SOCIAL NETWORKS

LECTURE #10

ADJACENCY MATRIX.

Every undirected graph $G=(V,E)$ with $|V|=n$ has associated with it a symmetric adjacency matrix

$$A \in \{0,1\}^{n \times n}$$

Binary $n \times n$ matrix A, in which

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ \& } v_j \text{ are adjacent} \\ & (v_i, v_j) \in E \\ 0 & \text{otherwise} \end{cases}$$

Since in an undirected graph

$$(v_i, v_j) \equiv (v_j, v_i)$$

$$\Leftrightarrow \quad a_{ij} \equiv a_{ji}$$

$$\Leftrightarrow \quad A^T = A$$

← A real-valued symmetric matrix.

Let $d_{ii} = d(v_i) = |\{v_j \mid (v_i, v_j) \in E\}|$
$= \text{Degree}.$

$$D = \begin{bmatrix} d_{11} & & & 0 \\ & d_{22} & & \\ & & \ddots & \\ 0 & & & d_{nn} \end{bmatrix} = \text{Diagonal Matrix}$$

$$\text{Trace } D = \text{Tr}(D) = \sum d_{ii}$$
$$= \sum_{v_i \in V} d_{v_i} = 2|E| = 2m.$$

<u>Boundary Matrix</u> $B \in \{-1, 0, +1\}^{m \times n}$

Columns are indexed by the vertices of G
Rows are indexed by the edges of G

$$B(e, v) = \begin{cases} +1 & \text{if } v \text{ is the head of } e \\ -1 & \text{if } v \text{ is the tail of } e \\ 0 & \text{otherwise} \end{cases}$$

Choose edge directions arbitrarily when G = undir.

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$$B^T B = \mathcal{L}$$

$$\mathcal{L}_{ii} = d(v_i) = \cancel{\sum B(v_i, e)^2} \ \sum_{e \in E} B(e, v_i)^2$$

$$\mathcal{L}_{ij} = \sum_{e \in E} B(v_i, e)\, B(v_j, e) = -a_{ij}$$

$$\therefore B^T B = \mathcal{L} = D - A.$$

$$\hookrightarrow \boxed{\text{LAPLACIAN.}}$$

Random Surfing on G

$$P(u,v) = \begin{cases} \frac{1}{d_v} & \text{if } (u,v) \in E \\ 0 & \text{otherwise.} \end{cases}$$

$$P = D^{-1} A$$

$$\mathcal{L} = D - A = D(I - D^{-1}A)$$

$$= D(I - P)$$

$$= D\Delta$$

$\Delta = I - P. \equiv$ (Discrete) Laplace Operator.

Your position within the social network assigns a social value...

$\Rightarrow$ RANK (e.g. page rank).

$f(x)$

$f : V \to R$ a scalar valued Rank function.

DIRICHLET SUM OF G.

$$\sum_{(u,v)\in E} \left(f(u) - f(v)\right)^2$$

Avoid the trival rank

$\forall x \; f(x) = \text{Const}.$

Choose ranks so that the Dirichlet sum is Minimized.

Focusing on the relative values

$$\Delta f(x) = \frac{1}{d_x} \sum_{(y,x)\in E} \left(f(x) - f(y)\right)$$

$\Rightarrow (I - P) f$

$\Rightarrow f(x) \approx \sum_{(y,x)\in E} f(y) / d_x$

Random Surfer Model.

Imagine a web surfer bouncing along randomly following the graph.

(e.g. hyperlink graph of the web).

- When a surfer arrives at a node he chooses at random hyper-links (directed edges) to a new node.
- Asymptotically the proportion of time the random surfer spends on a given $\approx$ weighted average of the proportion of time spent on neighboring vertices...

$\Rightarrow$ RELEVANCE $\propto$ time spent on a page.

(Relative Importance)...

PROBLEMS:

Dangling nodes - Sinks
+ Periodicity in the graph.

⇒ Stochastic Teleportation.

RANDOM SURFER WITH STOCHASTIC TELEPORTATION
⇒ PAGE RANK.