OF MEN AND MATHEMATICS

A Plea for the Historical Sense in Mathematics

MAA Meeting, Annapolis Md.

April 27, 1963

Philip J. Davis

Professor Saslaw. Ladies and Gentlemen:

I am honored by the opportunity to address you this morning. It surely adds an additional fillip to the occasion that I am addressing you in the institution where Professor Chauvenet taught for so many years.

In accepting the Chauvenet Prize this January in Berkeley, I thought it appropriate to say a few words about the value of the study of the history of mathematics and to make a plea for humanism in mathematics. Our Chairman caught me afterwards in the corridor and asked me to elaborate these ideas at the present meeting.

After I have done my technical work for the day — which at the moment consists of reading galley proofs —I’ve been in the habit of relaxing with the lightweight scientific literature that come across my desk. This consists of trade journals, company journals, newsletters, press releases, poop sheets; I’m sure you know the type. In reading this literature, I find myself exposed to a barrage of ideas and catch words that are currently tossed around. Here are some of them: population explosion, student explosion, information explosion, revised curricula, programmed learning, intelligence amplifiers, theorem proving by machine, the computer revolution, human engineering, operations analysis, criterion formulation, black boxes, the mathematics of mass destruction, men as images, men as symbols, men as molecules, creating substances with preassigned properties, creating people with preassigned properties, communication, the analysis of genius, the scientific method, the science of science, the abundance of abundance, the abundance of non-abundance, crash programs, contracts, kilo and megabucks, challenge, target, destination, go, go, go.

This is the current song that the locusts are singing. It is exciting. It is alive. It is science at its ripest. It points outward to space and inward to the elementary particles and toward the minds of men. It points everywhere.

But it also points nowhere. It is mad, intoxicated, frightening. I is depressing. It is dehumanized. The universe bends to our will, think of it; the universe bends to our will. If the Job story is still in the theatres on Broadway this is an anachronism: get a few bugs out of the system and all should be in order. But what really strikes me when I read these catch words is not the impact of ideas — either good or bad, but the push of equipment, the push of projects, the push of money to be spent and the push of money to be made, the push of mere push.

The ancient playwright Aeschylus had a word for this: hubris, or wanton arrogance and disregard of natural restraint, and it was axiomatic with him that hubris was followed by the tragedy of nemesis, the tragedy of retributive justice.

On such a lovely morning, when the birds are singing, if we care to listen to them, it is a bit cruel of me to tune you into this other type of background music. But I do it for a very personal reason: I have recently accepted an appointment at Brown University. And this impending change of position has inevitably led me to a bit of soul searching. I go to teach, but why, what, how? What is the significance of teaching seen against the present human condition? What should be the role of the mathematical education I am prepared to give.

Am I going to prepare students for big science, science unlimited? Am I going to prepare them, as Thoreau said, to be the tools of the tools they themselves will inevitably be fashioning? For let us make no mistake about it, mathematics has contributed its share to this dehumanized state of affairs, and promises to contribute much more. It does this through an internal structure and an attitude that stresses atomized thought, and the consequences of this atomization are touching us from technology to social policy. Or, on the other hand, am I going to fashion my students into the Dharma Bums of mathematics, creating exquisite miniatures of mathematical poetry and seeking personal salvation thereby? Should my instruction be — can it possibly be — something different than either of these?

The personal world of mathematics has changed enormously in the twenty five years since I studied analytic geometry from Osgood and Graustein. In those days, I did not suspect that I would make my living from mathematics. I loved my subject, but I would have settled for working as a short order cook. Today, a good living, a fine living can be made from mathematics. Mathematics has spread from the high school, the college and the insurance company. It has spread over industry and government. It has invaded old disciplines such as economics and sociology, biology and medicine. It has created new disciplines where none existed before. If mathematics has followed the pattern of other sciences, there should be about two to three times as much theory and application available as there was when I first began to study. It doubles every fifteen years. How does one swim in such a spreading sea of mathematics? These facts I must also take into consideration when I think through the role of the teacher.

As science has doubled, it has also split. What was once called Natural Philosophy, embracing all enquiries regarding nature, is now capital N specialties, listed by the scientific manpower commissions like names in a phone book. In the wake of this fragmentation, knowledge has replaced and has been confused with wisdom, and the scientist, to quote the Spanish philosopher Ortega y Gasset had become a

“learned ignoramus, which is a very serious matter, as it implies that he is a person that is ignorant, not in the fashion of the ignorant man, but with all the petulance of one who is learned.

In our own field, the fragmentation has solidified, creating the pure mathematician and the applied mathematician. Let me caricature them, cautioning you that of course there are no pure types.

The applied mathematicians are salesmen in striped suits and polka dot ties, embracing the world. The world is a differential equation of high order — partial. Society is a matrix with complex elements. Life — is not a bowl of cherries — it’s a Markov chain. The welfare state is to be run by a minimax condition. “Hiya, honey,” the applied mathematician says to the world, “you and me can make sweet music together. And then, to mix a metaphor, it turns out that the applied mathematician isn’t quite Prince Charming and the universe isn’t quite Cinderella and when the slipper doesn’t fit, cut the heel off and let it bleed.

Then there are the pure mathematicians. Feeding on honey dew and sucking on what they consider the milk of paradise, they know that mathematics is pure pattern arising from pure mind, and they mean to keep it that way. Isolated, finicky, and suspicious, they react with proper horror at applications and bury their head in the sands of overspecialization.

I reject the first position and I reject the second. Perhaps there are other extreme positions. I can’t claim there is a two-party system at work here. But from these extremes of positions there are created one, two, three, and sometimes four departments of mathematics at the same university, departments that are mutually antagonistic and increasingly non-communicating, where specialty is pile upon specialty like the neck of a giraffe growing longer. From these extremes is created an indifference, a killing boredom that sweeps across the sands like the hot sirocco, and finds admirable expression in this joke card, that I picked up in the drug store: “I may look interested, but I’m just being polite.” From these extremes is created a Berlin wall between mathematics and the humanities.

Where does the teacher fit her? Or is this an irrelevant question? Do I gather that university education has long ceased to be Mark Hopkins at one end of the log and the student at the other? Do I read the signs of the times correctly when I think it is more likely to be Mark 84 at the end of the line and the student — or even the student’s Mark 84 at the other? A banner headline in one of my trade journals reads, “Is it farewell, dear teacher”.

Mathematics has inherited a great liberal tradition, but this tradition seems to be slipping away. The spirit of men and the spirit of machines are both fostered by mathematics. It is up to us to deal with this irony by balancing the valid claims of both. That such a balance is possible is dramatized by the long careers of Bertrand Russell and Alfred North Whitehead who contributed so much to fragmented thought and who also have been the most eloquent spokesmen of humanism for two generations. While each of us has seek his own method for restoring balance, I would like to suggest to you a way that has been very useful to me; it is through the realization that our subject is not static, and that the very knowledge of its flow through the men who have created it acts as a great humanizing influence. It is through cultivating the sense of history in mathematics.

When I speak of cultivating a sense of history in mathematics, I am not speaking of formal courses in this subject. I am not speaking either of a remark following Euler’s theorem to the effect that Euler had 13 children and was blind in one eye. Nor am I speaking of the 100 best books approach to mathematics of education. I am speaking of something far more subtle and something that can be far more pervasive. Mathematics has been spreading, less as a result of its specific theorems than as a result of its point of view and attitude. By the same token, there is a historical attitude, quite apart from the individual facts and lessons of history, which deepens one’s outlook and allows one to see where one has been and where one is likely to be in the future.

History has for its subject man in all his aspects. It is proper that the attitudes of history penetrate the study of mathematics. I cannot conceive of mathematics n the absence of man; for me, pi exists, but not in the sky. History shows there is no finality to men’s positions. History teaches modesty and open mindedness. There is a wisdom in it that can be an antidote to the condition that Ortega complains about so bitterly.

The historical sense can show how mathematics is a living thing; how it is an organism that grow daily by the accretion of new information, changes daily by regarding itself and the world from new vantage point; how it maintains a regulatory balance by consigning to the oblivion of irrelevancy a fraction of itself.

The historical sense recognizes change as inevitable, but by acting as a conservative element, can help to soften the personal blows that may accompany change. The historical sense does not throw up its hands in horror and shame at the thought of rediscovery. It knows that the world thrives on this kind of redundancy and inefficiency. There is an oak tree in my yard and one in yours. But if I followed the pattern of some of our colleagues, I would complain to nature, asking her why she had produced your oak when she had already obtained the same result in my yard.

The historical sense explores and explodes wherever necessary the notion of progress in mathematics. When you undertake a problem and solve it, that, of course, is progress. Here the stage is small and the action clear cut. But when one considers long range changes in mathematics, it is not at all clear that what we have is progress. Perhaps what appears as progress is mere transformation. A simple example is conics. The Greeks had their conics, and we, following Descartes, have ours. Our methods presumably represent progress for they are vastly more powerful. And yet, strangely, the major theorems in conics are Greek.

Or consider determinants. To Leibniz, a determinant is *ab—cd*; simple. Ten thousand papers later, we arrive at Bourbaki’s definition: “The determinant of an endomorphism *u* of an *A*-module *E* having a basis of *n* elements is that scalar *λ* such that the anti-symmetric power *Δnu* is identical with the homothetic transformation *z→ λz* of *ΔnE*. Not so simple. The *coup de grace* comes when some people advocate throwing determinants into the ashcan altogether. Progress or transformation?

The historical sense recognizes that no one owns mathematics. It is not the property of the logicians or the philosophers, nor of the physicists. It isn’t even the property of the mathematicians. It exists as a common treasure, and all are free to draw on it without diminishing it, or to add to it without crushing it. There is no right way or unique way to speak its language. There are only dialects and the muse of the subject can understand anyone who has something to say or something to ask.

The historical sense realizes that it is important for different men to play different roles or to play different roles at different times. Some of us are pumps, making available the waters for irrigation. Some of us are sluices, directing the distribution. Some of us are dams, controlling the flow. I should think that the subject could be destroyed in 5 years if all of us sought to be pumps all of the time.

The historical sense leads us to the perception of creativity. How do ideas unfold? In what way are they related to one another, in what ways to human activity? The dimension of time is crucial here. The Elements of Euclid may be a model of mathematics as a deductive science, but in one respect the book is false. It lacks the time dimension. It presents the completed present and there is no suggestion of creativity in it. It is frozen material, a snapshop taken at 1/1000 of a second.

Yet, as we have agreed, mathematics is not a static subject. The simplest of its elements is in constant flux, and this flux is, itself, part of mathematics. “There is nothing less real than the present, wrote art historian Erwin Panofsky. “To grasp reality, we have to detach ourselves from the present.” To grasp creativity, we must equally detach ourself from the present.

But why bother with creativity in mathematics? Why not content ourselves with transmitting the perfected present? One reason has been given by our colleague R.C. Buck in an essay on teaching machines: “To look at mathematics without the creative side of it is to look at a black and white photograph of a Cézanne; the outlines may be there, but everything that matters is missing. To put this another way, the perception of creativity is what breathes life into the inert symbols on a page, what relates men to the deposits of ink they make.

\*\*\* \*\*\* \*\*\*

How can we imbue ourselves with a sense of history so that the consciousness of mathematical development is implicit in all our teaching? I have no formulas, just a number of passing thoughts.

First of all, a number of good and up to date book on the history of mathematics should be written. What is available can be counted on the fingers, and is not all good.

Running counter to this demand is the feeling that such things are vastly unimportant. The attitude of “getting down to brass tacks” say: what really counts is the perfected present. Another arguments says that, in order to write a good history, a person has to be a creative mathematician, i.e. a pump. If one writes history, one must stop pumping, and that would be a shame. The same argument has been used to explain a shortage of science administrators, text book writers, teachers, etc. I suggest that all of us have a duty to replace at the root for what we have received from the brahcn; the science has a right to claim a few man years now and again and dedicate them to such a purpose.

As preliminary to large scale studies of history, it would be time well spent, if our teachers and researchers were occasionally to do a historical study of some major idea of mathematics. Surely more might be learned in the long run from such a study than from “just another research paper of the usual kind.”

I am in favor of the proposed doctor of arts degree in mathematics that has been under discussion for a number of years. If properly supervised and administered, it could be a very good thing.

In a somewhat different vein — I have noticed at large technical talks how alive the audience is when the speaker talks about the origins of a problem tracing its genealogy back a few years. How glassy-eyed they become when the δ’s and ε’s appear. I notice, though, that even in dealing with difficult technical material it is possible to recapture some of the electricity of discovery if the discovery is presented as a creative act and not as a *fait accompli*.

But what I am really after does not lie in mathematics at all. What I am after, as an ideal goal, is that each of us shall have read as deeply in history, in literature, in art, and philosophy as, say, Russell and Whitehead. I am after more humanities for the scientist and not less.

Some mathematics teachers to whom I have suggested this have replied with a groan, “But they are asking us to teach more science, more mathematics, not less.” To this I reply that mathematics has a way of binding by abstraction diverse branches so that what was formerly taught in 3 or 4 hours can now be taught in one hour. Moreover, much mathematics becomes irrelevant as far as applications are concerned. B.O. Pierce’s Table of Integrals or its modern electronic equivalent, can also be a great time saver.

Some teachers of humanities to whom I have suggested this have replied, “But we are looking to the scientists for direction.” Perhaps they have become discouraged by the current state of art, honestly empty, or the current state of the novel, jaded and ill at ease with the civilization it describes. To these people I say: I am looking for a synthesis. It is as important to me to know what you have been thinking about and what you have considered worthwhile, as it is the other way around.

In conclusion, I will try to steal a bit of Frank Sinatra’s Academy Award thunder, and recall to your minds a picture. Not the Mona Lisa, but Michaelangelo’s magnificent fresco, “The Creation of Adam”. To the right is a philosophic god. To the left is Adam, supine, but potentially powerful. God and Adam have outstretched arms and are fingertip to fingertip. Adam waits for the divine flow that will make him human.

I would like to suggest this picture as symbolic of the proper role of teacher and student, symbolic also of the true structure of mathematics. The pupil awaits instruction from the teacher. This instruction includes elements that are intellectual, emotional, historical, and philosophical.

In view of this, how can it be “Goodbye, Mr. Chips!”

It must be “Hello, Mr. Adam!”

Bibliography

Bertrand Russell, “History as an Art”, pp. 176-193 of *Portraits from Memory and Other Essays*, George Allen and Unwin, London, 1956.