Sweeping Incremental Algorithm for Matrix Profile (SIAMP)

Reza Akbarinia

Problem Definition

- Let
  - A and B : times series of size n
  - m: size of sequences
  - $A_m[i]$ or $B_m[i]$: sequence of size m starting at position i in A (or B)
  - $D_{i,j}$: Square of Euclidean distance between $A_m[i]$ and $B_m[j]$

Goal:

- Compute J_{AB} : such that $J_{AB}[i]$ returns the position of the nearest sequence of B to $A_m[i]$

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- Main idea: compute the sequence distances incrementally
  - Each distance: in O(1) instead of O(m)
  - Thus, an amortized complexity of O(n) to find the nearest sequence of $A_m[i]$
- For this, we sweep the two time series in n-m steps
  - In each step, the distances are incrementally computed, and minimum distances updated
  - In step k, we compute the distance of $A_m[i]$ and $B_m[i+k]$
    - i.e., sequences of A and B that have a difference of k in their initial positions

Algorithm

- For i=0 to n-1 Min_D[i] := ∞ //initialize minimum distances
- For k=0 to n-m //sweep A and B in n-m steps
  - Compute $D_{0,k}$ using Euclidean function
  - For i=1 to n – k - 1
    - Incrementally compute $D_{i, i+k}$ using $D_{i-1, i+k-1}$ // O(1)
    - If (Min_D[i] > $D_{i, i+k}$) then
      - Min_D[i] := $D_{i, i+k}$
      - J_{AB}[i] := i+k

Incremental Distance computation

- $D_{i,j}$: Square of Euclidean distance between $A_m[i]$ and $B_m[j]$
- $A_m[i]$: $<a_i, \ldots, a_{i+m}>$
- $B_m[j]$: $<b_j, \ldots, b_{j+m}>$
- $D_{i,j} = \sum(a_i - b_j)^2$ for $1 \le i \le m$
- $D_{i-1,j-1} = \sum(a_{i-1} - b_{j-1})^2$ for $1 \le i \le m$

Thus, we have

- $\mathbf{D_{i,j} = D_{i-1,j-1} - (a_{i-1} - b_{j-1})^2 + (a_{i+m} - b_{j+m})^2}$

Analysis of SIAMP

- An exact algorithm for computing the matrix profile
- Time complexity: $O(n^2)$
- Space complexity: $O(n)$
- Simpler and faster than Keogh et al. algorithm whose complexity is $O(n^2 \log n)$
  - No need to Fourier transformations
  - No need to compute the mean and standard deviation of each sequence
  - No need to