

# Bottom-Up Syntax Analysis

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Compiler Construction  
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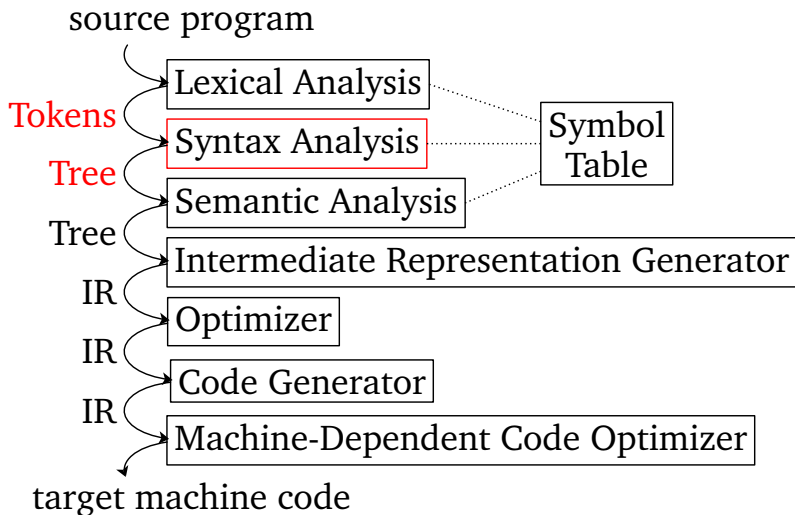


# Outline

- 1 Parsers (recap)
- 2 LR(0)
- 3 LR Parsing
- 4 LR(1) and LALR(1)



## Second compilation phase



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# A Left Recursive Grammar

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow ( E ) \mid \mathbf{id} \end{aligned} \tag{4.1}$$

## Bottom-Up Construction



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### Bottom-Up Construction



## Parser Categories

**L** Left (to right)

**R** (use) Rightmost derivation

**k** look-ahead count

LR(0) Easy parsers—include SLR, LALR.

LR(1) More powerful, less practical:

- ▶ most general non-backtracking method known
- ▶ detects errors early\*
- ▶ handles grammar superset of predictive parsers
- ▶ hard to hand-code
- ▶ hard to debug
- ▶ \*error recovery problematic



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# $LR(k)$

$LR(k)$  automaton:

- ▶ 2 basic moves: **shift** token, **reduce** production
- ▶ 2 other moves: **accept**, **error**

Concepts:

- ▶ An  $LR(0)$  item is a rule with a dot at some position:  
 $A \rightarrow .BCD, A \rightarrow B.CD, \dots$
- ▶ An  $LR(0)$  state is a set of items.
- ▶ Canonical  $LR(0)$  collection: FA used to make parsing decisions



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## Closure Construction

For Grammar  $G$  need—

- ▶ Augmented grammar  $G'$ 
  - ▶ Just add rules  $S' \rightarrow S$
- ▶ closure function for set of items  $I$ ,  $\text{closure}(I)$ :
  - 1 add  $I$  to  $\text{closure}(I)$ .
  - 2 if  $A \rightarrow \alpha.B\beta$  in  $\text{closure}(I)$  then add  $B \rightarrow \cdot\gamma$  (for each possible  $\gamma$ )
- ▶ GOTO function



# Example

- 1  $E' \rightarrow E$
- 2  $E \rightarrow E + T$
- 3  $E \rightarrow T$
- 4  $T \rightarrow T * F$
- 5  $T \rightarrow F$
- 6  $F \rightarrow (E)$
- 7  $F \rightarrow \mathbf{id}$



## Construction (I)

$$I = \{[E' \rightarrow .E]\}$$

$$I_0 = \text{closure}(I)$$

$$= \{[E' \rightarrow .E], [E \rightarrow .E+T], [E \rightarrow .T], [T \rightarrow .T*F], [T \rightarrow .F], \\ [F \rightarrow .(E)], [F \rightarrow .\mathbf{id}]\}$$





## Construction (II)

For items  $I$ ,  $\forall x$  in a set of symbols following dot.

$$\text{GOTO}(I, x) = \text{closure}(\{[A \rightarrow \alpha x \beta] \mid [A \rightarrow \alpha \cdot s \beta] \in I\})$$

gives

$$I_1 = \text{GOTO}(I_0, E) = \{[E' \rightarrow E.], [E' \rightarrow E. + T]\}$$

$$I_2 = \text{GOTO}(I_0, T) = \{[E \rightarrow T.], [T \rightarrow T. * F]\}$$

$$I_3 = \text{GOTO}(I_0, F) = \{[T \rightarrow F.]\}$$

$$I_4 = \text{GOTO}(I_0, () = \{[F \rightarrow (.E)], [E \rightarrow .E + T], [E \rightarrow .T], \\ [T \rightarrow .T * F], [T \rightarrow .F], [F \rightarrow .(E)], [F \rightarrow \mathbf{id}]\}$$

$$I_5 = \text{GOTO}(I_0, \mathbf{id}) = \{[F \rightarrow \mathbf{id}]\}$$



## Algorithm

```
initialize C = { closure({[S' → .S]}) }
repeat for all I ∈ C, for all x,
  if GOTO(I,x) ≠ ∅ and GOTO(I,x) ∉ C
    add GOTO(I,x) to C
```

In practice find repeating items, *i.e.*,  $GOTO(I_4, T) == I_2$

- ▶ A **final** item is one with the  $.$  at the end.
- ▶ If all final items are in states by themselves then the grammar is  $LR(0)$ ,
- ▶ otherwise there is a **shift-reduce** or **reduce-reduce** conflict in  $LR(0)$ .



## SLR Trick

Use **Follow Sets** to decide when to reduce:  $SLR(1)$ .  
If they overlap we can look **2** symbols ahead, *etc.*



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# Automaton

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## Shift-Reduce

STACK	INPUT	ACTION
\$	<b>id<sub>1</sub></b> * <b>id<sub>2</sub></b> \$	shift
\$ <b>id<sub>1</sub></b>	* <b>id<sub>2</sub></b> \$	reduce by $F \rightarrow \mathbf{id}$
\$ $F$	* <b>id<sub>2</sub></b> \$	reduce by $T \rightarrow F$
\$ $T$	* <b>id<sub>2</sub></b> \$	shift
\$ $T$ *	<b>id<sub>2</sub></b> \$	shift
\$ $T$ * <b>id<sub>2</sub></b>	\$	reduce by $F \rightarrow \mathbf{id}$
\$ $T$ * $F$	\$	reduce by $T \rightarrow T * F$
\$ $T$	\$	reduce by $E \rightarrow T$
\$ $E$	\$	accept



# LR components

- ▶ Input “pointer.”
- ▶ Driver.
- ▶ Action/GOTO table.
- ▶ Stack.



# Configuration

**Stack**  $s_0 \dots s_m$  (both productions and tokens)

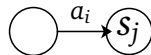
**Input**  $a_1 \dots a_n \$$





# Moves...

**shift  $j$**  consume symbol  $a_i$ , push  $s_j$ .

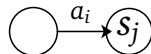


reduce  $A \rightarrow \beta$  pop  $|\beta|$  states from stack  
push  $\text{GOTO}(s_{m-|\beta|}, A)$  on stack.  
With  $A \rightarrow .BCD$ :

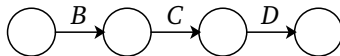


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# Example

Example.



## Remark

Do you see the bottom-up?



## Universal LR Driver

```
push S0 onto empty stack
read first input token
repeat:
  let s = state on top of stack
  if action(s,a) = shift t:
    push t onto stack
    a := next symbol
  else if action(s,a) = reduce  $A \rightarrow \beta$ 
    pop  $|\beta|$  states off stack
    let t be new top of stack
    push GOTO(t,A) onto stack
    synthesize attributes for  $A \rightarrow \beta$ 
  else if action(s,a) = accept
    break
  else
    call error recovery
```



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## LR(1) Items

- ▶ Extend LR(0) items with **look-ahead**:  $[A \rightarrow \alpha.\beta, a]$
- ▶ Start with  $[S' \rightarrow .S, \$]$
- ▶ Example:  $[E' \rightarrow .E, \$]$   
closure:  
 $[E \rightarrow .E+T, \$, +], [E \rightarrow .T, \$, +], [T \rightarrow .T*F, \$, +, *], \dots$



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## Algorithm (with Lookahead)

closure(I):

```
repeat for all  $[A \rightarrow \alpha.B\beta, a] \in I$ ,  
    for all  $B \rightarrow \gamma$  in  $G'$ ,  
    for all  $b$  in  $\text{FIRST}(\beta a)$   
    add  $[B \rightarrow \cdot\gamma, b]$  to I  
until no more items can be added
```

goto(I, x):

```
J =  $\emptyset$   
for all  $[A \rightarrow \alpha.x\beta, a] \in I$   
    add  $[A \rightarrow \alpha x.\beta, a]$  to J  
return closure(J)
```



## Remarks

- ▶ Unlike LR(0)  $\rightarrow$  SLR step no need to deal with ambiguous states of LR(1)
- ▶ For LALR(1), systematically *merge* certain LR(1) states  $\Rightarrow$  compact tables
- ▶ To build LALR from LR(0) consider *path that led to it*  $\Rightarrow$  efficient table construction



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## Summary

- ▶ LR( $k$ ) parsers are also called **bottom-up** parsers in contrast to LL( $k$ ) **top-down** parsers.
- ▶ LR(0) is efficient, LR( $k > 0$ ) is not.
- ▶ SLR and LALR improve LR(0) in important ways but remain efficient.
- ▶ SLR( $k$ ) and LALR( $k$ ) are both less than LR( $k$ ).
- ▶ The main source of conflicts are **shift-reduce** and **reduce-reduce** conflicts; resolving in nonstandard ways allows for (some) ambiguous grammars



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