

LOGIC

QUIZ #1

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11 September 2012

In the Island of Knights and Knaves, those called *knights* always tell the truth and *knaves* always lie. Furthermore, each inhabitant is either a knight or a knave.

Q1. [10] An anthropologist, visiting this island, came across three inhabitants, whom he called, *A*, *B* and *C*. He asked *A*: "Are you a knight or a knave?" *A* answered, but so indistinctly that the anthropologist could not understand what he said. He then asked *B*: "What did he say?" *B* replied: "He said that he is a knave." At this point, *C* piped up and said: "Don't believe that; it is a lie!"
What is *C*: a knight or a knave?

SOLN.1 Note that saying "I am a knave," is same as saying "I am a liar," which is self-referential and leads to a paradox (the so-called "Liar's paradox.") Thus *B* is lying, when he says that *A* said that he is a knave. Thus *B* is a knave and *C* is a knight. We can assume that *A* must have said "I am a knight," but that is not sufficient to determine if *A* is a knight or a knave.

Q2. [+10] In a slight variant of this story, the anthropologist asked *A* how many of the three are knaves.

Again *A* answered indistinctly. So the anthropologist asked *B* what *A* had said. *B* then said that *A* had said that *exactly two* out of the three were knaves. As before, *C* claimed that *B* was lying.

Can you now tell if *C* is a knight or a knave?

SOLN.2 First note that *B* and *C* contradict each other:

$$B \Leftrightarrow \neg C.$$

Thus only one out of the two (*B* or *C*) is a knave. Thus if *A* says "exactly two out of the three are knaves," he is claiming to be a knave, which is impossible. Thus *B* is a knave and *C* is a knight. Since *A* could not have also said that all three of them are knaves, he must have said "only one of us is a knave." It is not possible to determine if *A* is a knight or a knave.