LOGIC
HW #2
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20 November 2012 (due in 2 weeks)

Q1. [10] The axioms of PA in $\mathcal{L}_{ar} := \mathcal{L}\{0, S, +, \cdot\}$ are as follows:

$$\forall x\ Sx \neq 0$$
$$\forall x\ x + 0 = x \qquad \forall x\ x \cdot 0 = 0$$
$$\forall xy\ (Sx = Sy \rightarrow x = y)$$
$$\forall xy\ x + Sy = S(x + y) \qquad \forall xy\ x \cdot Sy = x \cdot y + x$$
$$\phi^x_0 \wedge \forall x\ (\phi \rightarrow \phi^x_{Sx}) \rightarrow \forall x\ \phi \quad (IS)$$

Prove in PA the associativity, commutativity, and distributivity of $+, \cdot$.

Q2. [10] Define $\leq$ in $\mathcal{L}_{ar}$. Derive reflexivity and transitivity of $\leq$ in PA.