Homework 3 - Discrete Math - Due 07/16/2014

Assigned: 07/08/2014
Due: 07/16/2014

Please make sure to clearly write your name at the top of your hand-in. Also, indicate if you worked with anybody and also indicate how many hours total you worked on the homework. This looks like more homework than it is since many problems are quite simple and others have solutions in the back. Feel free to discuss any problems (including the bonuses) on the class mailing list. I am also required to remind all students of the academic integrity policy at `http://www.cs.nyu.edu/web/Academic/Graduate/academic_integrity.html`. Any violations of this policy may result in failure of the course and being reported to the head of the department.

Problem 1
Prove the following two propositions using a contradiction approach:

a) For any integer a and any integer k, if k divides a then k does not divide $(a+1)$.
b) the square root of 2 is irrational. Follow the approach in the book and make sur eyou comprehend it!

Problem 2 To prove a biconditional statement, prove both directions. That is, to prove $p \leftrightarrow q$, you have to prove both cases: $p \rightarrow q$ and $q \rightarrow p$. For the following statement, prove the forward direction using a direct proof and the reverse direction using proof by contraposition.

For any integer n, n^2 is odd if and only if n is odd.

Problem 3 Prove or disprove the following statements involving the floor and ceiling functions.

a) For all real x, $\lfloor \frac{\lfloor \frac{x}{2} \rfloor}{2} \rfloor = \lfloor \frac{x}{4} \rfloor$.
b) For positive integers n and k, $\lceil \frac{n}{k} \rceil = \lfloor \frac{(n-1)}{k} \rfloor + 1$.

Problem 4
If the following statements are true, prove them. If not, disprove via counterexample.

a) If a divides b and a divides c, then a divides (b+c).
b) If a divides (b+c), then a divides b and a divides c.

Problem 5

The following definitions follow from class:

For integers a and b (not equal to zero), the largest integer d that divides both a and b is called the *greatest common divisor* of a and b, and we write this as $gcd(a, b) = d$. For example, $gcd(24, 36) = 12$.

For integers a and b (not equal to zero), the smallest positive integer c that is divisible by both a and b is called the *least common multiple* of a and b, and we write this as $lcm(a, b) = c$. For example, $lcm(24, 36) = 72$.

Once you get a feeling for the above definitions (try some problems from the book), prove or disprove the following statement:

For positive integers a and b, $a \cdot b = gcd(a, b) \cdot lcm(a, b)$.

(Hint: Consider the prime factorizations of a and b. Namely, $a = p_1^{a_1} p_2^{a_2} ... p_n^{a_n}$ and $b = p_1^{b_1} p_2^{b_2} ... p_n^{b_n}$. Think about how the prime factorizations of a and b relate to $gcd(a, b)$ and $lcm(a, b)$.)

Problem 6

Assume you have a 16 by 16 checkerboard and 128 dominos, where each domino covers two squares perfectly. Convince yourself that you can cover the whole board with the 128 dominos (we will soon show you can do this by induction, but just convince yourself for now that it's true). Now, if we remove the top left and bottom right corners, can we still cover the whole board with 127 dominos?

BONUS Problems

I I arrange the first ten digits as 8, 5, 4, 9, 1, 7, 6, 3, 2, 0. What is the pattern?

II A woman walks into a deli and buys two cans of soda: root beer and gingerale. When she pays for the sodas, the clerk givers her change and a receipt. She then proceeds to draw a small triangle on the bottom of the receipt as well as the following equation: 3*13 = 39. The clerk notices al l of this, and says to the woman, "I notice that you're a firefighter." How did the clerk know this?

III

Mike sells Bennett his bicycle for 100 dollars. Bennett then decides to sell it back to Mike a few days later for 80 dollars. The following day, Kavitha buys Mike's bike for 90 dollars. What is Mike's total profit? Make sure you argue why.