Problem Set 5

Assigned: June 24
Due: July 1

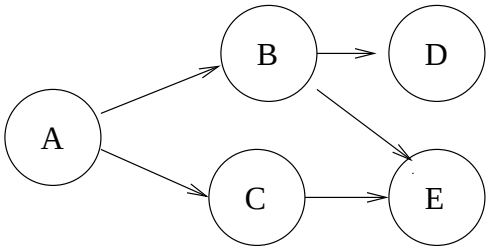
Problem 1.

Let G be a DAG. A vertex in G is a *sink* if it has no outarcs. A *forward path* from vertex U is a path that ends in a sink. Vertex V is a *terminus* of vertex U if V is a sink and there is a path from U to V.

A. Construct an algorithm `NumForPaths(G)` that computes the number of forward paths from every node in DAG G in linear time. If U is itself a sink, then `NumForPath(G)[U]` should be 1.

B. Construct an algorithm `NumTerminus(G,U)` that computes the number of terminuses for vertex U in DAG G in linear time.

For instance in the following graph G



`NumForPaths(G)[A]` = 3: A→B→D; A→B→E; and A→C→E
`NumTerminus(G,A)` = 2: D and E.

Problem 2

(Siegel). Write an algorithm that takes a DAG G as input and prints out *all* the possible topological sorts of G. For instance, given the graph in problem 1, the algorithm would output

```
A,B,C,D,E
A,B,C,E,D
A,B,D,C,E
A,C,B,D,E
A,C,B,E,D
```

It should print out each sort only once. Your algorithm does not have to produce the sorts in this order.

Problem 3

(Siegel). To see how essential marking is for graph traversal, consider the application of depth-first-traversal on a DAG G where node marking is not used. That is, the DFS code is rewritten in the form

```
procedure DFS(v) {
   for (each outarc v --> w) DFS(w)
}
```

Consider the complete DAG on n vertices; that is, the vertices are numbers $1 \ldots n$ and there is an arc from i to j for every pair $i < j$. What is the running time of this modified DFS on that graph?

Problem 4.

Let G be a DAG where the vertices are labelled with numerical values.

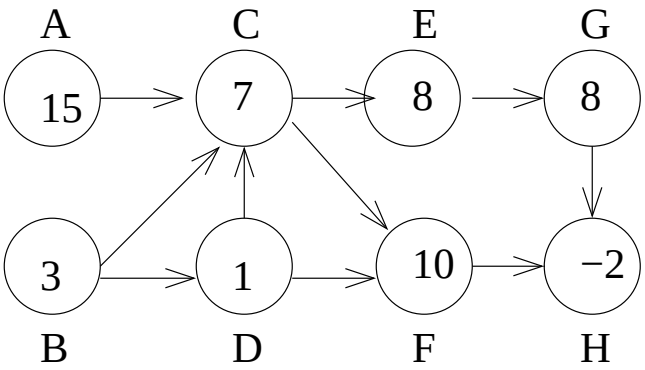
A. Write a function `MaxReachable(u)` which returns the maximum label on a vertex reachable from vertex `u` (including `u` itself.)

B. Write a function `TotalReachable(u)` which returns the sum of the labels on vertices reachable from vertex `u`.

C. Write a function `MaxPathFrom(u)` which returns the maximum sum of the labels on any path starting at `u`.

All these should run in time linear in the size of G.

For example, in the graph below:



`MaxReachable`(B) = 10 (corresponding to F).
`TotalReachable`(B) = 35 (B+C+D+E+F+G+H)
`MaxPathFrom`(B) = 27 (corresponding to B-D-C-E-G).