Homework 1 - Discrete Math - Due 06/21/16

Assigned: 06/06/16
Due: 06/21/16

Please make sure to clearly write your name at the top of your hand-in. Also, indicate if you worked with anybody and also indicate how many hours total you worked on the homework. This looks like more homework than it is since many problems are quite simple and others have solutions in the back. Feel free to discuss any problems (including the bonuses) on the class mailing list. I am also required to remind all students of the academic integrity policy at `http://www.cs.nyu.edu/web/Academic/Graduate/academic_integrity.html`. Any violations of this policy may result in failure of the course and being reported to the head of the department.

Reading

- Read Chapter 1 for general review (this is more of an intro to topic)
- Read Sections 2.1-2.3 and 2.5 for content (read this with more focus), section 2.4 for general interest if time allows.
- Read Sections 3.1-3.4

Problem 1

Let p,q *and* r be the propositions:
p: You have the flu
q: You miss the final exam
r: You pass the course
Express each of the following as a sentence in English:

a) $p \rightarrow q$
b) $q \rightarrow p$
c) $\neg q \leftrightarrow r$
d) $p \vee q \vee r$
e) $(p \rightarrow \neg r) \wedge (q \rightarrow \neg r)$
f) $(p \wedge q) \vee (\neg q \wedge r)$

Problem 2

Let p,q *and* r be the propositions:
p: You get an A on the final exam
q: You do every exercise in the book
r: You get an A in the class

Write each of the following using p, q and r:

a) You get an A in the class, but you do not do every exercise in the book
b) You get an A on the final, you do every exercise in the book, and you get an A in the class
c) To get an A in the class, it is necessary for you to get an A on the final
d) You get an A on the final, but you don't do every exercise in the book. Nevertheless, you get an A in the class
e) Getting an A on the final exam and doing every exercise in the book is not sufficient for getting an A in the class
f) You will get an A in the class if and only if you either do every exercise in this book or you get an A on the final

Problem 3 State the converse, contrapositive and inverse of the following sentence: *If it snows tonight, then I will stay home.* Explain why the contrapositive is equivalent to the original statement. Also, explain why the converse and inverse are not equivalent to the original statement, but why they are equivalent to each other.

Problem 4
Construct a truth table for the following statements:

a) $(p \wedge q) \vee \neg r$
b) $(p \rightarrow q) \vee (\neg p \rightarrow r)$

Problem 5
Construct truth tables to verify the following associative, distributive laws, and De Morgan's laws:

a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$
b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
c) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Problem 6 Use a truth table to verify the following implication is a tautology:

$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Problem 7 For each of the following sets of premises, what relevant conclusion(s) can be reached? Explain which rules of inference are used.

a) "If I play hockey, then I am sore the next day", "I use the whirlpool if I am sore", "I did not use the whirlpool"

b) "I am dreaming or hallucinating", "I am not dreaming", "If I am hallucinating, I see elephants smoking"

Problem 8 Five friends enjoy IMing with each other, and you want to determine who is currently IMing, given the following information. Either K or H , or both are IMing. Either R or V, but not both are IMing. If A is IMing, so is R. V and K are are either both IMing are neither is. If H is IMing, then so are A and K. What can you conclude?

Problem 9 Here is a puzzle by Lewis Carroll. What is the conclusion of the following premises?

(a) No interesting poems are unpopular among people of real taste.
(b) No modern poetry is free from affectation.
(c) All your poems are on the subject of soap-bubbles.
(d) No affected poetry is popular among people of real taste.
(e) No ancient poem is on the subject of soap-bubbles.

Problem 10 Convert the following numbers from decimal to binary notation.

a) 231
b) 4532
c) 10101

Problem 11 Convert the following numbers from binary to decimal notation.

a) 11011
b) 1110111110
c) 1010110101
d) 11111000001111

Problem 12 Integers can be represented as **one's complement**s to simplify computer arithmetic. To present positive and negative integers less than 2^{n-1}, a total of n bits are used: The left-most bit is used to represent the sign. That is, a zero in this position implies a positive integer while a 1 represents a negative integer. For positive integers, the remaining bit positions are identical to a normal binary expansion. For negative integers, we first find the binary

expansion of the absolute value of the integer, and then take the complement of each of the bit positions(1 becomes a 0, and 0 becomes a 1), except for the leftmost.

A similar representation is the **two's complement** representation of integers (more commonly used). To represent an integer x where $-2^{n-1} \leq x \leq 2^{n-1} - 1$, n bit positions are used. The leftmost bit again represents the sign, where a 0 implies a positive integer, and a 1 represents a negative integer (same as for one's complement). For a positive integer, the remaining bits are the same as for a normal binary expansion. For a negative integer, the remaining bits are the binary expansion of $2^{n-1} - |x|$.

Use the above information to answer the following questions:

a) Fine the one's complement and two's complement of the following integers: 22, 31, -7, -19

b)What integer do the following complement representations of length five represent? Answer for both one's and two's complement expansions:

11001, 01101, 10001, 1111

c) How is the one's complement representation of the sum of two integers obtained from the one's complement representations (that is, given a one's complement representation for integers x_1 and x_2, how do we obtain the one's complement representation for $x_1 + x_2$ without converting x_1 or x_2 to integers)? Also, answer this question for two's complement.

d) Do some research on the Internet and figure out where and when two's complements are actually used. You need not write anything up for this (it's for your own edification).

Problem 13

Determine the truth value of the following statements:

a) $\exists x(x^2 = 2)$
b) $\forall x(x^2 + 2 \geq 1)$
c) $\forall x(x^2 \neq x)$

Problem 14 For the following propositions, write them using quantifiers, then express the negation using quantifiers, and express the negation in English.

a) Some drivers do not obey the speed limit.
b) All Swedish movies are serious.
c) There is noone in the class who does not have a good attitude.
d) There is no dog that can read.

Problem 15 Determine if the following are logically equivalent and explain why (or why not).

a) $\exists x(P(x) \vee Q(x))$ and $\exists x P(x) \vee \exists x Q(x)$
b) $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$
c) $\forall x(P(x) \vee Q(x))$ and $\forall x P(x) \vee \forall x Q(x)$
d) $\exists x(P(x) \wedge Q(x))$ and $\exists x P(x) \wedge \exists x Q(x)$

Problem 16

Let $F(x, y)$ be the statement "x can fool y" where the domain D consists of all people on Earth. **First**, use quantifiers to express the following statements. **Second**, write the negation of the quantified statements using quantifiers. **Third**, express the negation of the statements below in English.

a) Everybody can fool Fred.
b) Evelyn can fool everybody.
c) Everyone can fool someone.
d) There exists noone who can fool everyone.
e) Everybody can be fooled by somebody.
f) Someone can fool Fred or Jerry but nobody can fool both Fred and Jerry.
g) Nancy can fool exactly two people.
h) There is exactly one person who can be fooled by everybody.
i) Nobody can fool themselves.
j) There is someone who can fool exactly one person and that person can fool exactly one other different person.

Problem 17

Determine the truth value of the following statements if the domain is the set of real numbers $\mathbf{R}$

a) $\forall x \exists y(x^2 = y)$
b) $\forall x \exists y(x = y^2)$
c) $\exists x \forall y(xy = 0)$
d) $\exists x \exists y(x + y \neq y + x)$
e) $\forall x(x \neq 0 \rightarrow \exists y(xy = 1))$
f) $\exists x \forall y(y \neq 0 \rightarrow xy = 1)$
g) $\forall x \exists y(x + y = 1)$
h) $\exists x \exists y((x + 2y = 2) \wedge (2x + 4y = 5))$
i) $\forall x \forall y \exists z(z = \frac{x+y}{2})$

Problem 18 Are the following statements true? If so, state generally why (do not try to prove it). If not, prove why they are false via counterexample. Remember, that to show something is false for a "for all" type statement, you

can take the negation of the statement to get a "there exists" statement and find a counterexample.

a) $\forall x \exists y (x = \frac{1}{y})$
b) $\forall x \forall y (x^2 \neq y^3)$
c) $\forall x \forall y ((x^2 = y^2) \rightarrow (x = y))$
d) $\forall x \exists y ((y^2 - x) < 100)$

Problem 19 If we were trying to show that $\forall x (P(x) \vee Q(x))$ is equivalent to $(\forall x P(X) \vee \forall x Q(X))$, is the following proof valid (Hint: No since they are not equivalent). Find the error or errors

1. $\forall x (P(x) \vee Q(x))$ (Premise)
2. $P(c) \vee Q(c)$ for some value c (Universal Instantiation (1))
3. $P(c)$ (Simplification from (2))
4. $\forall x P(x)$ (Universal Generalization from (3))
5. $Q(c)$ (Simplification from (2))
6. $\forall x Q(x)$ (Universal Generalization from (5))
7. $(\forall x P(x) \vee \forall x Q(x))$ (Conjunction from (4) and (6))

Problem 20 Given the following sets of premises, what can you you infer or conclude. What rules of inference did you use?

a) All men are mortal. Socrates is a man.
b) No man is an island. Manhattan is an island.
c) All insects have 6 legs. Dragonflies are insects. Spiders do not have 6 legs. Spiders eat cheeseburgers.

Problem 21 Prove the following statements via direct proof (using the construction methods we covered in class - you can use the sum of two even numbers being even as an example guide).

a) The negation of an odd integer is an odd integer.
b) The product of two odd integers is odd.

BONUS Problems Try working on these to the best of your ability

I) The following puzzle is attributed to Einstein and is known as the *zebra puzzle*:

Five men from different countries and different jobs live in consecutive houses on the same street. These houses are painted different colors, and the men have different types of pets and different favorite beverages. Determine who owns a zebra and whose favorite beverage is seltzer given the following: The Englishman lives in the red house, the Spaniard owns a dog, the Japanese man is a painter, the Italian drinks tea, the Norwegian lives in the first house on the left, the green house is immediately to the right of the white one, the photographer breeds snails, the diplomat lives in the yellow house, milk is the favorite drink of the person living in the middle house, the owner of the green house drinks coffee, the Norweigan's house is next to the blue one, the violinist drinks O.J., the fox is the pet in a house next to the physician's house, and a horse is the pet in a house next to the diplomat.

II) Do you think the following argument is valid?

"If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent. If he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist."

State which rules of inference you use, and use p, q, r variables to verify your claim.

III) Here's one that's somewhat unrelated, but it's good for your brain skills: Imagine you have two hourglasses which can tell you when 4 minutes have passed and when 7 minutes have passed. However, you want to know when 9 minutes have passed. Sitting there with your 7 and 4 minute hourglasses, how will you know when 9 minutes have passed? Draw pictures if you need help explaining how you do it.

IV If I haven't already, I will show you guys how to solve the mouse/cat problem with 100 mice and two different colored hats. Now, assume there are n mice and k different colored hats. Can you generalize a solution for the minimum number of mice saved for arbitrary n and k?

V You want to move into a new apartment, but you don't get paid for a month, so you cannot put down a security deposit. Fortunately, you have a bar of gold that is 31 cm long (it is very thin, or your apartment is very expensive) and worth one month's rent, so you make a deal with the landlord. You will give her 1cm of your gold everyday as a deposit for each day. However, every time you cut the bar, it costs you 5 bucks, so you want to cut it as little as possible. Your landlady suggests that you give her 1cm on the first day, 1cm on the second day, and on the third day, you give her one 3cm piece, and she

will give you back the two 1cm pieces. On the fourth day you won't have to do any cuts since you will have the two 1cm pieces. But, by the sixth day, you would have to cut again. Obviously, you want to minimize the number of cuts to save money, but you want to make sure you add 1cm to your landlord's pile everyday. Assuming she holds onto all of the gold pieces you give her (so you can trade back and forth), what is the smallest number of cuts you will have to make for the 31 days?