Problem Set 7

Assigned: July 6
Due: July 13

Problem 1

(This was problem 4.A of HWK6 which I cancelled last week, but I'll explain in class how to do this kind of problem.)

Consider the following problem. You are given a directed graph G with two disjoint subsets A and B. A path is considered invalid if it goes first through a vertex in A and then through a vertex in B. For example A and B may be points in enemy countries, and B may prohibit travellers whose passport has a visa to A. Or in an epidemic of a communicable disease, one may want to block people who have been through A from entering B.

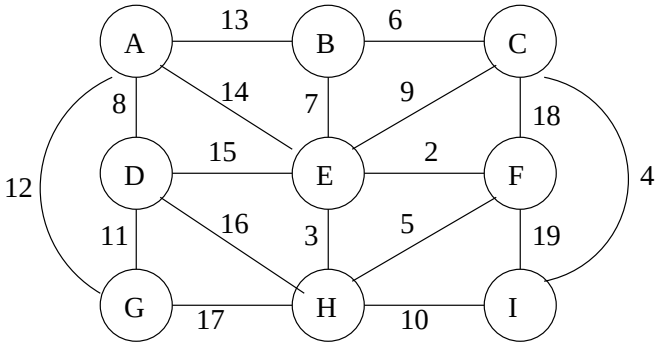
Modify Dijkstra's algorithm so that it returns the optimal valid paths.

Problem 2

A. Use Prim's algorithm to find the minimum spanning tree in the graph below.

B. Use Kruskal's algorithm for the same problem.

In both parts of the problem, all you need to show in your answer is the order in which the algorithm adds edges to the tree.



Problem 3

Let G be an undirected graph and let X be a subset of the vertices of G. A *connecting* tree on X is a tree composed out of the edges of G that contains all the vertices in X. One way to compute a connecting tree consists of two steps: (1) Compute a minimum spanning tree T over G. (2) Delete all the edges out of G not needed to connect vertices in X.

A. Give an algorithm to carry out step 2 above above in time $\theta(N)$ where N is the number of vertices in G.

B. The *Steiner tree* for X is the minimum cost connecting tree. Give an example to show that the above algorithm does not always return the Steiner tree.

Problem 4

Suppose we are given a collection of equations of the form variable = variable + constant, such as

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x = y+2
a = b+4
c = f-1
a = f+3
c = a-2
...
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We wish to check whether these equations are consistent. Show how the tree implementation of MFSETs can be modified so that this check can be carried out in time $\Theta(n\alpha(n))$. (You will need to augment the tree with certain extra information. Be sure to explain how this augmentation is modified when merging and path compression are performed.)