LECTURE #1.

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Administrivia.

HEURISTIC PROBLEM SOLVING.
CSCI-GA.2965-001.

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Text Book:

[MF] Z. Michalewicz and D. Fogel
How to Solve It: Modern Heuristics

[C] V. Chandra
Geek Sublime.

[B] J. Bentley
Programming Pearls

[MM] C. Moore & S. Mertens
The Nature of Computation.

◇ Introduction:

a) Computational Thinking:

"Stuff doing stuff to other stuff."

Examples:

Linguistics / Chomsky /

"Terminal and non-terminal symbols
doing syntactic transformations
to other terminals and nonterminal
symbols."

Quantum Mechanics / Quantum Computers / Feynman

" Entangled Quantum states (Qbits)
doing unitary transformations to
other Qbits."

System Biology / xS-systems

"Genes and proteins
doing activation and inhibition
transformations on
other genes and proteins."

Internet / Signaling Games / RV-systems.

"Senders and receivers
signaling other
senders and receivers."

user → Google → Ad-Exchange
→ Advertizer → User.

b) How do good "computation" arise and survive?

LONGEVITY → "MEME"

What does longevity depend on?

- Truth (Correctness, Unification, Efficiency)
- Beauty (Symmetry, Simplicity, Maintainability)
- Usefulness (Utility)
- Depth (Complexity)
- Interrelatedness

Truth ... Satisfeability: 2-SAT vs 3-SAT or K-SAT ($K>2$)

Beauty ... Shortest Vector: Geometry of numbers. (Crypto / Copy Number)

Usefulness ... Genome Mapping ... Optical Mapping

Depth ... Unique Games Conjecture ... MaxCut Semidefinite Relaxation.

PROBLEM 1.

SAT.

Satisfiability: (Propositional Satisfiability)

Data: $x_1, x_2, \ldots, x_n$ = Set of Boolean variables

$\neg x_i$, x_i = literals = Unnegated or negated variables.

$x_{i_1} \vee \neg x_{i_2} \vee x_{i_3}$ = Disjunction of literals

= Clause = C_j

$C_{j_1} \wedge C_{j_2} \wedge \cdots C_{j_K}$ = Conjunction of clauses

= Formual = f ← Boolean (Propositional) formula

$\mathcal{A}: \{x_1, \ldots, x_n\} \to \{T, F\}^n$ = Truth Assignment.

Desiderata: Does there exist an assignment $\mathcal{A}$ s.t.

$$\mathcal{A} \models f$$

i.e. under that truth assignment, the formula evaluates to true.

PROBLEM 2.

SVP.

Lattice: $v_1, v_2, \ldots, v_n$ = Basis for $\mathbb{R}^n$ (or $\mathbb{Q}^n$)

$$\Lambda = \left\{ \sum_{i=1}^{n} a_i v_i \mid a_i \in \mathbb{Z} \right\} = \text{Lattice}$$

N = Norm, e.g. $\mathcal{L}_2$-norm.

Data: A lattice Λ, represented by the vectors, $v_1, v_2, \ldots, v_n$.

Desiderata:

Find the shortest non-zero $v \in \Lambda$

$$\min \left\{ \|v\|_N \mid v \in \Lambda,\ v \neq 0 \right\}$$

PROBLEM 3

OPTMAP

$\mathcal{G} = [0, L]$ a genome of length L
with n ordered restriction sites
$x_1, x_2, \cdots, x_n$

$C_j \doteq$ Clone of length c, with offset o

$$\langle x_j, \ldots, x_{j+k} \rangle = [o, o+c] \cap \langle x_1, \ldots, x_n \rangle$$

Uninformative Sample (Oriented) + Noise

$$\langle \widehat{(x_j - o)}, \ldots \widehat{\langle x_{j+k} - o \rangle} \rangle = \langle \hat{x}_j', \ldots, \hat{x}_{j+k}' \rangle$$

Uninformative Unoriented Sample + Noise

$$\langle \hat{x}_j', \ldots, \tilde{x}_{j+k}' \rangle$$

or

$$\langle c - \hat{x}_{j+k}, \ldots, c - \hat{x}_j' \rangle = \langle \hat{x}_j', \ldots, \hat{x}_{j+k}' \rangle^R$$

Data: M uninformative Unoriented Sample + Noise

Desiderata: Restriction Map of the genome:

$$\langle \hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n \rangle$$

PROBLEM 4

UGC

PROMISE PROBLEM.

Label Cover with unique constraints.

Alphabet of size k, $\Sigma = [K] \to$ "Colors"

A graph $G = (V, E)$, with a collection of permutations one for each edge of the graph:

$$\pi_e : [K] \to [K]$$

An assignment gives to each vertex v of G a value in the set $[K]$.

An edge constraint is satisfied for an edge $e = \langle u, v \rangle$ by an assignment A, if $\pi_e(A(u)) = A(v)$.

Data: Given a graph $G = (V, E)$ together with the unique constraints per edge $\{\pi_e \mid e \in E\}$ such that number of edges satisfying the constraints is at least $(1-\varepsilon)$ of total number of edges $[\varepsilon > 0]$.

Desiderata: Find an assignment/labeling/Coloring of the graph s.t. at least a δ fraction of the edges satisfy the constraints.