Problem Set 4

Assigned: June 15
Due: June 22

Problem 1

Describe a data structure for a set of integers of arbitrary size that supports the operations listed below with the given running times. You may assume that you are given an upper bound on the possible size of the set $|S|$. Give a brief description of what is involved in each operation. You may assume the data structures that have been described in class. (Thus, for instance, if you use a modified heap, you may say "Use the insert operations on heaps," without detailing what that involves.)

ADD(x,S) – Worst case time $O(\lg(\mid S \mid))$
DELETE(x,S) – Worst time $O(\lg(\mid S \mid))$
MEMBER(x,S) – Avg. time $O(1)$. Worst case time $O(\lg(\mid S \mid))$
MIN(S) – Worst case time $O(1)$.
MAX(S) – Worst case time $O(1)$.
NEXT(x,S) – Given an element x of S, find the next larger element in S. Avg. time $O(1)$.
  Worst case $O(\lg(\mid S \mid))$
PRED(x,S) – Given an element x of S, find the next smaller element in S. Avg. time $O(1)$.
  Worst case $O(\lg(\mid S \mid))$.
COUNT(x,y,S) – Returns the number of elements in S between x and y (inclusive).
  Worst case time $O(\lg(\mid S \mid))$.

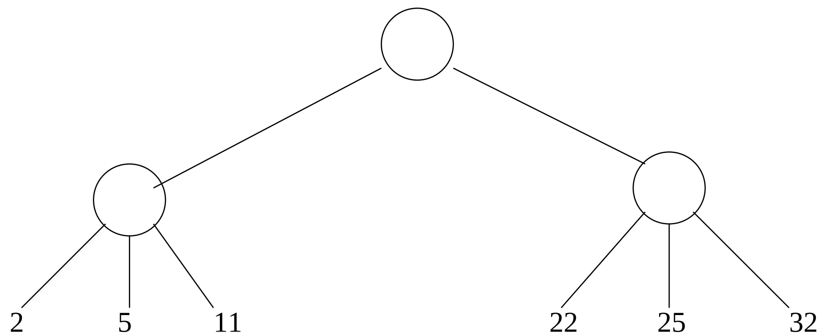
Problem 2

Suppose that we use a hash table of size m=13 and we insert the keys 10, 24, 20, 31, 49, 45, 33, 28. For convenience, use zero-based indexing (i.e. the indexes go from 0 to 12). Show the final state of the hash table, assuming that we use:

A. Chaining, with the hash function $h(k) = k \bmod 13$.

B. Linear probing, with with $h(k,i) = (k+i) \bmod 13$.

C. Double hashing, with the hash function $h(k,i) = (k + i \cdot (1 + k \bmod 11)) \bmod 13$.



Problem 3

Show what happens to the 2-3 tree above when the following operations are performed in sequence:

ADD(7), ADD(23), DEL(2), DEL(23).

Show the state of the tree after each operation. You need not show the transient states of the tree during the middle of the operations.