Problem 1
A bowl contains 1000 blue balls and 1000 red balls.

a) How many must be chosen before you have three of the same color?

b) How many must be chosen until you have 10 of the same color?

c) How many must be chosen until you have at least 3 blue balls and at least 3 red balls?

Problem 2
If there are nine students in a class, show that at least 5 must be male or at least 5 must be female. Also, show that at least three are male or at least 7 are female.

Problem 3
Prove that at a party with at least two people, that there are two people who know the same number of people there (not necessarily the same people - just the same number) given that every person at the party knows at least one person. Also, not that nobody can be his or her own friend. You can solve this with a tricky use of the Pigeonhole Principle.

Problem 4 There are six different airlines that fly from New York to Denver and seven that fly from Denver to San Antonio. How many different possibilities
are there for a trip from New York to San Antonio via Denver, when an airline is picked for the flight to Denver and an airline is picked for the continuing flight to San Antonio?

**Problem 5** How many license plates can be made considering that there are 50 different states in the U.S., assuming that each plate can only be made using either three letters followed by three digits or four letters followed by two digits?

**Problem 6** Every student in a Discrete Math class is either a computer science major or a mathematics major or is a double major in Math/C.S.. If there are 38 C.S. majors, 23 Math majors, and 7 double majors in the class, how many students are there total?

**Problem 7** A professor writes 40 true/false questions for a test, and 17 of them are true. If the questions can be positioned in any order, how many different answer keys are possible?

**Problem 8** Seven women and nine men are on the faculty in the Mathematics department at a school. How many ways are there to select a committee of five members of the department if at least one must be a woman and at least one must be a man?

**Problem 9** How many strings of length 10 consisting of lower-case letters from the English alphabet contain the letters a and b, where a is somewhere to the left of b in the string, with all the letters distinct?

**Problem 10** Find the coefficient of $x^5 y^8$ in the expansion of $(x + y)^{13}$.

**Problem 11** Prove the Binomial Theorem using Mathematical Induction. This should be easy to locate online or in a book, so make sure you understand the proof, and cite your source if you found the proof elsewhere.
Problem 12  A croissant shop has plain, cherry, chocolate, almond, apple, cheese, and broccoli croissants. How many ways are there to choose two dozen croissants with at least one plain, at least two cherry, at least three chocolate, at least one almond, at least two apple, and no more than three broccoli croissants?

Problem 13  How many ways are there to distribute 12 identical balls into six bins, numbered one through six (the balls are indistinguishable, but the bins are distinguishable)?

Problem 14  How many ways are there to distribute five distinguishable balls into three indistinguishable bins?

Problem 15  Which is more likely probability-wise: rolling a total of 8 when two dice are rolled, or rolling a total of 8 when three dice are rolled?

Problem 16  From a deck of randomly well-shuffled 52 cards, what is the probability of getting a full-house (three of a kind plus two of a kind)?

Problem 17  If we randomly select a permutation of the 26 lowercase letters of the English alphabet, what is the probability that the first 13 letters of the permutation are in alphabetical order? What is the probability that $z$ precedes both $a$ and $b$?

Problem 18  What is the conditional probability that a family with two children has two boys, given they have at least one boy? Assume that each of the four possibilities (BB, BG, GB and GG) are equally likely).

Problem 19  Find the probability that a family with five children does not have a boy, if the sexes of the children are independent of one another, and if:

- A boy and a girl are equally likely;
- The probability of having a boy is 0.51;
• The probability that the $i^{th}$ child is a boy is $0.51 - \frac{i}{100}$.

**BONUS Problems**

A) How many ways can you spell "WAS IT A CAT I SAW" from the following:

B) In a laser fight, you shoot with 33% accuracy, player B shoots with 50% accuracy, and player C shoots with 100% accuracy. The player with the worst accuracy (you) gets to shoot first (there's nothing to hide behind here), followed by player B and then C. Once a player is "shot", they are out. The game continues in this fashion each round (worst percentage shooter shoots first, etc.) until one player is left standing. So for example, if you shoot at player C and miss (2/3 probability this happens), player B shoots at you and hits (1/2 probability) and the player C aims at B and hits (1/1 probability), the probability of this outcome would be $(2/3)(1/2)(1/1) = (1/3)$. So there is a (1/3) chance of this outcome. What is the best strategy for you to possibly become the winner?