Homework 3 - Discrete Math - Due 07/07/15

Assigned: 07/07/15
Due: 07/21/15

Please make sure to clearly write your name at the top of your hand-in. Also, **indicate if you worked with anybody and also indicate how many hours total you worked on the homework**. Feel free to discuss any problems (including the bonuses) on the class mailing list. I am also required to remind all students of the academic integrity policy at `http://www.cs.nyu.edu/web/Academic/Graduate/academic_integrity.html`. Any violations of this policy may result in failure of the course and being reported to the head of the department. **Not acknowledging collaboration or sources for answers may result in failure of homework or the class along with possible additional review or dismissal by the Department and University**.

Problem 1

As in class, we discussed **Russell's Paradox**, so this should be simple review, but I want you to be able to put the asnwer into plain English. Let S be a set that contains all sets X such that $X \notin X$. That is, $S = \{X | X \notin X\}$.

a) Show why assuming $S \in S$ leads to a contradiction.
b) Show why assuming $S \notin S$ leads to a contrdiction.

Problem 2a Prove the following via direct proof. That is, show that each side of the equation is a subset of the other side (two cases) using an arbitrary particular element x as we did in class for DeMorgan's Law for two sets.

$A^c \cup B^c \cup C^c = (A \cap B \cap C)^c$

Problem 2b Show how the identity above in 2a can be proved using two steps of DeMorgan's Law along with some other basic set rules (An algebraic proof).

Problem 3

Draw Venn diagrams for the following combinations of sets A, B, and C.

a) $A \cap (B \cup C)$
b) $A^c \cap B^c \cap C^c$
c) $(A - B) \cup (A - C) \cup (B - C)$

Problem 4

What can you conclude about A and B if the following are true?

a) $A \cup B = A$
b) $A - B = A$
c) $A \cap B = B \cap A$
d) $A - B = B - A$
e) $A \cap B = A$

Problem 5

Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

Problem 6a

Determine which of the functions are bijective from the reals to the reals. To do this, first prove (or disprove) they are one-to-one, and then prove (or disprove) they are onto.

a) $f(x) = -3x + 4$
b) $f(x) = -3x^2 + 7$
c) $f(x) = \frac{x+1}{x+2}$
d) $f(x) = x^5 + 1$

Problem 6b

Repeat problem 6a, but determine if the functions are bijective from the integers to the integers.

Problem 7

Prove or disprove the following statements concerning compositions functions., for functions f and g

a) The composition of two one-to-one functions (f and g) is on-to-one.
b) The composition of two onto functions (f and g) is onto.

Problem 7

A bowl contains 1000 blue balls and 1000 red balls.

a) How many must be chosen before you have three of the same color?
b) How many must be chosen until you have 10 of the same color?

c) How many must be chosen until you have at least 3 blue balls and at least 3 red balls?

Problem 8

If there are nine students in a class, show that at least 5 must be male or at least 5 must be female. Also, show that at least three are male or at least 7 are female.

Problem 9

Find the least number of cables required to connect 8 computers to 4 printers such that four computers can directly access four different printers (assume you are not on a network, and a printer can only be accessed if connected to directly). Justify your answer.

Problem 10

Prove that at a party with at least two people, that there are two people who know the same **number** of people there (not necessarily the same people - just the same number) given that every person at the party knows at least one person. Also, not that nobody can be his or her own friend. You can solve this with a tricky use of the Pigeonhole Principle.

BONUS Problems

A)You go to a party to take a break from all of your Discrete Math homework and drink away your math blues. Unfortunately, at the party you cannot shake your desire to do some problem-solving (and who can blame you when it's so fun!!). You notice that everyone at the party has shaken hands with three other people except for you - you've shaken the hands of only one other person (the host). You start to wonder:

What is the smallest number of people that could be at the party?

Could 21 people be at a party such as this?

Is there a pattern for how many people could be at this party?

Is doing math problems resulting in nobody wanting to shake your hand?

B)You want to move into a new apartment, but you don't get paid for a month, so you cannot put down a security deposit. Fortunately, you have a bar of gold that is 31 cm long and worth one month's rent, so you make a deal with the landlord. You will give her 1cm of your gold everyday as a deposit for each day. However, every time you cut the bar, it costs you 5 bucks, so you want to cut it as little as possible. Your landlady suggests that you give her 1cm on the first day, 1cm on the second day, and on the third day, you give her one 3cm

piece, and she will give you back the two 1cm pieces. On the fourth day you won't have to do any cuts since you will have the two 1cm pieces. But, by the sixth day, you would have to cut again. Obviously, you want to minimize the number of cuts to save money, but you want to make sure you add 1cm to your landlord's pile everyday. Assuming she holds onto all of the gold pieces you give her (so you can trade back and forth), what is the smallest number of cuts you will have to make for the 31 days?

C) 8 players want to play chess against each other in order to rank who is the best. It is assumed if player A beats player B and B beats C then A wo uld beat C. Playing one match takes 1 hour and players can play several matches in a row if need be (they are very strong mentally). Overall, you want to rank them from best to worst in 20 hours or less, but you only have one table. Can you do it? Can you do it in less than 18 hours? How about less than 17 hours? The chess gods have come to the rescue and delivered 3 more tables, so now you have 4 tables. Can you rank the players in 6 hours or less? (The answer is yes - come up with the solution). One 6-hour solution has 2 only tables being used during two of the hours. Is there any way to rank the players in 5 hours? (As far as I know, this problem is unsolved, so don't go crazy).