the second time | want to go left second time) + prob(go left the first time|want to go right) \* prob(go right the second time | want to go left) = 0.9\*0.9 + 0.1\*0.1 = 0.81 + 0.01 = 0.82

Planner with feedback:

Try to go right the first time.

If I did in fact go right,

then I will try in the second time to go left

else I will try to go right the second time

prob(go right|try to go right the first time)\*prob(go left|try to go left) + prob(go left|try to go right the first time)\*prob(go right| try to go right) = 0.9\*0.9 + 0.1\*0.9 = 0.9

Statistics is Easy: trying to infer probabilities. Infer even probabilities (null hypothesis).

Nash equilibrium: Social Games

p. 15:

Alice High Alice Low

Bob high 3 (Bob),3(A) 0,4(A)

Bob low 4(Bob), 0(A) 1,1

Nash equilibrium: state (place in this four element matrix from which neither has an incentive to leave). Free market will go to the Nash equilibrium.

Environmental protection: higher number is better for society. What will happen is that we get to low numbers.

Alice to go to low: 0.9\*4 + 0.1\*-5 = 3.6 – 0.5 = 3.1.

Alice Honest Alice cheats

Bob H 5,2 0,4

Bob C 4,0 1,1

What’s going to happen? Both go to the lower right hand corner.

Decision trees: Sweet Tooth

Marie and Jeremie

Two square cakes of identical size and content. Jeremie says:

I will cut the first cake in two somehow. You can choose whichever of the two pieces you want. Or you can decline to choose in which case I will choose.

After you make your decision and take the piece that goes to you, I will cut the second cake.

If you made a choice on the first cake, then I get to make the choice on the second cake.

Otherwise you get to make the choice on the second cake.

What can Jeremie guarantee to get? What is his strategy to get as much cake as possible assuming Marie is a rational (cake-maximizing) actor?

Jeremie cuts first cake f, 1-f (f > 1-f)

Marie chooses f, Jeremie gets 1-f

Jeremie will get whole second cake

(1-f + 1 = 2-f)

Marie declines to choose, so Jeremie gets f

Jeremie will get ½ of the second cake (f + ½)

2-f = f + ½

2-1/2=2f

3/2 = 2f

¾ = f

Polling: ask four people and three say A one says B. Null hypothesis is that A and B have the same support. In that case, how likely is it to have three wanting A and one wanting B?

Simulate prob of ½ for A and prob of ½ of B. 10,000 times we take samples of 4 imaginary people and flip a coin 4 times.

See how often we get 3 or 4 As out of 4. Let’s say that you get 3 or 4 As 2,500 times then p-value of the null hypothesis (A and B have equal support) is 25%.

Information Theory

Decision Tree learning