Problem Set 11

Assigned: April 23
Due: April 30

Problem 1

(Siegel Ex. 11.39). Let $A[1 \ldots n, 1 \ldots n]$ be an $n \times n$ array of signed numbers. Find the rectangular subarray with the largest sum of elements. A trivial solution is just to consider all $\Theta(n^4)$ rectangular subarrays; your algorithm should be much faster.

For example, if A is the matrix

$$\begin{bmatrix} 1 & 6 & -8 & 3 \\ 3 & -10 & 4 & 5 \\ -2 & 2 & -1 & 6 \\ 1 & 1 & 7 & -1 \end{bmatrix}$$

the solution is

$$A[2-4, 3-4] = \begin{bmatrix} 4 & 5 \\ -1 & 6 \\ 7 & -1 \end{bmatrix}$$

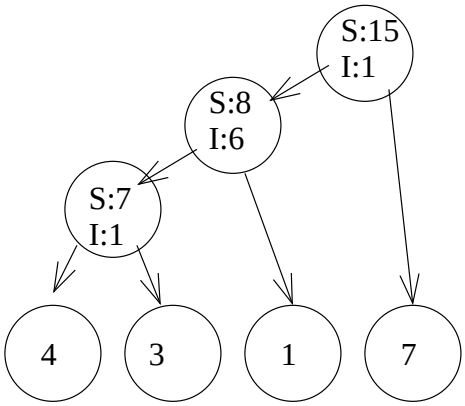
Problem 2

Let T be a binary tree whose leaves are labelled with numbers. For any internal node N of T, we define the *imbalance* of N to be the absolute value of the difference between the sum of values in the left subtree and the sum in the right tree. Define the *overall imbalance* of T to be the maximum imbalance of all internal nodes of T.

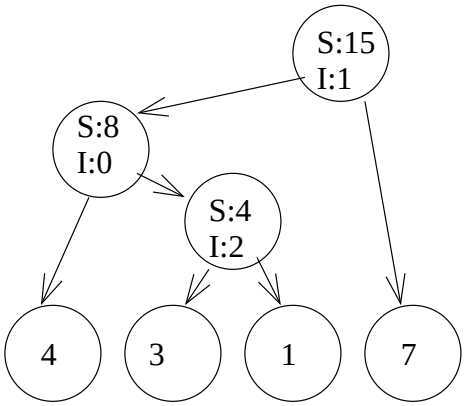
Given a sequence of numbers S, the *most balanced* tree for S is the tree whose leaves are S in the specified order with the smallest overall imbalance.

Write a dynamic programming algorithm that computes the most balanced tree for any sequence S. Hint: Use two arrays. $S[i, j]$ is the sum of elements i through j. $B[i, j]$ is the overall imbalance of the best subtree spanning elements i through j.

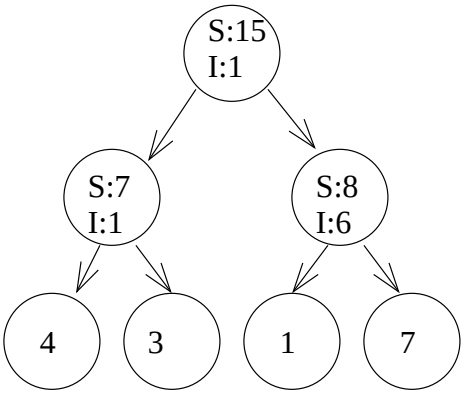
For instance, for the sequence $S = 4, 3, 1, 7$ there are five possible trees. The picture below shows the tree, with each node labelled with its sum S and its imbalance I. The most balanced tree is T2, with an overall imbalance of 2.



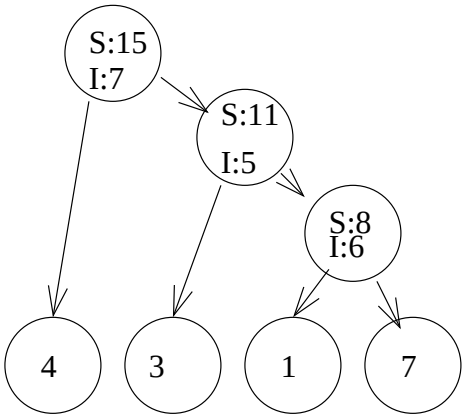
T1. Overall Imbalance:6



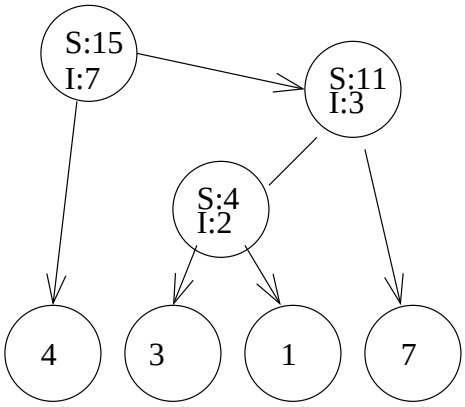
T2. Overall Imbalance:2



T3:. Overall Imbalance:6



T4. Overall Imbalance:7



T5. Overall Imbalance:7