Homework 2 - Discrete Math - Due 6/23/14

Assigned: 06/23/2014
Due: 07/02/2014

Please make sure to clearly write your name at the top of your hand-in. Also, indicate if you worked with anybody and also indicate how many hours total you worked on the homework. This looks like more homework than it is since many problems are quite simple and others have solutions in the back. Feel free to discuss any problems (including the bonuses) on the class mailing list. I am also required to remind all students of the academic integrity policy at `http://www.cs.nyu.edu/web/Academic/Graduate/academic_integrity.html`. Any violations of this policy may result in failure of the course and being reported to the head of the department.

Problem 1
Determine the truth value of the following statements:

a) $\exists x(x^2 = 2)$
b) $\forall x(x^2 + 2 \geq 1)$
c) $\forall x(x^2 \neq x)$

Problem 2 For the following propositions, write them using quantifiers, then express the negation using quantifiers, and expressthe negation in English.

a) Some drivers do not obey the speed limit.
b) All Swedish movies are serious.
c) There is noone in the class who does not have a good attitude.
d) There is no dog that can read.

Problem 3 Determine if the following are logically equivalent and explain why (or why not).

a) $\exists x(P(x) \vee Q(x))$ and $\exists x P(x) \vee \exists x Q(x)$
b) $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$
c) $\forall x(P(x) \vee Q(x))$ and $\forall x P(x) \vee \forall x Q(x)$
d) $\exists x(P(x) \wedge Q(x))$ and $\exists x P(x) \wedge \exists x Q(x)$

Problem 4

Let $F(x, y)$ be the statement "x can fool y" where the domain D consists of all people on Earth. **First**, use quantifiers to express the following statements. **Second**, write the negation of the quantified statements using quantifiers. **Third**, express the negation of the statements below in English.

a) Everybody can fool Fred.
b) Evelyn can fool everybody.
c) Everyone can fool someone.
d) There exists noone who can fool everyone.
e) Everybody can be fooled by somebody.
f) Someone can fool Fred or Jerry but nobody can fool both Fred and Jerry.
g) Nancy can fool exactly two people.
h) There is exactly one person who can be fooled by everybody.
i) Nobody can fool themselves.
j) There is someone who can fool exactly one person and that person can fool exactly one other different person.

Problem 5

Determine the truth value of the following statements if the domain is the set of real numbers **R**

a) $\forall x \exists y (x^2 = y)$
b) $\forall x \exists y (x = y^2)$
c) $\exists x \forall y (xy = 0)$
d) $\exists x \exists y (x + y \neq y + x)$
e) $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$
f) $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$
g) $\forall x \exists y (x + y = 1)$
h) $\exists x \exists y ((x + 2y = 2) \wedge (2x + 4y = 5))$
i) $\forall x \forall y \exists z (z = \frac{x+y}{2})$

Problem 6 Are the following statemenst true? If so, state generally why (do not try to prove it). If not, prove why they are false via counterexample. Remember, that to show something is false for a "for all" type statament, you can take the negation of the statement to get a "there exists" statement and find a counterexample.

a) $\forall x \exists y (x = \frac{1}{y})$
b) $\forall x \forall y (x^2 \neq y^3)$
c) $\forall x \forall y ((x^2 = y^2) \rightarrow (x = y))$
d) $\forall x \exists y ((y^2 - x) < 100)$

Problem 8 If we were trying to show that $\forall x(P(x) \vee Q(x))$ is equivalent to $\forall xP(X)\forall xQ(X)$, is the following proof valid (Hint: No since they are not equivalent). Find the error or errors

1. $\forall x(P(x) \vee Q(x))$ (Premise)
2. $P(c) \vee Q(c)$ for some value c (Universal Instantiation (1))
3. $P(c)$ (Simplification from (2))
4. $\forall xP(x)$ (Universal Generalization from (3))
5. $Q(c)$ (Simplification from (2))
6. $\forall xQ(x)$ (Universal Generalization from (5))
7. $\forall x(P(x) \vee \forall xQ(x))$ (Conjunction from (4) and (6))

Problem 9 Given the following sets of premises, what can you you infer or conclude. What rules of inference did you use?

a) All men are mortal. Socrates is a man.
b) No man is an island. Manhattan is an island.
c) All insects have 6 legs. Dragonflies are insects. Spiders do not have 6 legs. Spiders eat cheeseburgers.

Problem 10 Prove the following statements via direct proof (using the construction methods we covered in class - you can use the sum of two even numbers being even as an example guide).

a) The negation of an odd integer is an odd integer.
b) The product of two odd integers is odd.

BONUS Problems Try working on these to the best of your ability

I Here's one that's somewhat unrelated, but it's good for your brain skills: Imagine you have two hourglasses which can tell you when 4 minutes have passed and when 7 minutes have passed. However, you want to know when 9 minutes have passed. Sitting there with your 7 and 4 minute hourglasses, how will you know when 9 minutes have passed? Draw pictures if you need help explaining how you do it.

II If I haven't already, I will show you guys how to solve the mouse/cat problem with 100 mice and two different colored hats. Now, assume there are n mice and k different colored hats. Can you generalize a solution for the minimum number of mice saved for arbitrary n and k?

III You want to move into a new apartment, but you don't get paid for a month, so you cannot put down a security deposit. Fortunately, you have a bar of gold that is 31 cm long (it is very thin, or your apartment is very expensive) and worth one month's rent, so you make a deal with the landlord. You will give her 1cm of your gold everyday as a deposit for each day. However, every time you cut the bar, it costs you 5 bucks, so you want to cut it as little as possible. Your landlady suggests that you give her 1cm on the first day, 1cm on the second day, and on the third day, you give her one 3cm piece, and she will give you back the two 1cm pieces. On the fourth day you won't have to do any cuts since you will have the two 1cm pieces. But, by the sixth day, you would have to cut again. Obviously, you want to minimize the number of cuts to save money, but you want to make sure you add 1cm to your landlord's pile everyday. Assuming she holds onto all of the gold pieces you give her (so you can trade back and forth), what is the smallest number of cuts you will have to make for the 31 days?

IV You have decided to start an Ant Circus (lucky you!), and you have trained 21 ants to do something quite magical. There are 21 circles drawn on a table numbered 1-21, and while an accordion plays, each ant dances in his or her circle. Further, there is a line with an arrow going out of each circle to another circle, and a line with arrow coming in from another circle. Each circle has exactly one arrow coming in and one going out (so that you can't have the same arrow going out and coming into a circle). So, there are 21 distinct arrowed lines. Now, every time you blow your magic whistle, an ant does a flip from his circle to the next circle pointed to by the arrow. As you can expect, this portion of the program is very exciting, and people throw money at you. Now, when all of the ants are back in their circles all at the same time, the music stops, and the ants stop dancing. The question is: how many times can you blow the whistle before they stop?

Hint: As with all problems of this nature, it's easier to think of a smaller problem first. How about 3 ants? 4 ants? 5 ants? and then try to extrapolate. Also, when I say they all have to be back at the same time, that means that if ant 1 returns to circle 1, but ant 2 is in circle 3 and ant 3 is in circle 2, the music keeps going until each ant is paired with their original circles. i.e., (1,1), (2,2), (3,3), ... (21,21).