Upstart Puzzles

Ice Trapping

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Imagine a 500 by 500 checkerboard surrounded by fixed walls. A frictionless hockey puck is moving diagonally above the checkerboard in a northwest, northeast, southeast or southwest direction (you don't know which and you don't know where the puck starts), at a speed of one diagonal square in each time unit.

You can put up horizontal (east-west) or vertical (north-south) additional walls of any size, but your total wall length is limited to some length T.. In order to build your first wall of length L, you must wait at least ceiling(L/10) time units from the beginning of the game. To build any subsequent wall of length L', you must wait at least ceiling(L'/10) time units from the time you built your last wall. Once you are allowed to build a wall, it is built instantly. You may tear down your built walls instantly and at any time.

If the puck hits one of your walls or a side, it bounces to the reflecting diagonal. So for example, if the puck moves southeast and hits your north-south wall, it will bounce to a southwest direction one square south of the square from which it hit the wall (thus changing colors).

[We want a figure showing this]

We'll consider three detection scenarios:

1. If the puck hits any one of the walls you put up, you know which wall was hit, on which side, and where. You don't learn when the puck hits the side walls.

2. If the puck hits any one of your walls, you know which wall was hit but that's all.

3. If the puck hits any one of your walls, you know only that a wall has been hit, but not which one.

For each scenario, your goal is to guarantee to confine the puck to a region of area 1 by 1 in as little time as possible, no matter where the puck begins and no matter in which direction it is going.

How would you guarantee to achieve this goal given only a length of 502 in wall length in the second most difficult scenario (scenario 2)?

Solution: The following solution sketch is due to NYU freshman Dan Simon. Let's fix the southwest checker to be (0, 0), where the first coordinate represents the north-south direction. After waiting 50 units, put up a north-south wall from (0,1) to (499, 1). Eventually, the puck will hit that wall. If it hits soon after again, then you have trapped the puck in the narrow alleyway between (0,0) and (499,0). Otherwise the puck is traveling east, so ten units later, you can tear down that wall and put up another at (0,13) to (499, 13). That will reflect the puck back. You can tear down that wall. 13 units later, you can put up a wall from (0,2) to (499,2).

You have now trapped the puck in an alleyway of one unit thickness. Now put a single barrier at (1, 499). The puck will hit that barrier in at most 500 time units. If it hits that barrier again very soon (within three time units), then you have trapped the puck. If not, then destroy that wall and put up another one at (4, 499). Now at each time unit move south until you've trapped the puck in the southeast corner.

Upstart 1: The minimum conceivable wall length is 3 if you could somehow force the puck into a corner with the fixed walls. Can you achieve that? If not, what can you guarantee to achieve?

Upstart 2: Find optimal worst case guarantees in all three detection scenarios.

All are invited to submit their solutions to upstartpuzzles@cacm.acm.org; solutions to upstarts and discussion will be posted at http://cs.nyu.edu/cs/ faculty/shasha/papers/cacmpuzzles.html