Card Counters

BY DENNIS E. SHASHA

Alice and Bob play a game using 14 cards—the ace through seven of hearts and the ace through seven of spades. Bob picks a number between one and seven, then tells Alice the number. Alice deals that many cards, finishing the first round. Bob turns over the last card dealt; the number on that card determines how many cards Alice deals in the second round (an ace is considered one). Again, Bob turns over the last card dealt, and Alice then deals that many more cards. Alice wins if the last card dealt in some round is the very last of the 14 cards and if it is the ace of spades. Bob wins otherwise.

First, consider a warm-up problem: How can Alice arrange the cards in advance so that no matter which number Bob chooses at the beginning, the game will end with the ace of spades as the last card turned up? One possible answer is shown in illustrations A through E below.

Here's a distinctly harder problem. Suppose that Bob arranges all the cards that bear numbers greater than four (the five through seven of hearts and spades). Then Alice, without looking at what Bob has done, arranges the remaining cards and puts her cards after Bob's in the deck. Can she still force a win?

I've saved the hardest problem for last. Say that Bob takes any seven of the cards except the ace of spades. Alice may look at what Bob has done and insert the remaining cards among Bob's cards, but she cannot change their order. For example, if Bob orders the cards as shown in illustration F, Alice can win the game by making the right insertions (I'll leave it to you to figure out how). But can Alice win in general, no matter how Bob arranges the cards? My conjecture is yes, but I know of no proof. Perhaps some clever reader can enlighten me. SA

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Answer to Last Month's Puzzle

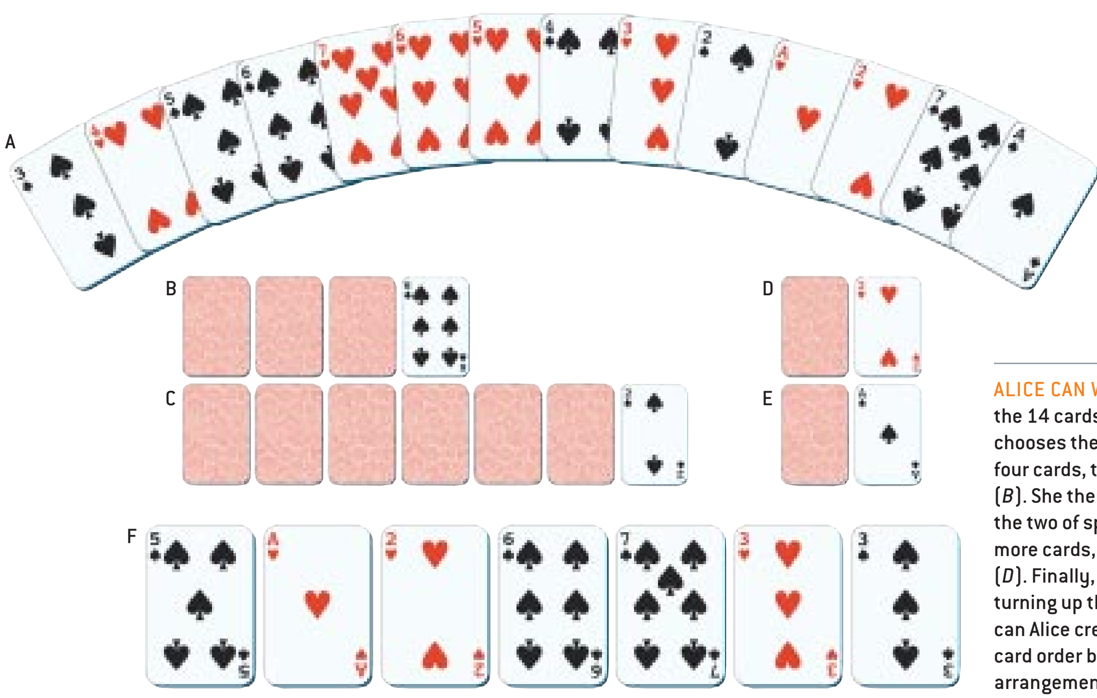
Of the nine lies told by the shifty witnesses, at least five must falsely assert innocence ("the suspect has no drugs"). The only possible scenario that will yield this result is if suspects 1, 4, 5, 7, 8 and 10 have drugs and the rest do not. For a more detailed explanation, go to www.sciam.com

Correction

In the solution to the "Truck Stop" puzzle [November 2001], the number of traversals required across both BC and CD is 12, not 11.

Web Solution

For a peek at the answer to this month's problem, visit www.sciam.com



ALICE CAN WIN the game by ordering the 14 cards as shown (A). Suppose Bob chooses the number four. Alice deals four cards, turning up the six of spades (B). She then deals six cards, turning up the two of spades (C). She deals two more cards, turning up the two of hearts (D). Finally, she deals two cards, turning up the ace of spades (E). But can Alice create another winning 14-card order by making insertions into the arrangement shown (F)?

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