Problem Set 6

Assigned: June 29
Due: July 6

Problem 1

Construct an algorithm to solve the following problem: Given a graph $G = \langle V, E \rangle$, and a subset of vertices, $K \subset V$ compute, for every vertex $v \in V - K$, the shortest path from the closest vertex in K to V. For instance, each vertex in K might be a fire station, and each vertex in $V - K$ a potential address of a fire. You want to compute, in advance, for every address v, what fire station is nearest to v and what is the shortest path that a fire truck should take from that station to v. The running time of your algorithm should be $O(E \cdot \lg(V))$

Hint: Modify the graph by adding an additional source vertex that has an arc to every vertex in K.

Problem 2

Someone has proposed the following algorithm to solve the single-source shortest path problem for graphs with negative cost arcs but no negative weight cycles.

```
function SingleSourceShortestPath(G:directed graph; s: source vertex)
{ M = the minimum cost of any edge in G;
  if (M < 0) add −M to every edge in G.
  since G now has no negative cost edges,
        apply Dijkstra's algorithm to solve the shortest path problem from s.
}
```

Obviously, the result here does not give the correct costs of the shortest paths, but give an example to show that it does not even find the shortest paths.

Problem 3

Someone has proposed the following algorithm to solve the all-pairs shortest path problem in a directed graph.

```
function AllPairsShortestPath(G:directed graph);
{ for (each vertex v in G)
      use Dijkstra's algorithm to solve the single-source shortest path problem from v.
}
```

Under what circumstances if any does this work better than the Floyd-Warshall algorithm?

Problem 4

Consider the following problem. You are given a directed graph G with two disjoint subsets A and B. A path is considered invalid if it goes first through a vertex in A and then through a vertex in B. For example A and B may be points in enemy countries, and B may prohibit travellers whose passport has a visa to A. Or in an epidemic of a communicable disease, one may want to block people who have been through A from entering B.

A. Modify Dijkstra's algorithm so that it returns the optimal valid paths.

B. Modify the Floyd-Warshall algorithm so that it returns the optimal valid paths.