

## Coach McGraw's Problem

With four tennis courts, we wish to generating a complete ranking of eight players in *five* hours. We divide the players into groups A and B, with four in each.

### Hour 1.

The four players in each group play one another in pairs.

### Hour 2.

The stronger players in each group from the first hour play each other, and the weaker players in each group from the first hour play each other.

### Hour 3.

The top player in group A plays the top player in group B, and the bottom player in group A plays the bottom player in group B. The middle two players in group A play each other if necessary, and the middle two players in group B play each other if necessary.

At this point, we have a complete ranking of groups A and B:  $(A_1, A_2, A_3, A_4)$  and  $(B_1, B_2, B_3, B_4)$ . Moreover, we know who is the strongest player overall and who is the weakest player overall. There are two scenarios.

**Scenario X.** The strongest and the weakest players overall are from the same group. Without loss of generality, we may assume that they are  $A_1$  and  $A_4$ .

### Hour 4.

$A_2$  plays  $B_2$  and  $A_3$  plays  $B_3$ .

### Hour 5.

**Case 1.**  $A_2$  is stronger than  $B_2$  and  $A_3$  is stronger than  $B_3$ .

Then  $A_2$  plays  $B_1$  for second and third places.  $A_3$  plays  $B_2$  for fourth and fifth places.  $B_3$  is in sixth place and  $B_4$  is in seventh place.

**Case 2.**  $A_2$  is stronger than  $B_2$  and  $A_3$  is weaker than  $B_3$ .

Then  $A_2$  plays  $B_1$  for second and third places.  $B_2$  is in fourth place and  $B_3$  is in fifth place.  $A_3$  plays  $B_4$  for sixth and seventh places.

**Case 3.**  $A_2$  is weaker than  $B_2$  and  $A_3$  is stronger than  $B_3$ .

Then  $B_1$  is in second place,  $B_2$  is in third place,  $A_2$  is in fourth place,  $A_3$  is in fifth place,  $B_3$  is in sixth place and  $B_4$  is in seventh place.

**Case 4.**  $A_2$  is weaker than  $B_2$  and  $A_3$  is weaker than  $B_3$ .

Then  $B_1$  is in second place and  $B_2$  is in third place.  $A_2$  plays  $B_3$  for fourth and fifth places.  $A_3$  plays  $B_4$  for sixth and seventh places.

**Scenario Y.** The strongest and the weakest players overall are from different groups. Without loss of generality, we may assume that they are  $A_1$  and  $B_4$ .

**Hour 4.**

$A_2$  plays  $B_2$  and  $A_3$  plays  $B_3$ .

**Hour 5.**

**Case 1.**  $A_2$  is stronger than  $B_2$  and  $A_3$  is stronger than  $B_3$ .

Then  $A_2$  plays  $B_1$  for second and third places.  $A_3$  plays  $B_2$  for fourth and fifth places.  $A_4$  plays  $B_3$  for sixth and seventh places.

**Case 2.**  $A_2$  is stronger than  $B_2$  and  $A_3$  is weaker than  $B_3$ .

Then  $A_2$  plays  $B_1$  for second and third places.  $B_2$  is in fourth place,  $B_3$  is in fifth place,  $A_3$  is in sixth place and  $A_4$  is in seventh place.

**Case 3.**  $A_2$  is weaker than  $B_2$  and  $A_3$  is stronger than  $B_3$ .

Then  $B_1$  is in second place,  $B_2$  is in third place,  $A_2$  is in fourth place and  $A_3$  is in fifth place.  $A_4$  plays  $B_3$  for sixth and seventh places.

**Case 4.**  $A_2$  is weaker than  $B_2$  and  $A_3$  is weaker than  $B_3$ .

Then  $B_1$  is in second place and  $B_2$  is in third place.  $A_2$  plays  $B_3$  for fourth and fifth places.  $A_3$  is in sixth place and  $A_4$  is in seventh place.