SOCIAL NETWORKS

LECTURE # 14

ULTIMATUM GAME.

- Two anonymous players bargain to divide a fixed amount between them.
- PLAYER 1 ; (PROPOSER) offers a division of the amount.
- PLAYER 2 ; (RESPONDER) decides whether to accept the offer.

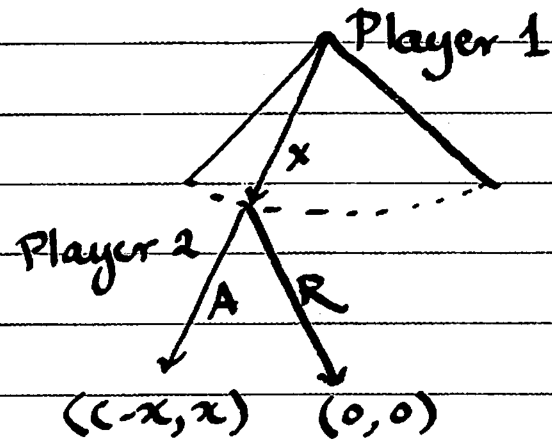
If accepted both players receive their agreed-upon shares ; otherwise, they receive nothing.

Total Amount to divide $= c$

1 offers to 2 some amount $0 \leq x \leq c$

→ If 2 accepts the payoff is $(c-x, x)$

→ If 2 rejects the payoff is $(0, 0)$



Extensive Form Game.

Subgame Perfect (Nash) Equilibrium:
(SPE)

"Optimal" not only at the beginning of the game, but also after every history.

Backward Induction:

Player 2 → (1) $x>0$, Accept; (2) $x=0$, Indiff between Accept/Reject.

$\Rightarrow$ Ⓐ $x \geq 0$, A; Ⓑ $x>0$, A; $x=0$, R

Player 1 → Ⓐ $x=0$ → $x=0$.

Player 1 → Ⓑ $\sup_{x>0} (c-x) \to x$ (or ε if x is not divisible)

Non-Credible Threat:

Player 2 → (1) $x > \bar{x}$, Accept; (2) $x \leq \bar{x}$, Reject.

Player 1 → Offers $\bar{x}$.

For every $\bar{x} \in [0, c]$, there exists a NE in which 1 offers $\bar{x}$.

STRATEGIC FORM GAMES

Notation:

$$S \equiv \prod_i S_i$$ ⇒ Strategy Profile.

$s_i \in S_i$ = Strategy of the player $i \in I$

$$S_{-i} = \prod_{j \neq i} S_j$$ ≡ Strategy Profile for all players except $i \in I$.

$$\therefore \quad S = S_i \times S_{-i} \; ; \quad s_{-i} \in S_{-i}$$

$$s_{-i} = \langle s_j \rangle_{j \neq i}$$ ≡ Vector of all strategies/actions for all players excluding player $i \in I$.

$$\langle s_i, s_{-i} \rangle$$ = STRATEGY PROFILE.

PLAY OF THE GAME

Each player chooses a strategy s_i

$$\Downarrow$$

Strategy Profile $= \langle s_1, s_2, \ldots, s_\ell \rangle \equiv S$

$$\Downarrow$$

Utility $\equiv u_i(s)$.

If $S^* = \langle s_1^*, s_2^*, \ldots, s_\ell^* \rangle =$ "BEST"
then

$$\forall_{i \in I} \; \forall_{s_i \in S_i}$$

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*).$$

$$\Downarrow$$

BEST RESPONSE.

① STABILITY: No player can profitably deviate from the chosen strategy, given the strategy of the other players.

② FIXED POINT UNDER CKR:
Each player chooses a strategy (s_i^*) expecting all other players to choose rationally.

Example BoS.

- Greedy Strategy $\not\Rightarrow$ NE.

S_F = Opera, S_M = Football

$\Rightarrow$ Pay-off = (0,0).

$\therefore$ Either S_F should deviate to Football (payoff $0 \to 2$)

Or S_M should deviate to Opera (payoff $0 \to 2$)

- Ultra-Altruistic Strategy $\not\Rightarrow$ NE.

S_F = Football, S_M = Opera

$\Rightarrow$ Pay-off = (0, 0)

Either S_F should deviate to Opera (payoff $0 \to 3$)

Or S_M should deviate to Football (payoff $0 \to 3$).

Two Nash Equilibria (Pure Strategy).

<O, O> <Opera, Opera>

<F, F> <Football, Football>.

F enjoys both the opera & M's company.

M gains utility by being in F's company.

Rock-Paper-Scissors. $\approx$ Matching-Pennies.

$P_1 \backslash P_2$	R	P	S
R	(0,0)	(-1,1)	(1,-1)
P	(1,-1)	(0,0)	(-1,1)
S	(-1,1)	(1,-1)	(0,0)

$\not\exists$ P.S.N.E

Zero-Sum Game.

$\Downarrow$

MIXED STRATEGIES

- Σ_i = Probability Measure over Pure Strategies S_i

$\sigma_i = (p_{i_1}, p_{i_2}, \dots, p_{ik}) \in \Sigma_i$

$\hookrightarrow p_{i_j} = \Pr[s_{i_j} \in S_i \text{ is played}]$

$\forall_j \; p_{i_j} \geq 0 \qquad \sum p_{ij} = 1.$

$\Sigma = \prod_i \Sigma_i \triangleq$ Mixed Strategy Profile.

$\sigma \in \Sigma.$

Utility $\equiv$ Expected Pay-off

$= \sum p_{i_j} u_i(s_{i_j}, \sigma_{-i}) = u_i(\sigma_i, \sigma_{-i})$

MIXED STRATEGY NASH EQUILIBRIUM

$$\sigma^* \in \Sigma \equiv \text{M.S.N.E.}$$

iff

$$\forall_{i \in I} \ \forall_{\sigma_i \in \Sigma_i} \ u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*)$$

- Nash Thm: ($\because$ KAKUTANI FIXED POINT THM.)

  Every finite game has a Mixed Strategy Nash Equilibrium.