Homework 4 - Discrete Math - Due 7/31/19

Assigned: 07/23/2019
Due: 07/31/19

Please make sure to clearly write your name at the top of your hand-in. Also, indicate if you worked with anybody and also indicate how many hours total you worked on the homework. This looks like more homework than it is since many problems are quite simple and others have solutions in the back. Feel free to discuss any problems (including the bonuses) on the class mailing list. I am also required to remind all students of the academic integrity policy at http://www.cs.nyu.edu/web/Academic/Graduate/academic_integrity.html. Any violations of this policy may result in failure of the course and being reported to the head of the department.

Problem 1
As in class, we discussed Russell’s Paradox, so this should be simple review, but I want you to be able to put the answer into plain English. Let $S$ be a set that contains all sets $X$ such that $X \notin X$. That is, $S = \{X | X \notin X\}$.

a) Show why assuming $S \in S$ leads to a contradiction.
b) Show why assuming $S \notin S$ leads to a contradiction.

Problem 2 Prove the following via direct proof. That is, show that each side of the equation is a subset of the other side (two cases) using an arbitrary particular element $x$ as we did in class for DeMorgan’s Law for two sets.

$$A^c \cup B^c \cup C^c = (A \cap B \cap C)^c$$

Problem 3 Show how the identity above in the prior problem can be proved using two steps of DeMorgan’s Law along with some other basic set rules (An algebraic proof).

Problem 4
Draw Venn diagrams for the following combinations of sets $A$, $B$, and $C$.

a) $A \cap (B \cup C)$
b) $A^c \cap B^c \cap C^c$
c) $(A - B) \cup (A - C) \cup (B - C)$
Problem 5
What can you conclude about $A$ and $B$ if the following are true?

a) $A \cup B = A$
b) $A - B = A$
c) $A \cap B = B \cap A$
d) $A - B = B - A$
e) $A \cap B = A$

Problem 6
Find the sets $A$ and $B$ if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

Problem 7
Determine which of the functions are bijective from the reals to the reals. To do this, first prove (or disprove) they are one-to-one, and then prove (or disprove) they are onto.

a) $f(x) = -3x + 4$
b) $f(x) = -3x^2 + 7$
c) $f(x) = \frac{x + 1}{x^2 + 2}$
d) $f(x) = x^5 + 1$

Problem 8
Repeat problem 7, but determine if the functions are bijective from the integers to the integers.

Problem 9
Prove or disprove the following statements concerning compositions functions for functions $f$ and $g$

a) The composition of two one-to-one functions ($f$ and $g$) is on-to-one.
b) The composition of two onto functions ($f$ and $g$) is onto.

Problem 10
Evaluate the following:

a) $\log_4 16$
b) $\log_2 16$
c) $\log_{\frac{1}{3}} 25$
d) \( \log_4 81 \)
e) \( \log_2 \frac{27}{8} \)
f) \( \log_{10} 1000 \)
g) \( \log_{10} \frac{1}{100} \)
h) \( \ln \frac{e}{2} \)
i) \( \ln \sqrt{e} \)
j) \( \log_8 1 \)

**Problem 11**
Simplify/expand the following:

a) \( \log_4 (x^3 y^5) \)
b) \( \log_{10} (\frac{x^2 y^3}{z^2}) \)
c) \( \ln \sqrt{xy} \)
d) \( \log_x (\frac{(x+y)^2}{x^2+y^2}) \)

**Problem 12**
Bring the following into a single logarithm function

a) \( 7 \times \log_{12} x + 2 \times \log_{12} y \)
b) \( 3 \times \log_{10} x - 6 \times \log_{10} y \)
c) \( 5 \times \ln(x + y) - 2 \times \ln y - 8 \times \ln x \)

**Problem 13**
Evaluate the following. To do this, show how to convert to a commonly-used logarithm (base-10, base-2 or natural ln) and then evaluate with a calculator. Please show your work.

\( \log_{5} 72 \)

**BONUS Problems**

I) You go to a party to take a break from all of your Discrete Math homework and drink away your math blues. Unfortunately, at the party you cannot shake your desire to do some problem-solving (and who can blame you when it’s so fun!!). You notice that everyone at the party has shaken hands with three other people except for you - you’ve shaken the hands of only one other person (the host). You start to wonder:

What is the smallest number of people that could be at the party?
Could 21 people be at a party such as this?
Is there a pattern for how many people could be at this party?
Is doing math problems resulting in nobody wanting to shake your hand?