Assignment 2
Game Theory

Due: September 9th, 2019 at the beginning of class.

For any doubts or queries regarding assignment please attend office hours on Monday 9:30-10:30 in CIWW 412. The assignment can we written or typed. Please write your name and netID on the assignment.

1. While at separate workplaces, Cameron and Jessie must choose to attend either the opera or a movie. Both individuals would rather spend the evening together than apart, but Cameron would rather be at the opera while Jessie would rather be at the movie. Please draw an according matrix and explain if there are Nash equilibria.

2. Dynamic equilibrium models involve firms and consumers meeting repeatedly under identical circumstances. Consider two firms, labelled 1 and 2. Each firm has three strategies, to produce 'low', 'middle' or 'high' quantities of output. The payoffs are represented in the matrix:

		Firm 2		
		L	***M***	***H***
Firm 1	***L***	15, 15	5, 27	5, 10
	M	27, 5	12, 12	2, 5
	H	10, 3	5, 2	0, 0

   Find the Nash equilibria for a single game.

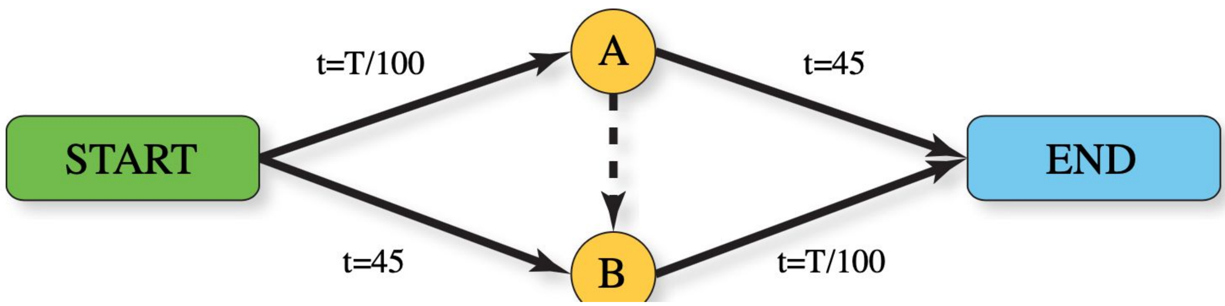
   If this were a **repeated** game, what strategy will lead to a better outcome? (Hint : Cooperation is sometimes better)

3. There are three bags:
   a. a bag containing two apples,
   b. a bag containing two oranges,
   c. a bag containing one apple and one orange.

   Suppose you randomly choose a bag and draw an apple. What is the probability that the other fruit in the bag is an apple?

Hint: Read about the Monty Hall problem and the Three Prisoners Problem.

4. Consider a road network as shown in the adjacent diagram on which 4000 drivers wish to travel from point Start to End. The travel time in minutes on the Start-A road is the number of travelers (T) divided by 100, and on Start-B is a constant 45 minutes (likewise with the roads across from them). If the dashed road does not exist (so the traffic network has 4 roads in total), the time needed to drive Start-A-End route with 'A' drivers would be 45 + A/100. The time needed to drive the Start-B-End route with 'B' drivers would be 45+ B/100. If either route took less time, it would not be a Nash equilibrium: a rational driver would switch from the longer route to the shorter route. As there are 4000 drivers, the fact that A+B = 4000 can be used to derive the fact that A=B=2000 when the system is at equilibrium. Therefore, each route takes 45+ 2000/100 = 65 minutes.



Now suppose the dashed line A-B is a road with an extremely short travel time of approximately 0 minutes. Suppose that the road is opened and one driver tries Start-A-B-End. To his surprise he finds that his time is 2000/100 + 2001/100 = 40.01 minutes, a saving of 25 minutes. Soon, more of the 4000 drivers are trying this new route. The time taken rises from 40.01 and keeps climbing.

A. At what number N drivers taking the new road Start-A-B-End, does the time take 65 minutes. Assume that the remaining 4000-N drivers take Start-B-End. (Notice that no one takes the road A-> End as it takes 45 minutes and clearly A->B->End will always be lesser than that.)
B. If N drivers are taking the new road. What happens to the remaining 4000-N drivers taking the road through B?
C. Observe that the other 4000-N drivers are incentivised to take the new road through A-B. Compare the time of travel between the two paths.
D. What is the new Nash Equilibrium route with respect to time?