Numerical Random Variables:

exercises

1 Discrete random variables

Exercise 1 We roll a fair die. Let \( X \) be the random variable taking the obtained result. Which is the distribution of \( X \)? Compute its expected value and its variance.

Exercise 2

A drug maker decides to save money on the advertising mails. For this, 1 mail over 3 is franked at the “urgent” rate, 2 mails over 3 are franked at the “normal” rate. Mails franked at the urgent rate are chosen randomly.

1. For a given mail, let \( X \) be the random variable taking value 0 if it is franked at normal rate, 1 otherwise. What is the law of \( X \)? what is its expected value, its variance?

2. Four mails are sent to a medical center of four physicians. Let \( Y \) be the numerical random variable taking the value of the number of mails urgent received by the medical center.

2. a/ What is the law of \( Y \)? What is its expected value? What is its standard deviation?

2. b/ Let \( A \) be the event “at least one physician receive a mail at rate urgent” and \( B \) be the event “exactly two physicians receive a mail at rate urgent”. Compute the probability of events \( A \) and \( B \).

3. The drug maker sends 100 mails. Give the probability that 56 are sent at urgent rate.

Exercise 3

A flight has 120 places. According to statistics, 18% of customers that booked do not take the flight. The flight is fully booked. Let \( X \) be the random variable that takes the value of the number of passengers taking the flight. What is the law of \( X \)? its expected value?

Exercise 4
The final exam of Mathematical Techniques for Computer Science Applications will be graded over 60. Suppose (it will certainly not be the case) it consists of 30 questions. For each question, 3 possible answers are proposed, only one is true. A correct answer give 2 points. For each wrong answer, 1 point is taken from the final grade.

A student answers randomly to each question. Let $X$ be the random variable taking the value of the final grade of this student. What is the expected value of $X$?

*Hint:* Let $Y$ be the random variable equal to the number of correct answers. Express $X$ in function of $Y$.

## 2 Continuous random variables

### Exercise 5

A firm constructs metal balls having a theoretic diameter of $8\text{mm}$, but the fabrication process yields some errors. The clients accept balls having diameter between $7.97\text{mm}$ and $8.03\text{mm}$. The firm wants to estimate the ratio of balls that will be accepted. For this, the firm decides to constructs 100 balls, and to measure on this sample the mean $\mu$ and the standard deviation $\sigma$ of the diameters. The obtained values are $\mu = 8\text{mm}$, $\sigma = 0.02$. Engineers decide that the diameter $X$ of a ball follows a Gaussian law.

1. Give an expression of $\hat{P}(X = 8)$.
2. Give an expression of $P(X \leq 7.97)$.
3. Give an expression of $P(X \geq 8.03)$.
4. Give the ration of accepted balls.

*Hint:* use the function `erf` of `Matlab` to evaluate the cumulative distribution function $P(X \leq t)$ for a given $t$. 
