

LOGIC

QUIZ #7

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Once again, we visit several Islands in the Archipelago of Knights and Knaves along with our Antropologist. In these islands, those called *knights* always tell the truth and *knaves* always lie. Furthermore, each inhabitant is either a knight or a knave.

Notation: k = knight, s = smoker.

Q1. [5] On one of these islands, all the inhabitants said the same thing: namely, "All of us here are the same type." What can be deduced about the inhabitants of that island?

Soln1. Let the statement S be $S \equiv (\forall x x = k) \vee (\forall x x = \neg k)$. Since all of them asserted S (which is either true or false), they are all of same type. Hence $S = \text{true}$ and the island's inhabitants are knights.

Q2. [5] On another of these islands, all the inhabitants said the same thing: namely, "Some of us here are knights and some are knaves." What can be deduced about the inhabitants of that island?

Soln2. Let the statement S' be $S' \equiv (\exists x x = k) \wedge (\exists x x = \neg k)$. Since all of them asserted S' (which is either true or false), they are all of same type (either all knights or all knaves). Hence $S' = \text{false}$ and the island's inhabitants are knaves.

Q3. [10] On yet another of these islands, all the inhabitants said the same thing: namely, "All knights on this island are smokers." What can be deduced about the inhabitants of that island?

Soln3. Let the statement Q be $Q \equiv \forall x (x = k) \Rightarrow (x = s)$. Since all of them asserted Q (which is either true or false), they are all of same type (either all knights or all knaves). If they are all knaves, then Q is false, i.e., $\exists x x = k \wedge x \neq s$ is true – which is impossible since there is no knight in the island. If they are all knights, then $Q = \text{true}$, and they are all knights and they all smoke.