Exercises: Continuous Random Variables

Exercise 1 – We pick up randomly a number in interval [2, 4] and another one, independently, in interval [3, 6]. Each time, any number in the interval has the same likelihood to appear.

1.1 – Give the expected value and the variance of the sum of these two numbers.
1.2 – Give the PDF of the sum of the two numbers.

Exercise 2 – A firm constructs metal balls having a theoretic diameter of 8mm, but the fabrication process yields some errors. The clients accept balls having diameter between 7.97mm and 8.03mm. The firm wants to estimate the ratio of balls that will be accepted. For this, the firm decides to construct 100 balls, and to measure on this sample the mean $\mu$ and the standard deviation $\sigma$ of the diameters. The obtained values are $\mu = 8mm$, $\sigma = 0.02$. Engineers decide that the diameter $X$ of a ball follows a Gaussian law of mean $\mu = 8mm$ and standard deviation $\sigma = 0.02$.

2.1 – Give an expression of $P(X \leq 7.97)$.
2.2 – Give an expression of $P(X \geq 8.03)$.
2.3 – Give the ratio of accepted balls.

Hint: use the function erf of Matlab to evaluate the cumulative distribution function $P(X \leq t)$ for a given $t$.

Exercise 3 – A firm constructs steel sheets meant to be stacked.

3.1 – The width of a constructed sheet follows a normal (Gaussian) law of mean $\mu = 0.3mm$ and standard deviation $\sigma = 0.1mm$. Give the probability that the width of a sheet is less than 0.36mm, and the probability that it is between 0.25 and 0.35 mm.

3.2 – One chooses randomly $n$ sheets, index them from 1 to $n$ and stack them. Let $Z$ be the width of the obtained stack. Give the law of $Z$, its mean and its standard deviation.