♣ VISUALIZING $\mathbb{R}^N$ AND SOME NEW DUALITIES

Alfred Inselberg
School of Mathematical Sciences
Tel Aviv University
Tel Aviv, Israel

With parallel coordinates the perceptual barrier imposed by our 3-dimensional habitation is breached enabling the visualization of multidimensional objects. The highlights, interlaced with interactive demonstrations, are intuitively developed. M-dimensional objects are recognized recursively from their $(M-1)$-dimensional subsets. It emerges that *a hyperplane in N-dimensions is represented by $(N-1)$ indexed points*. Points representing lines have two indices, those representing planes three indices and so on. In turn, this yields powerful geometrical algorithms (e.g. for intersections, containment, proximities) and applications including classification.

A smooth surface in 3-D is the envelope of its tangent planes each represented by 2 planar points. As a result it is represented by two planar regions, and a hypersurface in N-dimensions by $(N-1)$ regions. This is equivalent to *representing a surface by its normal vectors*. Developable surfaces are represented by curves revealing the surface characteristics. *Convex surfaces in any dimension* are recognized by hyperbola-like regions. Non-orientable surfaces yield stunning patterns unlocking new geometrical insights. Non-convexities like folds, bumps, concavities are visible. The patterns persist in the presence of errors. Intuition gained from the $\mathbb{R}^3$ representations leads to generalization for $\mathbb{R}^N$ with beautiful new dualities like **cusp in** $\mathbb{R}^N \leftrightarrow (N-1)$ **"swirls" in** $\mathbb{R}^2$**, "twist" in** $\mathbb{R}^N \leftrightarrow (N-1)$ **cusps in** $\mathbb{R}^2$**.**