Social Networks
Solution to Homework 1

Q1. [10] Exercise #3 ([EK], pp 40)

When we think about a single aggregate measure to summarize the distances between the nodes in a given graph, there are two natural quantities that come to mind. One is the diameter, which we define to be the maximum distance between any pair of nodes in the graph. Another is the average distance, which Ñ as the term suggests Ñ is the average distance over all pairs of nodes in the graph. In many graphs, these two quantities are close to each other in value. But there are graphs where they can be very different.

(a) Describe an example of a graph where the diameter is more than three times as large as the average distance.

Answer: Think of a complete graph of n nodes extended by a single path with 3 nodes on it. In this graph, diameter is 4. If n is sufficiently large, the average distance will approximately equal to 1. In fact,

$$\text{Average Distance} \quad = \frac{\frac{n(n-1)}{2}+1+2+3+4*(n-1)+1+2+3*(n-1)+1+2*(n-1)}{\frac{(n+3)(n+2)}{2}} = \frac{n^2+17n+2}{n^2+5n+6}$$

If $n \geq 31$, average distance will be less than 4/3. And the diameter will be more than three times as large as the average distance.

(b) Describe how you could extend your construction to produce graphs in which the diameter exceeds the average distance by as large a factor as you'd like. (That is, for every number c, can you produce a graph in which the diameter is more than c times as large as the average distance?)

Answer: Now we have a complete graph of n nodes extended by a single path with c nodes on it.

$$\text{Average Distance} \quad = \frac{\frac{n(n-1)}{2}+\frac{c(c+3)}{2}(n-1)+\frac{c(c+1)(c+2)}{6}}{\frac{(n+c)(n+c-1)}{2}}$$

Diameter = c+1

Let Diameter/Average Distance > c, then with this formula we can change the factor as large as we like.

Q2. [5] Exercise #4 ([EK], pp 75)

In the social network depicted in Figure 3.23 with each edge labeled as either a strong or weak tie, which two nodes violate the Strong Triadic Closure Property?

Provide an explanation for your answer.

Answer: A node violates the Strong Triadic Closure Property if it has strong ties to two other nodes and there is no edge between those two nodes. The two nodes that violate the Strong Triadic Closure Property are C and E. Node C has strong ties to B and E, but there is no edge between B and E; E has strong ties to C and D, but there is no edge between C and D. Therefore, C and E violate the Strong Triadic Closure Property.

Q3. [5] Exercise #5 ([EK], pp 75)
In the social network depicted in Figure 3.24, with each edge labeled as either a strong or weak tie, which nodes satisfy the Strong Triadic Closure Property from Chapter 3, and which do not? Provide an explanation for your answer.

Answer: A node satisfies the Strong Triadic Closure Property if it doesn't violate it. C violates the Strong Triadic Closure Property because C has strong ties to A and E, but there is no edge between A and E. A,B,D,E satisfy the Strong Triadic Closure Property. A has strong ties to B and C, there is an edge between B and C. B has strong ties to A and C, there is an edge between A and C. Node D and Node E don't have two strong ties to other nodes. Therefore, A,B,D,E satisfy the Strong Triadic Closure Property and C violates the Strong Triadic Closure Property.