# Corrigendum to "Efficient similarity search and classification via rank aggregation" by Ronald Fagin, Ravi Kumar and D. Sivakumar, Proc. SIGMOD'03.


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In this corrigendum, we correct an error in the paper [FKS03]. The error was discovered by Alexandr Andoni, and the corrected theorem is due to the three authors of [FKS03], along with Alexandr Andoni and Mihai Pătraşcu.

Theorem 4 of [FKS03] states:

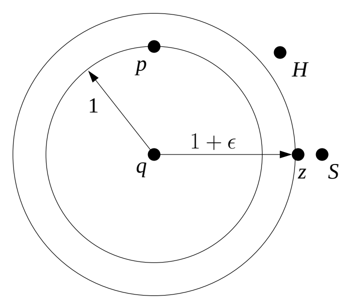
> *Let $D$ be a collection of $n$ points in $\mathbb{R}^d$. Let $r_1, \ldots r_m$ be random unit vectors in $\mathbb{R}^d$, where $m = \alpha\epsilon^{-2}\log n$ with $a$ suitably chosen. Let $q \in \mathbb{R}^d$ be an arbitrary point, and define, for each $i$ with $1 \leq i \leq m$, the ranked list $L_i$ of the $n$ points in $D$ by sorting them in increasing order of their distances to the projection of $q$ along $r_i$. For each element $x$ of $D$, let $medrank(x) = median(L_1(x), \ldots, L_m(x))$. Let $z$ be a member of $D$ such that $medrank(x)$ is minimized. Then with probability at least $1 - 1/n$, we have $d(z,q) \leq (1+\epsilon)d(x,q)$ for all $x \in D$.*

As stated, the above theorem does not hold, but a slight modification of it holds. Below, we first give a counter-example to the original theorem, and then present a modified theorem.

## 1. A COUNTER-EXAMPLE

In our counter-example, we give a specific set of $n$ points in 2-dimensional space. Consider the following point set for very small $\epsilon$, illustrated in Fig. 1:

- point $q = (0,0)$, the query;
- point $p = (0,1)$, the nearest neighbor;
- point $z = (1+\epsilon, 0)$, the false nearest neighbor;
- set $H$ of 10 points $h$ all at distance $(1+\epsilon)^2$ from $q$, specifically $h = (1+\epsilon)^2 \cdot (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$;
- set $S$ of the rest $n - 12$ points, all situated at $s = ((1+\epsilon)^2, 0)$.

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Let $r$ be random unit vector in $\mathbb{R}^2$, $L$ is the list of the pointset $D$ sorted by increasing distance from $q$; and $rank(x)$ is the rank of point $x$ in $L$. Then we have the following two claims.

Claim 1.1. $\Pr_r[rank(z) \leq 2] > 1/2 + \Omega(\epsilon)$.

The claim follows immediately from Lemma 3 of [FKS03].

Claim 1.2. $\Pr_r[rank(p) \geq |H|] \geq 1/2 + 1/16$.

It is sufficient to consider $r$'s with non-negative $x$ coordinate, and identify $r$'s by their angle with the $Ox$ axis. First, $rank(p) \leq rank(z)$ if $r \in [\alpha, \beta]$, where $\alpha$ is angle formed by the perpendicular to the line connecting $q$ to midpoint of $pz$, and $\beta$ is the angle formed by the perpendicular to $pz$. We can estimate $\alpha$ and $\beta$:

$$\alpha = \arctan\frac{p_y + z_y}{p_x + z_x} - \pi/2 = \arctan(1+\epsilon) - \pi/2 = -\pi/4 - \Theta(\epsilon)$$

$$\beta = \arctan\frac{z_x - p_x}{p_y - z_y} = \arctan(1+\epsilon) = \pi/4 + \Theta(\epsilon).$$



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Thus, if $r \in [\alpha, \beta]$, $rank(p) \geq rank(z)$, and then $rank(p) \geq |S| + 1$.

Moreover, as we will see, if the angle of $r$ is around $-\pi/4$, then $rank(p) > rank(h)$. Indeed, consider any angle $\gamma \in [-\pi/4, -\pi/4 + \pi/16]$. Then, $|\langle p, r\rangle| = |\sin\gamma| \geq 0.5$ and $|\langle h, r\rangle| = |(1+\epsilon)^2 \frac{1}{\sqrt{2}} \cdot (\sin\gamma + \cos\gamma)| \leq 0.2(1+\epsilon)^2$.

Thus, if the angle of $r$ is in the range $(-\pi/2, -\pi/4 + \pi/16)$ or $(\beta, \pi/2)$, $rank(p) \geq |H|$, and this happens with probability at least $\frac{\pi/4+\pi/16-\Theta(\epsilon)}{\pi/2} \geq 1/2 + 1/16$.

Standard high concentration bounds will yield that $medrank(z) < medrank(p)$ with high probability. For completeness, we include one such lemma, due to Indyk:

Lemma 1.3 (cf. [Ind00], Lemma 2). *Let $\mathcal{D}$ be a distribution on $\mathbb{R}$ and $F$ be its cumulative distribution function. Then, for $\epsilon, \delta > 0$ and $k = O(\frac{\log 1/\delta}{\epsilon^2})$, if $X_1 \dots X_k$ are iid from $\mathcal{D}$, then $X = median\{X_1, \dots X_k\}$ satisfies $\Pr[F(x) \in (1/2 - \epsilon, 1/2 + \epsilon)] \geq 1 - \delta$.*

## 2. A NEW ALGORITHM

To correct the theorem, we propose to use the following new function $medrank(x)$:

$$medrank(x) = median_i(|xr_i - qr_i|).$$

The resulting algorithm is presented in Fig. 2. Next, we show that this algorithm gives a $1+\epsilon$ nearest neighbor data structure.

We note that, for Step 2 of the query algorithm, we can use also other aggregation functions instead of the median function. In particular, if we use $\ell_2$ norm of the *score* vector instead of the median, then the same lemma as above holds, implied by Johnson-Lindernstrauss lemma [JL84]. Furthermore, if use $\ell_1$ norm of the *score* vector, then again the same lemma as above holds, and is implied by the $\ell_2$ to $\ell_1$ embedding of [JS82].

## 3. REFERENCES


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**Preprocessing.** Input: a set of points $P \subset \mathbb{R}^d$, $|P| = n$, and $\epsilon > 0$.

1. Choose $k = O(\frac{\log n}{\epsilon^2})$ vectors $r_i \in \mathbb{R}^d$, $i = 1 \dots k$, where each coordinate of $r_i$ is drawn from a Gaussian $N(0,1)$ distribution. Vectors $r_i$ represent random projections.
2. Construct $k$ lists, where the $i^{th}$ list contains all the points from $p \in P$ sorted according to the value $p \cdot r_i$.

**Query.** Input: a query point $q \in \mathbb{R}^d$.

1. For fixed $i$ and $p \in P$, define $score_i(p) = q \cdot r_i - p \cdot r_i$.
2. Find the point $p^* \in P$ that minimizes $median_{i \in [k]}\{|score_i(p^*)|\}$.
3. Return $p^*$.



Lemma 2.1. *The algorithm from Figure 2 returns a $1+\epsilon$ nearest neighbor of $q$ with probability at least $1 - 1/n$.*

Proof. Fix some $p$ and let $\Delta = \|p - q\|_2$. For each $i \in [k]$, $score_i(p)$ is distributed as $N(0, \Delta^2)$, normal distribution with standard deviation $\Delta$. We will once again use Lemma 1.3 for estimating the median of iid samples.

Let $M_p = median_{i\in[k]}\{|score_i(p)|\}$. Applying Lemma 1.3 to the distribution $N(0, \Delta^2)$, we conclude that $F(M_p) \in (1/2 - \epsilon, 1/2 + \epsilon)$ with probability at least $1 - 1/n^2$, where $F$ is the cumulative of $N(0, \Delta^2)$. Since the value $x$ that satisfies $F(x) = 1/2$ is $x = c\Delta$ where $c$ is an absolute constant, and $F$ has derivate $\Theta(1/\Delta)$ around this $x$, we conclude that $M_p \in (x - O(\epsilon\Delta), x + O(\epsilon\Delta))$. Thus, $M_p$ is a $1+\epsilon$ approximation to $\|q - p\|$ with probability at least $1 - 1/n^2$.

We conclude that $M_p$ is a $1+\epsilon$ approximation to $\|q-p\|$ for all $p$, with probability at least $1 - 1/n$. Thus the algorithm returns a $1+\epsilon$ approximate nearest neighbor with probability at least $1 - 1/n$. $\square$