

1. Construct an explicit basis for the lattice $\{\mathbf{x} \in \mathbb{Z}^n : \mathbf{x}_1 + \sum_{i=2}^n a_i \mathbf{x}_i \equiv 0 \pmod{p}\}$, where $a_i \in \mathbb{Z}_p$, p a prime.
2. Exercise 1 in Lecture 1.
3. Take $a_1, \dots, a_n \in \mathbb{N}$. The greatest common divisor of a_1, \dots, a_n , denoted $\gcd(a_1, \dots, a_n)$, is the largest integer d such that $d|a_i$ (meaning d divides a_i), for all $i \in [n]$. Show that there exists $z_1, \dots, z_n \in \mathbb{Z}$ such that $\gcd(a_1, \dots, a_n) = \sum_{i=1}^n z_i a_i$. Give a simple algorithm to compute $\gcd(a_1, \dots, a_n)$ (no need to analyze its complexity).