Exercise 6.8

Exercise 7.2

Exercise 7.4

Exercise 11.1

Rewrite Program 5 on p. 72 in MATLAB, saving the values of \( h \) and the computed values of \( \text{error} \) in vectors, say \( \text{h_vec} \) and \( \text{error_vec} \), of length 20, and then generate a plot like Fig. 11.1 (it might not be exactly the same) using the command \( \text{loglog(h_vec, error_vec, 'o')} \). You don’t have to declare the variables as “double” because this is the default in MATLAB. Then modify the program to use the central difference quotient (see p. 74) instead of the difference quotient used in Program 5. Do you observe that the discretization errors for larger values of \( h \) now decrease as \( O(h^2) \) instead of \( O(h) \)? Explain how you can see this in the plot. But as before, if \( h \) is too small, rounding error dominates. What is the best value of \( h \) now, approximately? Is it bigger than previously or is it smaller? Explain why this is the case. Print both plots along with your MATLAB code and submit them with your answers.

Exercise 12.1

Exercise 13.3

Exercise 13.4

- Let \( f(x) = \exp(x) - 1 \). What is the condition number of \( f \) at \( x \)? In particular, what is it for \( x = 10^{-6} \)?
- Based on this condition number, what does Rule of Thumb 12.1 tell us about how many digits should be correct when we compute \( f(x) \) for \( x = 10^{-6} \) using a stable algorithm in double precision?
There is a special function available to compute this function: \texttt{expm1}. What answer does it give for $x = 10^{-6}$ (using the default double precision)? To how many digits is this answer correct? To answer this, compare it with what you get \textit{by hand} using as much as you need of the Taylor series expansion on p. 89. \textbf{Use “format long” or “format long e” so that you see all 16 digits after the decimal point (17 total, but since $2^{-53} \approx 10^{-16}$, the last one may not be meaningful).}

A more obvious algorithm to compute $f$ is:

$$y = \exp(x); \quad f = y - 1$$

Run this in \texttt{matlab} for $x = 10^{-6}$. How many digits are correct? \textbf{Explain why this happens.} It will be helpful to print the value of $y$. \textbf{Again, print all double values to 16 digits.}

What can we conclude about the stability of this obvious algorithm?

\begin{itemize}
  \item (10 pts) Ex 13.12. \textbf{(You can use either C or \texttt{matlab}, but you have to figure out how to modify the program so no subtraction takes place.)} Run the program for $x = 10^{-j}, \; j = 1, \ldots, 10$ and compare the results with the ones you get from \texttt{expm1}: are they just as good? Why or why not? \textbf{If you use \texttt{matlab}, the numbers are doubles by default, and you should display them in the long format.} If you modify the C program directly, the numbers are floats, and then it makes sense to display only 7 digits. If you change them to doubles, display them to 16 digits, e.g., using \texttt{“%25.16e”} instead of \texttt{“%e”} in the output format.
\end{itemize}

Please remember: It is important that you do the homework yourself (not jointly with another student), but when you get stuck, I encourage you to consult with other students, or me, to get help when necessary. \textbf{However, when you get help, it’s important to acknowledge it in writing in your homework submission.} Passing off other people’s work as your own is called plagiarism and is not acceptable.

Penalty for not reporting your sources : grade of zero for the homework. Penalty for late homework: 20%. Homework will \textbf{not} be accepted more than one week late.
If you have any questions about homework, please post them to Piazza, either as a private note to me or as a public post to the class. But don’t post answers to the homework on Piazza! Helpful hints are OK!