Programming Languages

G22.2110-001

Assignment # 3: λ-calculus

Due date: June 22th

1. Using the definitions of `PLUS`, `ISZERO`, and `SUCC` from the lecture, determine the normal form of the following. Show each intermediate step (i.e., each β-reduction).

   (a) `PLUS` $\ulcorner 3 \urcorner \ulcorner 2 \urcorner$

   (b) `ISZERO` $\ulcorner 2 \urcorner$

   (c) `SUCC` $\ulcorner 3 \urcorner$

2. Show that application is not associative for closed terms. That is,

   $$M_1(M_2M_3) \neq (M_1M_2)M_3$$

   when M_1, M_2, and M_3 are closed terms. (A closed term is one which has no free variables.)

   Note: If A and B are two λ-calculus expressions, for purposes of these exercises, we say that $A = B$ if we can use β- and/or α-reductions to reduce A and B to the same expression.

   For this exercise, it is enough to show closed terms M_1, M_2, and M_3 for which the equality is not true.

3. Show that $(\lambda y \,.\, (\lambda x \,.\, M))\, N = \lambda x \,.\, ((\lambda y \,.\, M)\, N)$, for any N and M.

4. Draw syntax trees for the following λ-terms:

   (a) $\lambda x\, y \,.\, y\, (x\, y)$

   (b) $\lambda x\, y\, z \,.\, x\, y\, z$

   (c) $(\lambda x \,.\, x\, x)(\lambda x \,.\, x\, x)$

   Use the BNF syntax for lambda terms given on slide 3, after expanding out any shorthands in the terms above.

5. Using the definitions in the lecture slides, which of the following terms are in normal form? If a term is not in normal form, identify a redex.

   (a) `IF`

   (b) `AND`

   (c) `OR TRUE FALSE`

   (d) $\ulcorner 2 \urcorner$

   (e) `PLUS`

   (f) `PRED` $\ulcorner 0 \urcorner$

   (g) `FIX`

6. Suppose a symbol of the λ-calculus alphabet is 0.5 cm wide. Write down a λ-term less than 20 cm wide having a normal form with length at least $10^{10^{10}}$ lightyears. The speed of light is approximately $3 * 10^{10}$ cm/sec. (Please do not attempt to reduce this term to normal form on paper.)