SOCIAL NETWORKS

Nov 24 2015

LECTURE # 19

A General Description of Evolutionarily Stable Strategies (ESS)

		Organism 2	
		S	T
Organism 1	S	(a,a)	(b,c)
	T	(c,b)	(d,d)

Two-player Two-strategy Game (Symmetric).

Condition for (S,S) to be evolutionarily stable

$S = ESS$

Let $\epsilon > 0$

$1-\epsilon$ fraction uses S } Expected payoff for organism playing, S $= a(1-\epsilon) + b\epsilon$

ϵ fraction uses T.

} Exp. payoff for org playing, T $= c(1-\epsilon) + d\epsilon$

We need the following inequality

$$a(1-\epsilon) + b\epsilon > c(1-\epsilon) + d\epsilon$$

$$\lim_{\epsilon \to 0} \quad a > c$$

$\therefore$ $S = ESS$ if $a > c$.

$S \neq ESS$ if $a < c$.

if $a = c$, $S = ESS$, if $b > d$.

<u>Thm</u>: In a two player, two-strategy, symmetric game, S is evolutionarily stable

when either (i) $a > c$

or (ii) $a = c$ and $b > d$.

~

Note: (S, S) = Nash Equilibria

$$u_1(S,S) \geq u_1(T,S) \quad \boxed{\equiv \ a \geq c}$$

&

$$u_2(S,S) \geq u_2(S,T)$$

$\Rightarrow$

$S = ESS \Rightarrow (S,S) = N.E.$

Corollary:

If strategy S (in a two-player, two-strategy symmetric game) is ESS (evolutionarily stable), then (S,S) = Nash equilibrium (P.S.N.E.)

Not all Nash Equilibria are E.S.S.

Stag-Hunt Game.

Hunter 1 \ Hunter 2	Stag	Hare
Stag	4,4	0,3
Hare	3,0	3,3

$a = 4$
$b = 0$
$c = 3$
$d = 3$

$\left.\begin{matrix} a > c \\ d > b \end{matrix}\right\}$ Two ESS.

↙ Stag ↘ Hare.

Modify $c = 4$

	S	H
S	4,4	0,4
H	4,0	3,3

$a = 4$
$c = 4$
$b = 0$
$d = 3$

$a = c, \; b < d$
$\Rightarrow$ Stag $\neq$ ESS.

But $d > b \Rightarrow$ Hare = ESS

(H, H) = N.E.

Note: ESS and N.E. are similar but underlying mechanisms differ

N.E → Rationality (Fictious Play)

E.S.S. → Replicator Dynamics

Reasoning vs. Learning.

Evolutionarily Stable Mixed Strategies. (ESMS)

"How to handle games in which no strategy is ESS."

Hawk-Dove Game

Symmetric
2-player
2-strategy

Animal 2

Animal 1	D	H
D	(3,3)	(1,5)
H	(5,1)	(0,0)

$a = 3$
$b = 1$
$c = 5$
$d = 0$

$a < c$
$d < b$

Hawk = Aggressive
Dove = Passive

- No ESS
- But it has two P.S.N.E. $= \{(D,H), (H,D)\}$
- Neither D nor H is a best response to itself.
- Evolutionary Game in which strategy can be "*mixed*." (by any individual)

Probability, $p \in [0,1]$ ~~$Pr[S] \sim B$~~

$q = 1-p$

$$\left.\begin{aligned}&\text{Play } S \text{ with probability } p\\ &\quad\& \; T \text{ with } \;\;\text{"} \qquad\qquad q\end{aligned}\right\} \; S \sim Ber(p).$$

Organism 1 chooses S with prob p
X

Organism 2 chooses S with prob p'
Y

	X	Y
X	pp' (a,a)	pq' (b,c)
Y	qp' (c,b)	qq' (d,d)

Expected Pay-off for the player 1 ⇒

$$V(p,p') = pp'a + pq'b + qp'c + qq'd$$

$$\left\{\begin{aligned} V(p,p) &= p^2 a + 2pq(b+c) + q^2 d\\ V(p',p) &= pp'a + pq'c + qp'b + qq'd\\ V(p',p') &= p'^2 a + p'q'(b+c) + q'^2 d \end{aligned}\right.$$

$p \neq p'$

$$\forall_{p \neq p'} \exists_{y>0} \forall_{x<y}$$

$$(1-x)V(p,p) + x\,V(p,p')$$

$$> (1-x)V(p',p) + x\,V(p',p')$$

In the general symmetric game,
p is an evolutionarily stable mixed strategy,
if there is a (small) positive number y such that, when any other mixed strategy p′ invades p ($p \neq p'$) at any level $x<y$, the fitness of the organism playing p is strictly greater than the fitness of an organism playing p′.

* * *

$p = 1/3$ ≡ Probability of playing Dove

To be shown: $p = 1/3$ ≡ E.S.M.S.