LOGIC
QUIZ #8

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Once again, we return to the Island of Knights and Knaves. In this island, those called *knights* always tell the truth and *knaves* always lie. Furthermore, each inhabitant is either a knight or a knave.

Notation: K = knight. For any inhabitant x, we let Kx be the proposition that x is a knight. Then $\neg Kx$ is the proposition that x is a knave.

Q1. [10] Whenever an inhabitant x asserts a proposition $\mathcal{P}$, we know that if x is a knight then $\mathcal{P}$ is true, and if x is a knave the $\mathcal{P}$ is false — in other words, x is a knight *iff* $\mathcal{P}$ is true. Thus we translate "x asserts $\mathcal{P}$" as $Kx \equiv \mathcal{P}$. Suppose, each inhabitant asserted that all the inhabitants are of the same type: all knights or all knaves. Thus each inhabitant asserted

$$\forall x Kx \vee \forall x \neg Kx,$$

so for each x we have

$$Kx \equiv (\forall x Kx \vee \forall x \neg Kx).$$

We previously argued that, in this case, $\forall x Kx$ must hold and the following is logically valid:

$$\forall x (Kx \equiv (\forall x Kx \vee \forall x \neg Kx)) \Rightarrow \forall x Kx.$$

Give a formal proof.

Soln1. *We can use a Tableux method; start with assuming the statement* *false.*

$\mathbf{F}\ \forall x(Kx \equiv (\forall xKx \vee \forall x\neg Kx)) \Rightarrow \forall xKx$

$\rightarrow\quad \mathbf{T}\ \forall x(Kx \equiv (\forall xKx \vee \forall x\neg Kx))$

$\qquad \mathbf{F}\ \forall xKx$

$\rightarrow\quad \mathbf{T}\ \exists x\neg Kx$

$\qquad \mathbf{T}\ \neg Ka$

$\qquad \mathbf{F}\ Ka$

$\rightarrow\quad \mathbf{T}\ Ka \equiv (\forall xKx \vee \forall x\neg Kx))$

$\qquad \mathbf{F}\ (\forall xKx \vee \forall x\neg Kx))$

$\qquad \mathbf{F}\ \forall xKx$

$\qquad \mathbf{F}\ \forall x\neg Kx$

$\rightarrow\quad \mathbf{T}\ \exists xKx$

$\qquad \mathbf{T}\ \neg Ka \wedge Kb \wedge a \neq b$

$\rightarrow\quad \bot.$