1. **Missing claim from the GapCVPP algorithm.** Recall that we defined the function $f : \mathbb{R}^n \to \mathbb{R}$ by $f(x) := \rho(x+\Lambda)/\rho(\Lambda)$.
   (a) Show that f is Λ-periodic.
   (b) Show that for all $x \in \mathbb{R}^n$, $f(x) \geq \rho(\|x\|)$. (Hint: cosh)
   (c) Deduce that for any $c > 0$ there exists a $c' > 0$ such that if $\mathrm{dist}(x, \Lambda) \leq c\sqrt{\log n}$ then $f(x) \geq n^{-c'}$.

2. **Missing claim from the** $\mathsf{GapCVP}_{\sqrt{n}} \in \mathsf{coNP}$ **proof.**
   (a) Show that for all $f : \mathbb{R} \to \mathbb{R}$ (in $L^1(\mathbb{R})$), if $g = f'$ (and is also in $L^1(\mathbb{R})$) then $\hat{g}(y) = 2\pi i y \hat{f}(y)$.
   (b) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2\rho(x)$. Compute the Fourier transform of f.
   (c) Show that
   $$\mathrm{Exp}_x[x^2] \leq \frac{1}{2\pi},$$
   where x is chosen from the discrete Gaussian distribution D_Λ for a one-dimensional lattice Λ (i.e., the distribution on Λ that assigns mass proportional to $\rho(x)$ to any $x \in \Lambda$). Notice that there is an equality when x is chosen from the continuous Gaussian.
   (d) Extend this to the n-dimensional case by showing that for any n-dimensional lattice Λ and any unit vector $u \in \mathbb{R}^n$,
   $$\mathrm{Exp}_x[\langle x, u\rangle^2] \leq \frac{1}{2\pi},$$
   where x is chosen from the discrete Gaussian distribution D_Λ. (Suggestion: show that wlog we can take $u = e_1$ in which case $\langle x, u\rangle = x_1$.)

3. Show that for any $\varepsilon < \frac{1}{100}$ we have $\eta_\varepsilon(\Lambda) \geq (\lambda_1(\Lambda^*))^{-1} \geq \frac{1}{n}\lambda_n(\Lambda)$.

4. The *discrete Gaussian distribution* on Λ with parameter $s \geq 0$ and center $c \in \mathbb{R}^n$, denoted $D_{\Lambda,s,c}$, is defined as the probability distribution with support Λ that assigns to each $x \in \Lambda$ the probability $\rho_s(x-c)/\rho_s(\Lambda - c)$. Show that for $s \geq \sqrt{2}\eta_\varepsilon(\Lambda)$ where $\varepsilon \leq 1/100$, any $c \in \mathbb{R}^n$, and any $(n-1)$-dimensional hyperplane H,
   $$\Pr_{x \sim D_{\Lambda,s,c}}[x \in H] < 0.9.$$
   Hint: Notice that without loss of generality, we can assume that $H = \{x \in \mathbb{R}^n \mid x_1 = r\}$ for some $r \geq 0$. Then observe that it is enough to show that $\mathrm{Exp}_{x \sim D_{\Lambda,s,c}}[e^{-\pi(\frac{x_1 - r}{s})^2}] < 0.9$. Prove this using the Poisson summation formula.

5. Consider the following algorithm for sampling from $D_{\Lambda,s,c}$. Assume we have a good basis B of Λ. The algorithm samples a point from the *continuous* Gaussian distribution $\rho_s(x-c)/s^n$, rounds it to a nearby lattice point (say, using Babai's nearest plane algorithm), and outputs the result. Show that the output of this algorithm is statistically quite far from $D_{\Lambda,s,c}$, even for radii s that are polynomially bigger than the length of the given basis. Hint: Take the lattice $\mathbb{Z}$ and $c = 0$, and let s be, say, n^{10}. Show that the probability of outputting 0 is noticeably far from what it should be.